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SHORT COMMUNICATION

Lanczos potential for the Weyl tensor

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ABSTRACT

For arbitrary spacetimes with Petrov types O, N and III, we indicate general results about the Lanczos potential for the corresponding Weyl tensor.

***Keywords*:** Conformal tensor, Lanczos generator, Newman-Penrose formalism, Petrov classification, Weyl-Lanczos equations, 2-spinors, Spin coefficients

1. INTRODUCTION

We shall employ the notation and quantities explained in [1-6]. The Lanczos potential K_{abc} [7-12] is a generator for the Weyl tensor in four dimensions; in [9] was used the Newman-Penrose (NP) formalism [4, 6, 13-17] to determine the Lanczos spintensor for any spacetime of Petrov types [12-15, 18-21] N, O, and III, thus:

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$$S_{abc} = K_{abc} + i\ {}^*K_{abc} = \frac{2q}{3}[V_{ab}(-3\nu\, l_c - \pi\, n_c + 3\lambda\, m_c + \mu\, \bar{m}_c) + \qquad (1)$$

$$+ U_{ab}(\tau\, l_c + 3\kappa\, n_c - \rho\, m_c - 3\sigma\, \bar{m}_c) + M_{ab}(-\mu\, l_c + \rho\, n_c + \pi\, m_c - \tau\, \bar{m}_c)],$$

with $q = \frac{1}{2}$ and 1 for the types O, N, and III, respectively; besides [22]:

$$V_{ab} = l_a\ x\ m_b\,, \qquad U_{ab} = \bar{m}_a\ x\ n_b\,, \quad M_{ab} = m_a\ x\ \bar{m}_b + n_a\ x\ l_b\,, \qquad (2)$$

for the corresponding canonical null tetrad [15, 21]:

$$l^a \ \leftrightarrow\ o^A\, o^{\dot{B}}\,, \quad n^a\ \leftrightarrow\ \iota^A\, \iota^{\dot{B}}\,, \quad m^a\ \leftrightarrow\ o^A\, \iota^{\dot{B}}\,, \quad \bar{m}^a\ \leftrightarrow\ \iota^A\, o^{\dot{B}}\,, \quad o_A\, \iota^A = 1. \qquad (3)$$

In Sec. 2 we obtain the Lanczos spinor $L_{ABC\dot{D}}$ [2, 23-30] associated to the tensorial result (1):

$$L_{ABC\dot{D}} = \frac{1}{4}\sigma^a{}_{A\dot{E}}\, \sigma^b{}_B{}^{\dot{E}}\, \sigma^c{}_{C\dot{D}}\, S_{abc}\,, \qquad (4)$$

where $\sigma^r{}_{F\dot{G}}$ are the Infeld-van der Waerden symbols [19, 29-31], in accordance with [28].

We note that a better understanding of the Lanczos potential permits to know more about the Liénard-Wiechert field, for example, to obtain the physical meaning of the Weert generator [32-34] and to construct [35] a Petrov classification [12-15, 18-21] for the electromagnetic field produced by a point charge in arbitrary motion.

The Lanczos spintensor is known for arbitrary types O, N and III 4-spaces [9], Kerr geometry [36-41], Gödel cosmological model [42-44], plane gravitational waves [45], and several spacetimes [46-50] of interest in general relativity. The deduction of $K_{\mu\nu\alpha}$ for arbitrary types I, II and D is an open problem. Lanczos [7] determined his potential for weak gravitational fields, and in the corresponding calculations showed up the Dirac equation for spin-1/2 without the mass term, hence he had hoped that K_{abc} may be important in a future quantum gravity theory.

In Sec. 3, for arbitrary metrics of Petrov types III, N, O, and D (empty), we determine the Andersson-Edgar's generator [51, 52] for the Lanczos spinor.

