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Charged anisotropic models in a modified Tolman IV spacetime

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ABSTRACT

In this paper, we study the behaviour of compact relativistic objects with charged anisotropic matter distribution considering modified Tolman IV type potential for the gravitational potential *Z(x)* which depends on an adjustable parameter α and a particular form of the metric function *y(x)* used by Malaver (2009). The Einstein-Maxwell fields equations are integrated analytical and we obtain the radial pressure, energy density, charge density, anisotropy and the mass function. A physical analysis of electromagnetic field indicates that is regular at the origin and well behaved. The new stellar models satisfies all physical properties in a realistic star for any value of the adjustable parameter.

***Keywords*:** Compact relativistic objects, Gravitational potential, Adjustable parameter, Metric function, Tolman IV type potential, Electromagnetic field

1. INTRODUCTION

The Einstein´s theory of general relativity can be used to explain the behavior and structure of massive objects as neutron stars, quasars, pulsars and white dwarfs [1,2]. Between the pioneering theoretical works are very important the contributions of Schwarzschild [3], Tolman [4], Oppenheimer and Volkoff [5]. Schwarzschild [3] constructed within the framework of general relativity the simplest model of astrophysical relevance for a

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fluid configuration. Tolman [4] generated new solutions for static spheres of fluid and Oppenheimer and Volkoff [5] study of the gravitational equilibrium of masses of neutrons using the equation of state for a cold Fermi gas and general relativity.

The physics of configurations of superdense matter is not well understood and many researchs of strange stars have been performed within the framework of the MIT bag model [6]. In this model, the strange matter equation of state (EOS) has a simple linear form given by $P=\frac{1}{3}(\rho-4B)$, where is the energy density ρ, P is the isotropic pressure and B is the bag constant.

Many researchers have used a great variety of mathematical techniques to try to obtain exact solutions for quark stars within the framework of MIT-Bag model; Komathiraj and Maharaj [6] found two new classes of exact solutions to the Einstein-Maxwell system of equations with a particular form of the gravitational potential and isotropic pressure. Malaver [7,8] also has obtained some models for quark stars considering a potential gravitational that depends on an adjustable parameter. Thirukkanesh and Maharaj [9] studied the behavior of compact relativistic objects with anisotropic pressure in the presence of the electromagnetic field. Maharaj et al. [10] generated new models for quark stars with charged anisotropic matter considering a linear equation of state. Thirukkanesh and Ragel [11] obtained new models for compact stars with quark matter. Sunzu et al. found new classes of solutions with specific forms for the measure of anisotropy [12].

The first detailed models of strange stars, based on a more realistic EOS of strange quark matter, taking into account strange quark mass and the lowest order QCD strong interactions, were constructed by Haensel et al. [13], who considered also specific features of accretion on strange stars. Similar results were presented somewhat later by Alcock et al. [14], who additionally proposed scenarios of the formation of strange stars and analyzed strange stars with the normal crust.

Local isotropy, as it has been shown in recent years, is a too stringent condition, which may excessively constrain modeling of self-gravitating objects. Of course no astrophysical object is entirely composed of perfect fluid. The theoretical investigations [15-18] made in the last few decades back about more realistic stellar models show that the nuclear matter may be locally anisotropic at least in certain very high density ranges ($\rho > 10^{15}$ g cm^3), where the relativistic treatment of nuclear interactions in the stellar matter becomes important. Bowers and Liang [19] first generalized the equation of hydrostatic equilibrium for the case local anisotropy. Cosenza et al. [20,21], Bayin [22], Krori et al. [23], Herrera and León [24], Herrera et al. [25,26], Dev and Gleiser [27,28], Ivanov [29], Mak and Harko [30,31], Ivanov [32], Malaver [33-38] and Pant et al. [39] have studied the effect of local anisotropy on the bulk properties of spherically symmetric static general relativistic compact objects.

