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An Axiomatic Approach to Quantum Mechanics

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ABSTRACT

We have shown that the Schrödinger wave equation can be explained and derived from fundamental postulates that are based on the conservation of probability, significance of measurements at infinity and nature's tendency of maintaining a system as unbiased as possible. As a reasonable measure for the local randomness, Fisher information is considered. The presented approach provides an axiomatic derivation for the Schrödinger wave equation, avoiding imperfect models borrowed from classical mechanics such as direct application of the energy conservation, statistical mechanics or vibrating string models.

***Keywords*:** Fisher Information, Quantum Mechanics, Du Bois Raymond Lemma, Variational Principle, Schrödinger's Wave Equation

1. INTRODUCTION

Since the birth of quantum mechanics, it was interpreted in many ways and the so-called wave equations were derived based on different set of postulates [1-7]. Among all the attempts to interpret the quantum behavior, statistical postulates exhibit a reasonable approach of explaining how the nature works [2-4]**.** However, even the most renowned scientists such as Einstein had a dispute over this approach [8, 9], where they argued that there should be set of well-defined physical principles that determines the behavior of particles at quantum scale.

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Analysis on the available literature suggests that the term "well defined" in above statement, indirectly refer to postulates that are very closer in definition and notion to the classical mechanics.

The base of deriving the wave equation plays an important role in how we interpret quantum mechanics. Even though the quantum mechanics is conceptually far away from classical mechanics, Schrödinger wave equation was first derived from set of concepts burrowed from classical mechanics. Some of the later attempts to explain or derive the Schrodinger equation, often inserted the concepts from classical mechanics, resulting an imperfect or less rigorous theoretical ground. As examples, Morse and Feshbach uses an imperfect vibrating model [8, 9] while Nelson rely on the Brownian motion [1] to derive the Schrodinger wave equation. The construction of such models uses the practices in statistical mechanics, starting from a non-quantum dynamical systems and use necessary approximations to arrive Schrodinger's wave equation or equations that structurally resemble with the Schrodinger's wave equation. However, it is questionable to assume that classical mechanical principals are meaningful in the quantum domain. As we see, the main questions to raise against classical mechanical approach to the quantum mechanics has two components, (*a*) Wave particle duality: It is observed that the particles have a wave like behavior in quantum scale. Therefore, the notion of mass in classical mechanics cannot be directly applied in the quantum domain and (*b*) Uncertainty: In the quantum domain, the measurements are not exact and they are associated with a probability. There is no way to know the exact system dynamics in the quantum domain and the observations are dependent on the observer. Therefore, it is clear that, classical mechanics is not enough to explain quantum mechanics and such foundations for quantum mechanics, mixed up with classical principals is not perfect. Many attempts were recently made to axiomatize the quantum mechanics using more fundamental principles. However, almost all of them have a statistical origin. It is evident that these approaches were chosen heuristically, reviewing the experimental results and the facts given by existing quantum theories.

When describing an entity associated with uncertainty, entropic concepts or the measure of information is very useful and provide deeper insights [10, 14, 15]. Information is associated with the randomness or in another word, with the probability distribution. Considering the above, one can state that solving Schrodinger equation reveals the wave function that expresses the probability distribution, ultimately revealing the information measures. Going back in this deduction process suggests that, principles on information measures can determine the properties of the desired or allowed probability distribution functions, which might ultimately describe the behavior of quantum states. Reviews of the information theory leave us the belief of information theory based principles being a very reasonable and very fundamental base for quantum mechanics [10, 15].

Different attempts to derive quantum mechanical equation, using information theoretical approaches can be found in literature. In such attempts, Fisher information is used to derive a dynamic equation through variation principle. But to derive, the Schrodinger equation, one has to introduce a potential energy term to the equations. Frieden [2] uses the energy conservation law to derive a kinetic energy term and then to form a constraint optimization problem. Even though it solves the problem of introducing a potential term, this approach permits only real valued wave function solutions. Reginatto [3] uses the continuity equation as a constraint to derive an equation in the form of the Schrodinger wave equation. The approach does immediately result the wave equation for a free particle. But to arrive at the general form of the

Schrodinger wave equation, a potential term is introduced without proper justifications. Most of the works that uses Fisher information to derive the Schrodinger wave equation, introduce a Hamiltonian and add a potential term and the time derivative term to the equation, directly without serious justifications.