2. LANCZOS SPINOR

From (2) and (3) are immediate the relations:

$$o_A\, o_B = \frac{1}{2}\sigma^a{}_{A\dot{E}}\, \sigma^b{}_B{}^{\dot{E}}\, V_{ab}\,, \iota_A\, \iota_B = \frac{1}{2}\sigma^a{}_{A\dot{E}}\, \sigma^b{}_B{}^{\dot{E}}\, U_{ab}\,, o_A\, \iota_B + o_B\, \iota_A = -\frac{1}{2}\sigma^a{}_{A\dot{E}}\, \sigma^b{}_B{}^{\dot{E}}\, M_{ab}\,, \quad (5)$$

then (1), (4) and (5) imply:

$$L_{ABC\dot{D}} = \frac{q}{3}[o_A\, o_B\big((\mu\, \iota_c - 3\nu\, o_c)\, o_{\dot{D}} + (3\lambda\, o_c - \pi\, \iota_c)\, \iota_{\dot{D}}\big) + \\ + \iota_A\, \iota_B\big((\tau\, o_C - 3\sigma\, \iota_C)\, o_{\dot{D}} + (3\kappa\, \iota_C - \rho\, o_C)\, \iota_{\dot{D}}\big) \\ + (o_A\, \iota_B + o_B\, \iota_A)\big((\mu\, o_C + \tau\, \iota_C)\, o_{\dot{D}} - (\pi\, o_C + \rho\, \iota_C)\, \iota_{\dot{D}}\big), \tag{6}$$

which can be written in compact form; in fact, it is simple to deduce the following expression [4]:

$$\nabla_{C\dot{D}}\, (o_A\, \iota_B) = o_A\, o_B\, [(\nu\, o_C - \mu\, \iota_C)\, o_{\dot{D}} + (\pi\, \iota_C - \lambda\, o_C)\, \iota_{\dot{D}}] + \\ + \iota_A\, \iota_B\, [(\sigma\, \iota_C - \tau\, o_C)\, o_{\dot{D}} + (\rho\, o_C - \kappa\, \iota_C)\, \iota_{\dot{D}}]\,, \tag{7}$$

therefore (6) acquires the structure [2]:

$$L_{ABC}{}^{\dot{D}} = -q\, \nabla_{(C}{}^{\dot{D}}\, o_A\, \iota_{B)}\,, \qquad q = \tfrac{1}{2}\,, 1\,. \tag{8}$$

The equation (8) gives the Lanczos potential in terms of the spin coefficients associated to the corresponding canonical null tetrad for the Petrov types O, N, and III; we can verify (8) via its connection with the Weyl tensor [3, 28, 29]:

$$\psi_{ABCE} \equiv \psi_0\, \iota_A\iota_B\iota_C\iota_E - 4\psi_1\, o_{(A}\iota_B\iota_C\iota_{E)} + 6\psi_2\, o_{(A}o_B\iota_C\iota_{E)} - 4\psi_3\, o_{(A}o_Bo_C\iota_{E)} + \psi_4\, o_Ao_Bo_Co_E\,, \tag{9}$$

$$= 2\, \nabla_{(E}{}^{\dot{D}}\, L_{ABC)\dot{D}}\,. \tag{10}$$

We know [30] the formula $\nabla_E{}^{\dot{D}}\, \nabla_{C\dot{D}} = \frac{1}{2}\varepsilon_{CE}\, \nabla - \nabla_{EC}$, hence:

$$-\nabla_E{}^{\dot{D}}\, \nabla_{(C|\dot{D}|}\, \big(o_A\, \iota_{B)}\big) = \nabla_{(EC}\big(o_A\, \iota_{B)}\big); \tag{11}$$

thus (8), (10) and (11) imply:

$$\psi_{ABCE} = 2q\, [o^F\, \psi_{F(ABC}\, \iota_{E)} + \iota^F\, \psi_{F(ABC}\, o_{E)}]\,, \\ = 2q\, \big[-\psi_0\, \iota_A\, \iota_B\, \iota_C\, \iota_E + 2\psi_1\, o_{(A}\iota_B\iota_C\iota_{E)} - 2\psi_3\, o_{(A}o_Bo_C\iota_{E)} + \psi_4\, o_Ao_Bo_Co_E\big], \tag{12}$$

whose comparison with (9) gives $q = \frac{1}{2}$ and 1 for the Petrov types O, N, and III, respectively, about the canonical tetrad [15, 21].