Over the years the continuously growing interests enable researchers to develop mathematically simple exact analytical models of self-bound strange stars within the framework of linear equation of state based on MIT bag model together with a particular choice of metric potentials/mass function (Gokhroo and Mehra [40], Mak and Harko [41], Chaisi and Maharaj [42], Sharma and Maharaj [43], Varela et al. [44], Komathiraj and Maharaj [45], Takisa and Maharaj [46], Rahaman et al. [47], Kalam et al. [48], Thirukkanesh and Ragel [49-51], Ngubelanga et al. [52, 53], Thirukkanesh et al. [54], Bhar et al. [55], Murad [56], Malaver [57,58,61]).

In particular, Feroze and Siddiqui [59,60] and Malaver [33,35,61] consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity

The aim of this paper is to obtain new exact solutions to the Maxwell-Einstein system of field equations for charged anisotropic matter in static spherically symmetric spacetime using modified Tolman IV for the gravitational potential *Z(x)* used which depends on an adjustable parameter α and a particular form of the metric function *y(x)* proposed by Malaver [7].

We have obtained a new classes of static spherically symmetrical models for relativistic stars in presence of an electromagnetic field. This article is organized as follows, in Section 2, we present Einstein´s field equations of anisotropic fluid distribution. In Section 3, we make a particular choice of gravitational potential Z(x) and of the metric function *y(x)* that allows solving the field equations and we have obtained new models for charged anisotropic matter.

In Section 4, a physical analysis of the new solution is performed. Finally in Section 5, we conclude.

2. EINSTEIN FIELD EQUATIONS

We consider a spherically symmetric, static and homogeneous and anisotropic spacetime in Schwarzschild coordinates given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where $\nu(r)$ and $\lambda(r)$ are two arbitrary functions.

We are considering an anisotropic fluid in the presence of electromagnetic field. The components of energy momentum tensor is given by

$$T_{00} = -\rho - \frac{1}{2}E^2 \tag{2}$$

$$T_{11} = p_r - \frac{1}{2}E^2 \tag{3}$$

$$T_{22} = T_{33} = p_t + \frac{1}{2}E^2 \tag{4}$$

where: ρ is the energy density, p_r is the radial pressure, E is electric field intensity and p_t is the tangential pressure, respectively. The Einstein-Maxwell equations take the form

$$\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho + \frac{1}{2}E^2 \tag{5}$$

$$-\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\nu'}{r}e^{-2\lambda} = p_r-\frac{1}{2}E^2 \tag{6}$$

$$e^{-2\lambda}\left(\nu''+\nu'^2+\frac{\nu'}{r}-\nu'\lambda'-\frac{\lambda'}{r}\right)=p_t+\frac{1}{2}E^2 \tag{7}$$

$$\sigma=\frac{1}{r^2}e^{-\lambda}(r^2E)' \tag{8}$$

where primes represent differentiation with respect to r and σ is the charge density.

Using the transformations $x=cr^2$, $Z(x)=e^{-2\lambda(r)}$ and $A^2y^2(x)=e^{2\nu(r)}$ with arbitrary constants A and c>0 suggested by Durgapal and Bannerji [62], the Einstein field equations that governs the gravitational behaviour of a charged anisotropic sphere can be written as

$$\frac{1-Z}{x}-2\dot{Z}=\frac{\rho}{c}+\frac{E^2}{2c} \tag{9}$$

$$4Z\frac{\dot{y}}{y}-\frac{1-Z}{x}=\frac{p_r}{c}-\frac{E^2}{2c} \tag{10}$$

$$4xZ\frac{\ddot{y}}{y}+(4Z+2x\dot{Z})\frac{\dot{y}}{y}+\dot{Z}=\frac{p_t}{c}+\frac{E^2}{2c} \tag{11}$$

$$p_t=p_r+\Delta \tag{12}$$

$$\frac{\Delta}{c}=4xZ\frac{\ddot{y}}{y}+\dot{Z}\left(1+2x\frac{\dot{y}}{y}\right)+\frac{1-Z}{x}-\frac{E^2}{c} \tag{13}$$

$$\sigma^2=\frac{4cZ}{x}\left(x\dot{E}+E\right)^2 \tag{14}$$

$\Delta=p_t-p_r$ is the measure of anisotropy and dots denote differentiation with respect to x. With the transformations of [60], the mass within a radius r of the sphere take the form

$$M(x)=\frac{1}{4c^{3/2}}\int_0^x\sqrt{x}\left(\rho_*+E^2\right)dx \tag{15}$$

where $\rho_* = \left(\frac{1-Z}{x} - 2\dot{Z}\right)c$ and $\rho = \rho_* + E^2$