In the current work, we derive a form of Schrodinger wave equation considering a novel set of postulates or axioms having information theoretical and statistical origins. The basic statistical postulates are based on the conservation of probability, significance of measurements at infinity and nature's tendency of maintaining maximum disorder. As a reasonable measure for the disorder [10, 12], Fisher information is considered. During the derivation, the intermediate results suggest equation structures similar to classical mechanics.

2. THE AXIOMS

In this section, we suggest a set of axioms that are very much fundamental and having more of a mathematical origin compared to the postulates in classical mechanics. The present formulation is based on the position space for the simplicity. However, the moment space based derivation is possible. First we would like to discuss about categorizing the axioms based on their use and theoretical backgrounds. They are namely, conservation of probability, significance of measurements at infinity, local variation in probability distribution.

The conservation of probability can be considered as a basic principle or a definition. Considering an evolving probability density function in the space, over time, results a continuity equation. For the continuity equation, the conservation of probability:

$$\int_{-\infty}^{+\infty} \rho\, d^3 r = 1 \tag{1}$$

here ρ is the three-dimensional probability density function of space and time.

$$\frac{d\rho}{dt} + \nabla.(\rho \boldsymbol{u}) = 0 \tag{2}$$

Note that the flux of probability is defined as, $\rho\boldsymbol{u}$. Here $\boldsymbol{u}$ is the flow velocity of the probability. At this point, let's introduce a function $G(\boldsymbol{r},t)$ in such a way that:

$$\boldsymbol{u} = \frac{1}{m}\nabla G$$

Then eq. 2 becomes,

$$\frac{d\rho}{dt} + \nabla.\left(\rho \frac{1}{m}\nabla G\right) = 0 \tag{3}$$

At this point, we should think about how to interpret this equation. It is obvious that ∇G is having the dimensions of momentum. But it is not clear at this point how ∇G expresses something physically.

It is necessary to have an argument about, how significant, a physical condition at distance, contribute to shape the local dynamics of a particle. From some of the concepts borrowed from classical mechanics, one might postulate that an effect of a physical phenomena get reduced linearly or proportionally to some power of the distance measured (ex: Gravity). But in this domain, we cannot reasonably postulate a relationship between the distance and an influence of some physical phenomenon. Rather we would like to consider suggesting a "significance" of physical conditions that contribute to the dynamics of a quantum particle. We postulate a measure of significance is ρX, where ρ is the probability distribution describing the presence of a particle in space time while X being a measurement of some physical quantity. Then we postulate that the significance of X should be negligible at infinity or in another word, ρX must be zero as the distance goes to infinity.

Let us consider the case where the magnitude of a position vector $\boldsymbol{r}$ in three-dimension goes to infinity. As, $|\boldsymbol{r}| \to \infty$:

$$\lim_{|\boldsymbol{r}|\to\infty} \rho = 0 \tag{4}$$

This is because $\rho > 0$ and eq. 1 holds.

If X is the momentum defined by ∇G then:

$$\lim_{|\boldsymbol{r}|\to\infty} \rho \nabla G = 0 \tag{5}$$

If X is the kinetic energy defined by $\frac{1}{2m}\|\nabla G\|^2$, then:

$$\lim_{|\boldsymbol{r}|\to\infty} \frac{\rho}{2m}\|\nabla G\|^2 = 0 \tag{6}$$

If X is the time derivative of the field G defined by dG/dt, then:

$$\lim_{|\boldsymbol{r}|\to\infty} \rho \frac{dG}{dt} = 0 \tag{7}$$

Even though, we do not directly attach physical laws that are derived from Newtonian mechanics, we still would like to consider the definition of the Potential Energy. One of the main reasons to look at the definitions as such, is that the measurements that we use to describe our understandings of a system are dependent upon the definitions introduced by classical mechanics.

Define the quantity $\bar{\boldsymbol{P}}$ as follows,

$$\bar{\boldsymbol{P}} = -\int_{-\infty}^{+\infty} \rho \nabla G \, d^3 r \tag{8}$$

We note that $\bar{\boldsymbol{P}}$ represents some average momentum quantity with relevant units. Now define a quantity $\bar{\boldsymbol{F}}$ which is the time derivative of $\bar{\boldsymbol{P}}$:

$$\bar{\boldsymbol{F}} = \frac{d\bar{\boldsymbol{P}}}{dt} \tag{9}$$

Looking at the units, one can see that $\bar{\boldsymbol{F}}$ defines some kind of an average force. We cannot link this force with any physical entity at this moment.