3. ANDERSSON-EDGAR'S POTENTIAL FOR THE LANCZOS SPINOR

In [46] was obtained the Lanczos generator for an arbitrary empty spacetime of Petrov type D, with the following Newman-Penrose (NP) components:

$$\Omega_2 = \pi\, \psi_2^{-\frac{2}{3}} , \qquad \Omega_6 = \mu\, \psi_2^{-\frac{2}{3}} , \qquad \Omega_r = 0,\ r \neq 2, 6, \tag{13}$$

in terms of the spin coefficients associated to the canonical null tetrad, hence for the type D the Lanczos spinor is given by [53]:

$$L_{ABC\dot{D}} = \psi_2^{-\frac{2}{3}} (o_A o_B \iota_C + (o_A * \iota_B) o_C)(-\mu\, o_{\dot{D}} + \pi\, \iota_{\dot{D}}). \tag{14}$$

Similarly [9]:

$$\begin{aligned} &\Omega_0 = q\,\kappa\,, \qquad \Omega_3 = -q\,\lambda\,, \qquad \Omega_4 = q\,\sigma\,, \qquad \Omega_7 = -q\,\nu\,, \\ &\Omega_1 = \frac{q}{3}\rho\,, \qquad \Omega_2 = -\frac{q}{3}\pi\,, \qquad \Omega_5 = \frac{q}{3}\tau\,, \qquad \Omega_6 = -\frac{q}{3}\mu\,, \end{aligned} \tag{15}$$

for the types N ($q = \frac{1}{2}$) and III ($q = 1$), in the corresponding canonical tetrad, with the Lanczos spinor:

$$\begin{aligned} L_{ABC\dot{D}} = {} & \iota_A\, \iota_B\, \iota_C (\Omega_0\, \iota_{\dot{D}} - \Omega_4\, o_{\dot{D}}) + (\iota_A\, \iota_B\, o_C + (o_A * \iota_B)\, \iota_C)\, (-\Omega_1\, \iota_{\dot{D}} + \Omega_5\, o_{\dot{D}}) + \\ & + (o_A\, o_B\, \iota_C + (o_A * \iota_B)\, o_C)\, (\Omega_2\, \iota_{\dot{D}} - \Omega_6\, o_{\dot{D}}) + o_A\, o_B\, o_C (-\Omega_3\, \iota_{\dot{D}} + \Omega_7\, o_{\dot{D}}); \end{aligned} \tag{16}$$

for the type O we may employ $q = \frac{1}{2}$ or $q = 1$.

On the other hand, Andersson-Edgar [51, 52] proved that any Lanczos spinor can be generated via the relation:

$$L_{ABC\dot{D}} = \nabla^{E}{}_{\dot{D}}\, T_{ABCE}\,, \qquad T_{ABCE} = T_{(ABC)E}\,, \tag{17}$$

then we shall construct T_{ABCE} for the cases (14) and (16). Thus, in (17) we use the expansion:

$$\begin{aligned} T_{ABCE} = {} & \iota_A\, \iota_B\, \iota_C (\Lambda_0\, \iota_E - \Lambda_4\, o_E) + (\iota_A\, \iota_B\, o_C + (o_A * \iota_B)\, \iota_C)\, (-\Lambda_1\, \iota_E + \Lambda_5\, o_E) + \\ & + (o_A\, o_B\, \iota_C + (o_A * \iota_B)\, o_C)\, (\Lambda_2\, \iota_E - \Lambda_6\, o_E) + o_A\, o_B\, o_C (-\Lambda_3\, \iota_E + \Lambda_7\, o_E), \end{aligned} \tag{18}$$

to obtain the set of NP equations:

$$\begin{aligned} \Omega_0 &= \bar{\delta}\Lambda_0 - D\Lambda_4 + (\pi - 4\alpha)\, \Lambda_0 + 3\rho\, \Lambda_1 + (2\varepsilon + \rho)\, \Lambda_4 - 3\kappa\, \Lambda_5\,, \\ \Omega_1 &= \bar{\delta}\Lambda_1 - D\Lambda_5 - \lambda\, \Lambda_0 + (\pi - 2\alpha)\, \Lambda_1 + 2\rho\, \Lambda_2 + \pi\, \Lambda_4 + \rho\, \Lambda_5 - 2\kappa\, \Lambda_6\,, \\ \Omega_2 &= \bar{\delta}\Lambda_2 - D\Lambda_6 - 2\lambda\, \Lambda_1 + \pi\, \Lambda_2 + \rho\, \Lambda_3 + 2\pi\, \Lambda_5 + (\rho - 2\varepsilon)\, \Lambda_6 - \kappa\, \Lambda_7\,, \\ \Omega_3 &= \bar{\delta}\Lambda_3 - D\Lambda_7 - 3\lambda\, \Lambda_2 + (2\alpha + \pi)\, \Lambda_3 + 3\pi\, \Lambda_6 + (\rho - 4\varepsilon)\, \Lambda_7\,, \end{aligned} \tag{19}$$

$$\Omega_4 = \Delta\Lambda_0 - \delta\Lambda_4 + (\mu - 4\gamma)\,\Lambda_0 + 3\tau\,\Lambda_1 + (2\beta + \tau)\,\Lambda_4 - 3\sigma\,\Lambda_5\,,$$

$$\Omega_5 = \Delta\Lambda_1 - \delta\Lambda_5 - \nu\,\Lambda_0 + (\mu - 2\gamma)\,\Lambda_1 + 2\tau\,\Lambda_2 + \mu\,\Lambda_4 + \tau\,\Lambda_5 - 2\sigma\,\Lambda_6\,,$$

$$\Omega_6 = \Delta\Lambda_2 - \delta\Lambda_6 - 2\nu\,\Lambda_1 + \mu\,\Lambda_2 + \tau\,\Lambda_3 + 2\mu\,\Lambda_5 + (\tau - 2\beta)\,\Lambda_6 - \sigma\,\Lambda_7\,,$$

$$\Omega_7 = \Delta\Lambda_3 - \delta\Lambda_7 - 3\nu\,\Lambda_2 + (2\gamma + \mu)\,\Lambda_3 + 3\mu\,\Lambda_6 + (\tau - 4\beta)\,\Lambda_7,$$

hence (19) implies (15) for $\Lambda_2 = -\Lambda_5 = \frac{q}{3}$, $\Lambda_r = 0$, $r \neq 2, 5$, that is:

$$T_{ABCE} = \frac{q}{3}\left[(o_A\,o_B\,\iota_C + (o_A * \iota_B)\,o_C)\,\iota_E - (\iota_A\,\iota_B\,o_C + (o_A * \iota_B)\,\iota_C)\,o_E\right], \qquad (20)$$

is a generator of the Lanczos spinor (16) for arbitrary spacetimes of Petrov types O, N, and III, in the canonical null tetrad.

Let's remember that for type D vacuum geometries [3, 15]:

$$\kappa = \sigma = \lambda = \nu = 0, \qquad \psi_2 \neq 0, \qquad \psi_r = 0, \quad r \neq 2,$$
$$D\psi_2 = 3\rho\,\psi_2\,, \qquad \Delta\psi_2 = -3\mu\,\psi_2\,, \qquad \delta\psi_2 = 3\tau\,\psi_2\,, \qquad \bar{\delta}\psi_2 = -3\pi\,\psi_2\,, \qquad (21)$$

then (13) is consequence from (19) for the values $\Lambda_2 = -\frac{3}{2}\Lambda_5 = \frac{3}{5}\psi_2^{-2/3}$, therefore:

$$T_{ABCE} = \frac{1}{5}\,\psi_2^{-2/3}\left[3(o_A\,o_B\,\iota_C + (o_A * \iota_B)\,o_C)\,\iota_E - 2(\iota_A\,\iota_B\,o_C + (o_A * \iota_B)\,\iota_C)\,o_E\right], \qquad (22)$$

is a potential for the Lanczos spinor (14).

4. CONCLUSIONS

The construction of $L_{ABC\dot{D}}$ for arbitrary 4-spaces of Petrov types I, II, and D, is an open problem, and we consider that the equations (19) are important in such research. Besides, our approach shows the fundamental participation of the canonical tetrad for each Petrov type of the spacetime under analysis.

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