3. CHARGED ANISOTROPIC MODELS

Following Tolman [4], Thirukkanesh and Ragel [51] and Malaver [58], we take the form of the gravitational potential, $Z(x)$ as

$$Z(x) = \frac{(1+ax)^{\alpha}(1-bx)}{(1+2ax)} \tag{16}$$

where: *a* and *b* are real constants and *α* is an adjustable parameter. The potential is regular at the origin and well behaved in the interior of the sphere. The electric field intensity *E* is continuous, reaches a maximum and then it diminishes in the surface of the sphere and given by

$$E^2 = \frac{2acx}{(1+ax)^2} \tag{17}$$

We choose the particular form of the metric function *y(x)* proposed by Malaver [7]

$$y(x) = (a+ax)^2 \tag{18}$$

In this paper, we have considered the particular cases *α* = 1, 2.

For the case *α* = 1, using $Z(x)$, eq. (17) and eq.(18) in eq.(10) we obtain for the radial pressure

$$P_r = c\frac{\begin{bmatrix} -9a^3bx^4 + \left(-a^4b + 7a^3 - 27a^2b + 2a^2\right)x^3 + \left(-a^4 - 3a^3b + 2a^3 + 22a^2 - 27ab + a\right)x^2 \\ + \left(a^2 + 23a - 9b - 2a^3 - 3a^2b\right)x - a^2 - ab + 8 \end{bmatrix}}{(1+2ax)(a+x)(1+ax)^2} \tag{19}$$

Substituting (16) and (17) in eq. (9) the energy density can be written as

$$\rho = \rho_* + E^2 = \frac{c\left[6a^4bx^4 + \left(2a^4 - 19a^3b - 4a^3\right)x^3 + \left(7a^3 + 23a^2b - 4a^2\right)x^2 + \left(8a^2 + 13ab - a\right)x + 3(a+b)\right]}{(1+ax)^2(1+2ax)^2} \tag{20}$$

replacing (20) in (15) we have for the mass function

$$M(x) = \frac{\sqrt{x}}{2\sqrt{c}}\left[\frac{abx^2 + (a+b)x}{2ax+1} - \frac{(2ax+3)}{2a(ax+1)}\right] + \frac{3arctag\sqrt{2ax}}{4a\sqrt{ac}} \tag{21}$$

and for charge density

$$\sigma^2 = \frac{2ac^2(1-bx)(3+4ax+a^2x^2)^2}{(1+2ax)(1+ax)^5} \tag{22}$$

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as

$$e^{2\lambda} = \frac{(1+2ax)}{(1+ax)(1-bx)} \tag{23}$$

$$e^{2\nu} = A^2(a+x)^4 \tag{24}$$

The measure of anisotropy is given by

$$\Delta = c\frac{\begin{bmatrix} -24a^4bx^6 + (18a^4 - 18a^5b - 79a^3b - 8a^3)x^5 + (4a^5 - 22a^4b + 56a^3 - 98a^2b - 16a^4 - 8a^2)x^4 \\ +(2a^6 + a^5b + 4a^4 - 24a^3b + 66a^2 - 55ab - 8a^5 - 16a^3 - 2a)x^3 \\ +(4a^5 + 2a^4b - 4a^3 - 14a^2b + 36a - 12b - 8a^4 - 4a^2)x^2 + (2a^4 + a^3b - 4a^2 - 2a^3 + 8)x \end{bmatrix}}{(1+2ax)^2(a+x)^2(1+ax)^2} \tag{25}$$