Finally, define a potential energy U as,

$$\bar{\boldsymbol{F}} = \int_{-\infty}^{+\infty} \rho \nabla U \, d^3 r \tag{10}$$

If X is the potential energy U that was defined above, then considering our postulate of significance of measurements at infinity:

$$\lim_{|\boldsymbol{r}| \to \infty} \rho U = 0 \tag{11}$$

In our problem, we consider a probability distribution in the space and we try to come up with an axiom which helps us to select a set of probability distributions out of infinite number of possible distributions, that are possible and reasonable to describe a quantum behavior. Here we can say that our probability distribution cannot be biased to have lower or higher values in particular space coordinates without a reason.

That means in another words our probability distribution function ρ, cannot have local spatial variations unless there is a reason. To insert this idea into a mathematical construction, we should come up with a measure [10, p. 35].

Therefore, we consider a popular functional that measure the information content or the randomness of an entity which is called the Fisher Information [13]. Here we do not jump into selecting Fisher Information directly, the reason will be explained. It is however, noted that the Fisher Information has become popular in deriving quantum equation during last few decades [2, 4], [10, p. 112-127], [11, 12]. In such previous works, Fisher Information was chosen due to various reasons and justifications.

Fisher Information function I is defined in one-dimension as follows:

$$I = \int_{-\infty}^{+\infty} \frac{1}{\rho} \left(\frac{\partial \rho}{\partial x} \right)^2 dx \tag{12}$$

Take a function $R(x,t)$ such that:

$$R = \rho^{1/2} \tag{13}$$

Substituting eq. 12 in eq.13 leads to:

$$I = 4\int_{-\infty}^{+\infty}\left(\frac{\partial R}{\partial x}\right)^2 dx \tag{14}$$

The eq. 14 implies that I measures the overall variations in the probability distribution and to come up with a less biased probability distribution, it is needed to minimize.

3. CONSTRUCTION OF A DYNAMIC EQUATION

We start the construction of the dynamic equation in one dimension by considering the definitions in eq. 9 and 10 which result:

$$\frac{d\overline{\boldsymbol{P}}}{dt} = -\int_{-\infty}^{+\infty}\rho\frac{\partial U}{\partial x}dx \tag{15}$$

Equation 15 gets further expanded with the substitution of eq. 8:

$$\frac{d}{dt}\int_{-\infty}^{+\infty}\rho\frac{\partial G}{\partial x}dx = \int_{-\infty}^{+\infty}\frac{\partial}{\partial t}\left(\frac{\rho\partial G}{\partial x}\right)dx = -\int_{-\infty}^{+\infty}\rho\frac{\partial U}{\partial x}dx \tag{16}$$

Taking the left side of the equation 16 and performing the integration by parts

$$\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}\left(\frac{\rho\partial G}{\partial x}\right)dx = \int_{-\infty}^{\infty}\left(\frac{\partial\rho}{\partial t}\frac{\partial G}{\partial x}+\rho\frac{\partial^2 G}{\partial x\partial t}\right)dx$$

and substituting for $\partial\rho/\partial t$ from eq. 3:

$$\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}\left(\frac{\rho\partial G}{\partial x}\right)dx = \int_{-\infty}^{\infty}\left(-\frac{\partial}{\partial x}\left(\frac{\rho}{m}\frac{\partial G}{\partial x}\right)\frac{\partial G}{\partial x}+\rho\frac{\partial^2 G}{\partial x\partial t}\right)dx$$

$$\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}\left(\frac{\rho\partial G}{\partial x}\right)dx = \int_{-\infty}^{\infty}\left(-\frac{1}{m}\frac{\partial\rho}{\partial x}\left(\frac{\partial G}{\partial x}\right)^2-\frac{\rho}{m}\left(\frac{\partial^2 G}{\partial x^2}\right)\frac{\partial G}{\partial x}+\rho\frac{\partial^2 G}{\partial x\partial t}\right)dx$$

$$\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}\left(\frac{\rho\partial G}{\partial x}\right)dx = \int_{-\infty}^{\infty}\left(-\frac{1}{2m}\frac{\partial\rho}{\partial x}\left(\frac{\partial G}{\partial x}\right)^2-\frac{\partial}{\partial x}\left(\frac{\rho}{2m}\left(\frac{\partial G}{\partial x}\right)^2\right)+\rho\frac{\partial^2 G}{\partial x\partial t}\right)dx \tag{17}$$

Integrating by parts,

$$\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}\left(\frac{\rho\partial G}{\partial x}\right)dx = \int_{-\infty}^{\infty}\left(-\frac{1}{2m}\frac{\partial\rho}{\partial x}\left(\frac{\partial G}{\partial x}\right)^2+\rho\frac{\partial^2 G}{\partial x\partial t}\right)dx - \frac{\rho}{2m}\left(\frac{\partial G}{\partial x}\right)^2\Bigg|_{-\infty}^{+\infty} \tag{18}$$

Since $\rho\left(\frac{\partial G}{\partial x}\right)^2$ is zero at infinity from eq. 6:

$$\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}\left(\frac{\rho\partial G}{\partial x}\right)dx = \int_{-\infty}^{\infty}\left(-\frac{1}{2m}\frac{\partial\rho}{\partial x}\left(\frac{\partial G}{\partial x}\right)^2+\rho\frac{\partial^2 G}{\partial x\partial t}\right)dx \tag{19}$$

Again using integration by parts,

$$\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}\left(\frac{\rho\partial G}{\partial x}\right)dx = \int_{-\infty}^{\infty}\left(-\frac{1}{2m}\frac{\partial\rho}{\partial x}\left(\frac{\partial G}{\partial x}\right)^2-\frac{\partial\rho}{\partial x}\frac{\partial G}{\partial t}\right)dx-\rho\frac{\partial G}{\partial t}\Bigg|_{-\infty}^{+\infty} \tag{20}$$

and Since $\rho\,\partial G/\partial t$ is zero at infinity from eq. 7:

$$\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}\left(\frac{\rho\partial G}{\partial x}\right)dx = \int_{-\infty}^{\infty}\left(-\frac{1}{2m}\frac{\partial\rho}{\partial x}\left(\frac{\partial G}{\partial x}\right)^2-\frac{\partial\rho}{\partial x}\frac{\partial G}{\partial t}\right)dx \tag{21}$$

The right side of eq. 16:

$$-\int_{-\infty}^{+\infty}\rho\frac{\partial U}{\partial x}dx = -\int_{-\infty}^{+\infty}\left(\frac{\partial}{\partial x}(\rho U)-\frac{\partial\rho}{\partial x}U\right)dx = \int_{-\infty}^{+\infty}-\frac{\partial\rho}{\partial x}Udx-\rho U\Big|_{-\infty}^{+\infty} \tag{22}$$

Since from eq. 11 ρU is zero at infinity:

$$-\int_{-\infty}^{+\infty}\rho\frac{\partial U}{\partial x}dx = \int_{-\infty}^{+\infty}\frac{\partial\rho}{\partial x}Udx \tag{23}$$

Combining eq. 21 and 23 yield:

$$\int_{-\infty}^{+\infty}\left(\frac{1}{2m}\frac{\partial\rho}{\partial x}\left(\frac{\partial G}{\partial x}\right)^2+\frac{\partial\rho}{\partial x}\frac{\partial G}{\partial t}+\frac{\partial\rho}{\partial x}U\right)dx = 0 \tag{24}$$

Rearrange eq. 24:

$$\int_{-\infty}^{+\infty}\frac{\partial\rho}{\partial x}\left(\frac{1}{2m}\left(\frac{\partial G}{\partial x}\right)^2+\frac{\partial G}{\partial t}+U\right)dx = 0 \tag{25}$$

The result in eq. 4, will allow us use the Du Bois-Raymond Lemma on eq. 25:

$$\frac{1}{2m}\left(\frac{\partial G}{\partial x}\right)^2+\frac{\partial G}{\partial t}+U = C \text{ (constant)} \tag{26}$$

Since eq. 1 holds, eq. 26 results:

$$\int_{-\infty}^{+\infty}\rho\left(\frac{1}{2m}\left(\frac{\partial G}{\partial x}\right)^2+\frac{\partial G}{\partial t}+U\right)dx = C_0 \tag{27}$$

4. SHRODINGER WAVE EQUATION

Now we would like to consider our last postulate of the need of minimizing the Fisher Information I. For our construction, we can think of a one dimensional version as in eq.14.

$$I = 4\int_{-\infty}^{+\infty}\left(\frac{\partial R}{\partial x}\right)^2 dx \tag{28}$$

Now we are in a position to come up with an optimization problem. To find the minimum of

$$\int_{-\infty}^{+\infty}\left(\frac{\partial R}{\partial x}\right)^2 dx + \lambda\int_{-\infty}^{+\infty} CR^2 dx \tag{29}$$

with the use eq. 27 as a constraint where C is defined by eq. 26.