With $\alpha = 2$, we can find the following analytical model

$$P_r = c\frac{\begin{bmatrix} 16a^4bx^5 + (18a^4 - 2a^4b + 60a^3b)x^4 + (a^5 - a^5b + 86a^2b + 68a^3 - 2a^4b + 4a^2 - 4a^3b)x^3 \\ +(2a^4 - 2a^4b - 5a^3b + 2a^3 - 2a^2b + 98a^2 + 56ab + 2a)x^2 \\ +(a^3 + a^2 - 4a^2b - a^3b + 14b + 64a)x - ab + 16 \end{bmatrix}}{(1+ax)^2(1+2ax)(a+2x)} \tag{26}$$

$$\rho = c\frac{\left[10a^5bx^5 + (39a^4b - 6a^5)x^4 + (60a^3b - 17a^4 - 4a^3)x^3 + (46a^2b - 16a^3 - 4a^2)x^2 + (18ab - 5a^2 - a)x + 3b\right]}{(1+ax)^2(1+2ax)^2} \tag{27}$$

$$M(x) = \frac{\sqrt{x}}{\sqrt{c}}\left[\frac{2abx^2 + (3b-2a)x + 1}{8} - \frac{1}{16a}\left(\frac{2a+b}{2ax+1} + \frac{a(b-8)x + b - 12}{ax+1}\right)\right] + \frac{3arctag\sqrt{ax}}{4a\sqrt{ac}} \tag{28}$$

$$\sigma^2 = \frac{2ac^2(1-bx)(3+4ax+a^2x^2)^2}{(1+2ax)(1+ax)^4} \tag{29}$$

$$e^{2\lambda} = \frac{(1+2ax)}{(1+ax)^2(1-bx)} \tag{30}$$

$$e^{2\nu} = A^2(a+2x)^4 \tag{31}$$

$$\Delta = c\frac{\begin{bmatrix} -136a^5bx^7 + (96a^5 - 552a^4b - 40a^6b)x^6 + (388a^4 + 16a^6 - 888a^3b - 2a^7b - 144a^5b)x^5 \\ +(632a^3 - 712a^2b - 6a^6b + 52a^5 - 200a^4b)x^4 + (484a^2 - 288ab + a^6 - 6a^5b + 56a^4 - 192a^3b)x^3 \\ +(2a^5 + 20a^3 - 48b - 48a^2b + 192a + 2a^4b)x^2 + (a^4 - 8ab + 32)x \end{bmatrix}}{(1+2ax)^2(1+ax)^2(a+2x)^2} \tag{32}$$

The Figures 1, 2, 3, 4 and 5 represent the graphs of P_r, ρ, $M(x)$, σ^2 and Δ, respectively for the case $\alpha = 1$ with $c = 1$. The graphs has been plotted for a particular choice of parameters a = 0.04786 , b = 0 .00986 with a stellar radius of r = 1.8 km.