Now we define our Lagrangian as follows:

$$L = \left(\frac{\partial R}{\partial x}\right)^2+\lambda CR^2 = R'^2+\lambda CR^2 \tag{30}$$

Using the Euler Lagrange method, we can write,

$$\frac{d}{dx}\left(\frac{\partial L}{\partial R'}\right)-\frac{\partial L}{\partial R} = 0 \tag{31}$$

where $R' = \partial R/\partial x$. With the substitution for the Lagrangian, eq. 30 is obtained:

$$R'' - \lambda RC \quad\Rightarrow\quad \frac{\partial^2 R}{\partial x^2} = \lambda R\left(\frac{1}{2m}\left(\frac{\partial G}{\partial x}\right)^2+\frac{\partial G}{\partial t}+U\right) \tag{32}$$

At this point we can introduce a function,

(33)

$$\psi = R\mathrm{e}^{\alpha G}$$

Here α can be any real or imaginary constant.
Taking the second derivative of eq. 33 with respect to x:

$$\frac{\partial^2\psi}{\partial x^2} = \left(\frac{\partial^2 R}{\partial x^2} + 2\alpha\frac{\partial R}{\partial x}\frac{\partial G}{\partial x} + \alpha^2 R\left(\frac{\partial G}{\partial x}\right)^2 + \alpha R\frac{\partial^2 G}{\partial x^2}\right)\mathrm{e}^{\alpha G} \tag{34}$$

Taking the first derivative of eq. 33 with respect to t:

$$\frac{\partial\psi}{\partial t} = \left(\frac{\partial R}{\partial t} + \alpha R\frac{\partial G}{\partial t}\right)\mathrm{e}^{\alpha G} \tag{35}$$

Substitution of eq. 3 with eq. 13 $R = \rho^{1/2}$

$$\frac{\partial\rho}{\partial t} = -\frac{\partial}{\partial x}\left(\frac{\rho}{m}\frac{\partial G}{\partial x}\right) = 2R\frac{\partial R}{\partial t}$$

in eq. 35 result:

$$\frac{\partial\psi}{\partial t} = \left(-\frac{1}{2R}\frac{\partial}{\partial x}\left(\frac{R^2}{m}\frac{\partial G}{\partial x}\right) + \alpha R\frac{\partial G}{\partial t}\right)\mathrm{e}^{\alpha G}$$

$$\frac{\partial\psi}{\partial t} = \left(-\frac{1}{m}\frac{\partial R}{\partial x}\frac{\partial G}{\partial x} - \frac{R}{2m}\frac{\partial^2 G}{\partial x^2} + \alpha R\frac{\partial G}{\partial t}\right)\mathrm{e}^{\alpha G} \tag{36}$$

Eq. 34 and eq. 36 result:

$$\frac{1}{2m}\frac{\partial^2\psi}{\partial x^2} + \alpha\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(\frac{\partial^2 R}{\partial x^2} + \alpha^2 R\left(\frac{\partial G}{\partial x}\right)^2\right)\mathrm{e}^{\alpha G} + \alpha^2 R\frac{\partial G}{\partial t}\mathrm{e}^{\alpha G} \tag{37}$$

Further expansion,

$$\frac{1}{2m}\frac{\partial^2\psi}{\partial x^2} + \alpha\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(\frac{\partial^2 R}{\partial x^2} + 2m\alpha^2 R\left(\frac{1}{2m}\left(\frac{\partial G}{\partial x}\right)^2 + \frac{\partial G}{\partial t} + U\right)\right)\mathrm{e}^{\alpha G} - \alpha^2 RU\,\mathrm{e}^{\alpha G} \tag{38}$$

Since we can set any imaginary value for α in eq. 33, let,

$$\alpha = i\sqrt{\frac{\lambda}{2m}} \tag{39}$$

Please note that the selection of α defines our choice of ψ in eq. 33.
Then eq. 38 reduces to the following form:

$$\frac{1}{2m}\frac{\partial^2\psi}{\partial x^2} + \alpha\frac{\partial\psi}{\partial t} = -\alpha^2 RU\, \mathrm{e}^{\alpha G} \tag{40}$$

Substitution of eq. 33 in eq. 40 and rearranging the equation, result:

$$-\frac{1}{\alpha}\frac{\partial\psi}{\partial t} = \frac{1}{2m\alpha^2}\frac{\partial^2\psi}{\partial x^2} + \psi U \tag{41}$$

Letting $\lambda = \frac{-2m}{\hbar^2}$ and substituting in eq.39 would reduce the eq.41 to the following,

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \psi U \tag{42}$$

We immediately recognize that this is the one-dimension version of the Schrödinger wave equation.

5. HEISENBERG UNCERTAINITY PRINCIPLE

Here we would like to highlight the link between Heisenberg uncertainty principles in quantum mechanics and Fisher Information. The Cramer Rao lower bound serves as the basis of the uncertainty inequality incorporated with the definition of the momentum operator which is a direct result of the Schrödinger wave equation [17-18]. Following Stam's approach [18] we would like to show the important steps and results as follows.

Define $\psi(x)$ and $\varphi(p)$ to be position and momentum wave functions.

From the Schrödinger wave equation (eq. 42) ,it can be shown that there exists a Fourier relation between them as defined by eq. 43 and eq. 44.

$$\psi(x) = \frac{1}{\hbar}\int_{-\infty}^{+\infty}\varphi(p)\, e^{\frac{ipx}{\hbar}}\, dp \tag{43}$$

$$\varphi(p) = \frac{1}{\hbar}\int_{-\infty}^{+\infty}\psi(x)\, e^{-\frac{ipx}{\hbar}}\, dx \tag{44}$$

Following the eq.14, we can write,

$$I = 4\int_{-\infty}^{+\infty}\left(\frac{\partial\left|\psi(x)\right|}{\partial x}\right)^2 dx \tag{45}$$

Considering Stam's approach [18], we can have the following inequality,

$$I \leq \frac{4}{\hbar^2}\sigma^2{}_{\varphi} \tag{45}$$

where, $\sigma^2{}_{\varphi}$ is the variance of momentum.

Also the Cramer-Rao lower bound [19] suggests the relation between the variance of an unbiased estimator and the fisher information, which can be directly applied to our case as follows.

$$\sigma^2{}_{\psi} \geq \frac{1}{I} \tag{46}$$

where, $\sigma^2{}_{\psi}$ is the variance of position parameter.

Combining the eq.45 and eq.46 result the Heisenberg uncertainty principle as follows.

$$\sigma^2{}_{\varphi}\sigma^2{}_{\psi} \geq \frac{\hbar^2}{4} \tag{47}$$

6. CONCLUSIONS

The core concept delivered in the paper is that, nature always tries to keep a system as unbiased as possible. The physical realities or laws appear as a result of applying conditions on systems which are trying to stay as unbiased as possible. From our construction, it is clear that how we define such conditions such as the limits at infinity described by eq. 5, eq. 7 and eq. 11. In the quantum domain, the focus is more about the local information or the local randomness. This again justify Fisher information as a reasonable measure of the biasness in local context due to the derivative term defined in eq. 12 which is sensitive to local variations in the probability distribution function.

Our construction, is mainly based on the postulate that the nature tries to keep a system as unbiased as possible unless there is a proper reason for a particular biasness. Out of the triviality i.e. probability density function with no local variations, we try to derive physical phenomenon using conditions suggested as postulates herein which are dependent upon the observer. The equations resulted from the postulates regarding the observations at infinity are some way similar to the assumptions used in limit evaluations of integrals by Klein [4]. Klein also uses additional assumptions regarding the limits at infinity to simplify the integrations. However, Klein's approach goes in a different route with different set of assumptions mainly on independent parameters of equations describing motion to arrive the Schrodinger wave equation. The approach also has the lack of justifications in limiting the number of parameters at Klein's final assumption.

It should be also noted that our postulates defining conditions at infinity, also suggest a particular way to look at the notion of "distance" and "time". Notion of infinity distance or reaching to infinity is associated with the scale. If we consider, negligible influence of the conditions that occur at infinity suggests that time to receive information from the conditions at infinity is infinitely long. We might conclude that, how we define infinity in "time" also depends on the scale we are considering similarly to the case of infinity in "distance". This argument can be linked to the Scale Relativity [16].

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