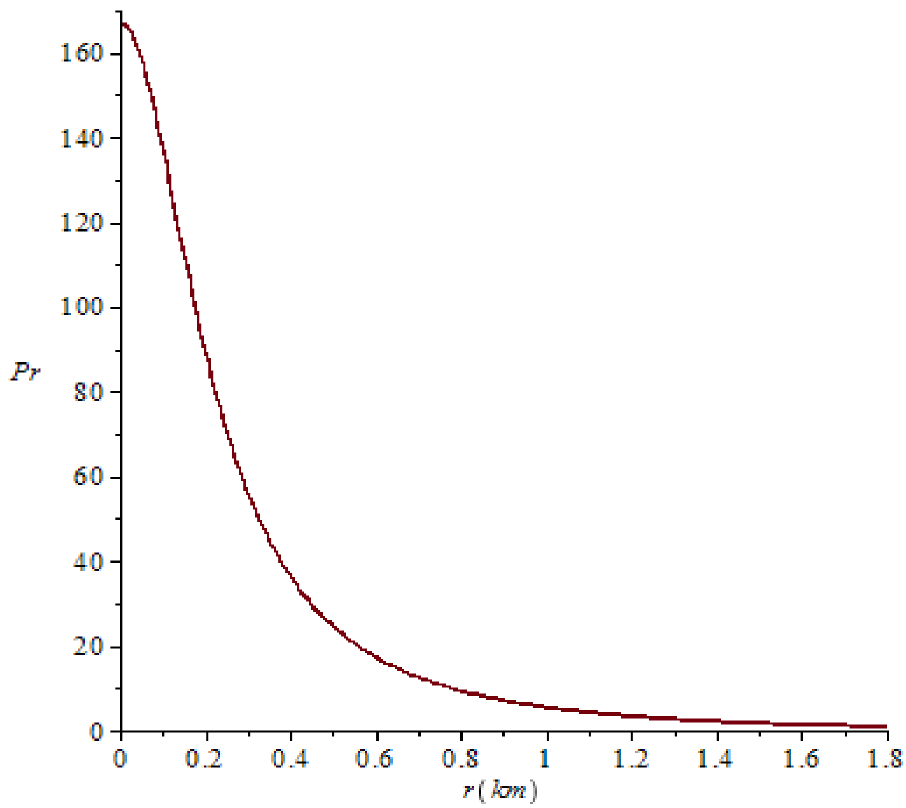

Fig. 1. *Radial Pressure*

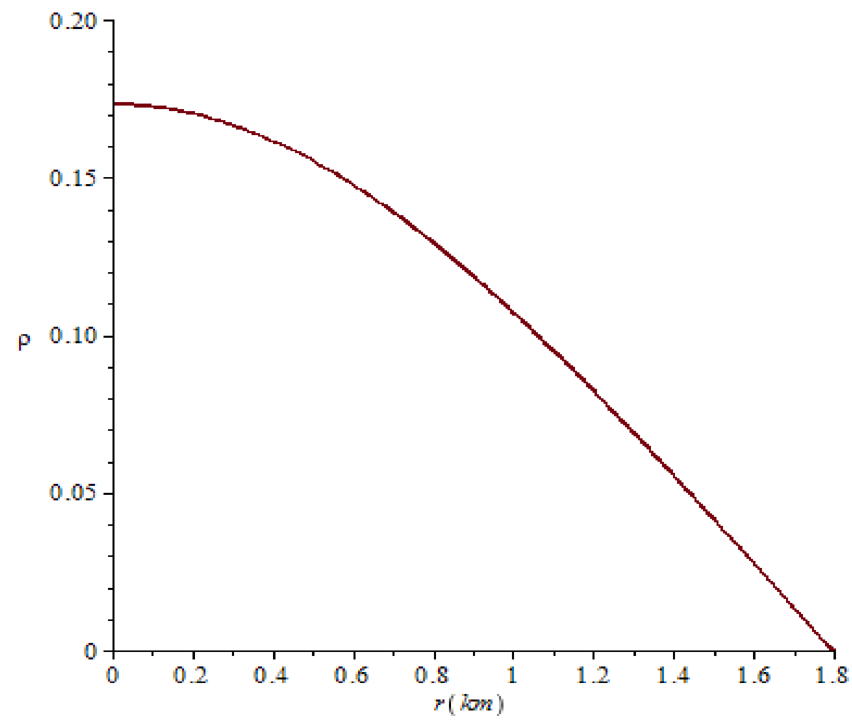

Fig. 2. Energy density

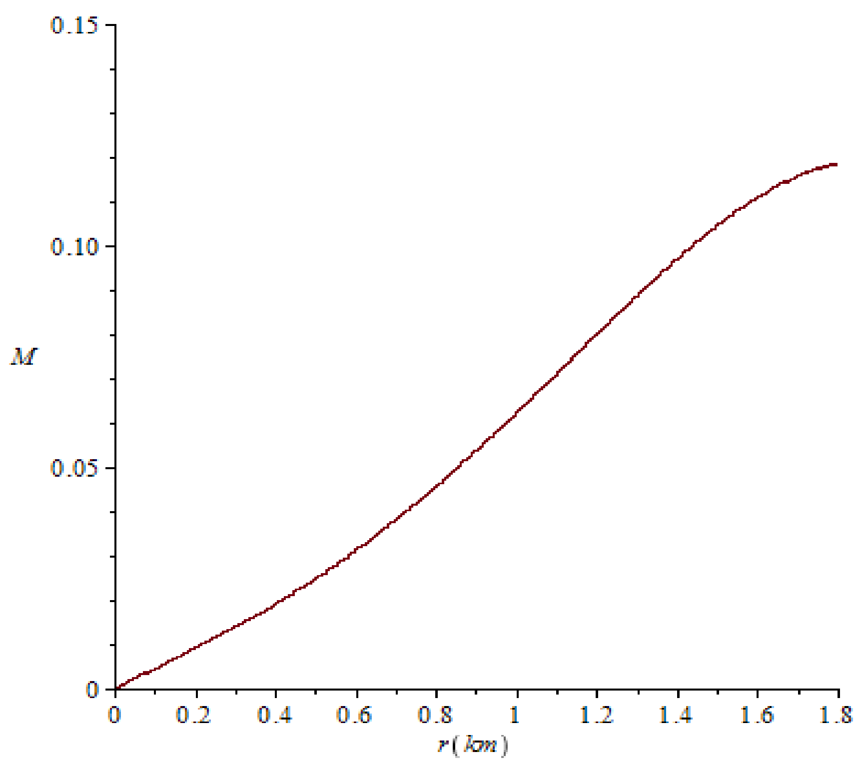

Fig. 3. Mass function

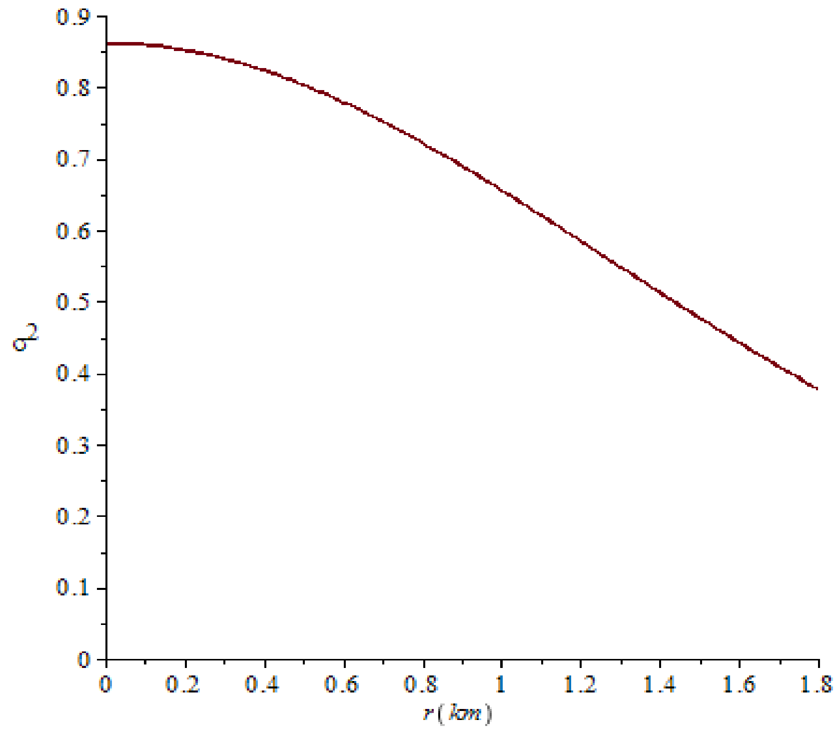

Fig. 4. Charge density

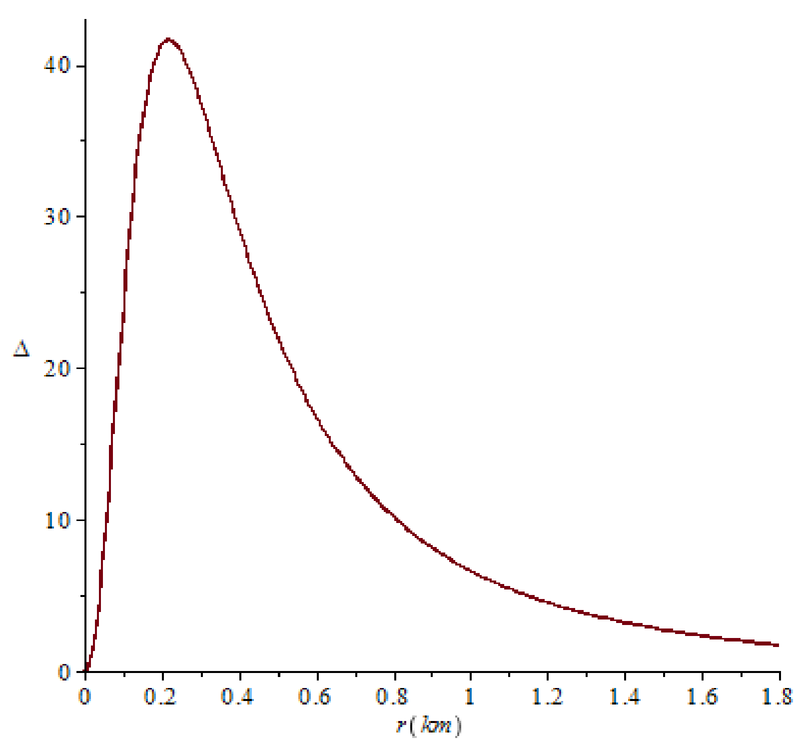

Fig. 5. Anisotropy

4. CONDITIONS OF PHYSICAL ACCEPTABILITY OF THE NEW MODELS

In order for a solution of Einstein field equations to be physically acceptable, the following conditions should be satisfied [49-51]:

(i). The metric potentials should be free from singularities in the interior of the star.

(ii). The radial pressure and density must be finite and positive inside of the fluid sphere.

(iii). $P_r > 0$ and $\rho > 0$ at the origin.

(iv). Monotonic decrease of the energy density and the radial pressure with increasing radius.

The new models satisfy the Einstein-Maxwell system of field equations (9)-(14) and constitute another new family of solutions for anisotropic charged matter. The physical variables are expressed in terms of elementary functions which allows a detailed analysis of the physical behavior of the compact star.

At the origin $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$ and $e^{2\lambda(0)}=1$, $e^{2\nu(0)}=A^2a^4$; these imply that the metric functions are regular in r = 0. For the case α = 1, $P_r(0)=c[8-a(a+b)]$, $\rho(0)=c[3(a+b)]$ and $\Delta(0)=0$, therefore the radial pressure, energy density and anisotropy factor will be non-.negative at the centre. For the case α = 2, $P_r(0)=c[16-ab]$, $\rho(0)=3bc$ and $\Delta(0)=0$, again P_r , ρ and Δ are non-negative in r = 0. In both cases, the mass function is a strictly increasing function, continuous and finite and the charge density is continuous and behaves well inside of the sphere.

In Figure 1, the radial pressure is finite and decreasing with the radial coordinate. In Figure 2, that represent energy density we observe that is continuous, finite and monotonically decreasing function. In Fig. 3, the mass function is increasing function, continuous and finite. In Figure 4, the charge density σ is not singular at the origin, non-negative and decreases. In Figure 5, the anisotropy factor Δ is increasing with the radial coordinate, reaches a maximum and then it diminishes in the surface of the star. The plots show that all the physical variables comply with the requirements of a realistic star.

5. CONCLUSIONS

In this paper, we have generated new exact solutions to the Einstein-Maxwell system of equations considering modified Tolman IV form for the gravitational potential Z which depends on an adjustable parameter α and a prescribed form of the metric function *y(x)* proposed by Malaver [7] used in the description of models of charged isotropic matter. The radial pressure, energy density and the coefficients of the metric are well defined. The charge density σ does not have singularities at the centre of the stellar object and the mass function is an increasing, continuous and finite function. The gravitational potentials are regular at the centre and well behaved. The relativistic solution to the Einstein-Maxwell system presented are physically reasonable. The new obtained model may be used to model relativistic charged compact objects in different astrophysical scenes as quark and neutron stars.

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