Technical Analysis of Price Formations with Fibonacci Sequence on Warsaw Stock Exchange

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ABSTRACT

Investing on the stock market is associated with taking risk. Contemporary investors use various methods that support their decisions. Apart from fundamental analysis, the technical analysis, in which a variety of tools is used allows for a precise study of diagrams and trends and, therefore, enabling forecasting future changes in the prices of financial instruments, becomes more and more popular. With that, investors are able to adequately react and maximise their profits or minimise risks on the stock market. The purpose of this article is the presentation and description of technical analysis tools based on Fibonacci sequence with a particular emphasis on price patterns created using Fibonacci numbers. In the first part of the article the author presents Fibonacci sequence and the golden ratio, then pointing out the possibilities of its use on financial markets. Moreover, the attempt of finding the two main types of price patterns in the actual realities of stock market is conducted, within markings of polish index WIG20. The article ends with conclusions concerning the use of Fibonacci numbers and the golden ratio in the technical analysis, and the possibility of effective price pattern application in the actual investment.

Keywords: Technical analysis, price patterns; price formations; Fibonacci sequence; Fibonacci numbers; stock market; WIG20
1. INTRODUCTION

Investing on the stock market is associated with taking risk. Investors search for viable methods to maintain the control of the risk and decide effectively about their investments (Barber and Terrance, 2011).

There is a great number of tools helping the decision-making process of investors. Among them, there is the use of the Fibonacci number in the technical analysis. Some researchers of the financial markets propose the replacement of Elliot’s approach with the research on Fibonacci number without paying attention to the order of the waves. It is supported by the elimination of an uncertainty factor replacing it with clearly defined and testable rules (R. Fischer, 2015). Developing this concept may lead to the introduction of discipline to the process of investment decision-making and to the automation of transaction as well.

In the article a large emphasis is put on the analysis of Fibonacci sequence with the particular consideration of Fibonacci numbers. Fibonacci sequence is not a mathematical game. It is rather a way to approach natural phenomena. Fibonacci numbers are used as a geometric tools to analyse the share prices movements.

Fibonacci numbers can be primarily used as a prognostic tool based on the levels of targeted correction waves. However, appropriate usage based on a variety of available methods, the Fibonacci analysis may be applied to create short-term investing strategies on the stock market.

The Fibonacci sequence, therefore, allows for the elaboration of methods of price-time analysis, allowing the determination of market peaks and troughs with an incredible precision. It is expressed through a number of price patterns created while using this sequence, supporting investment decisions on the capital market. In the article those formations have been described with the attempt of applying them in the actual investing process.

Geometry based on Fibonacci numbers is characterised by the versatility of its application. To check the effectiveness of using the Fibonacci sequence on the financial markets, the author attempts to find some price patterns received by using Fibonacci numbers, within marking of polish index WIG20.

2. THE FIBONACCI NUMBER AND THE GOLDEN RATIO

The Fibonacci sequence was found in XIII century by the Italian mathematician Leonardo of Pisa, known as Fibonacci (1170-1240) (Posamentier and Lehmann, 2007). It is the sequence of natural numbers defined recursively, which is described in two ways, with and without the 0 as a first digit in the sequence. The first digit equals 0, the second one equals 1, and each next digit is the sum of the previous two. Therefore formally, the notation of Fibonacci sequence with the 0 as a first digit looks as follows (Beck and Geoghegan, 2010):

\[
F_n := \begin{cases} 
0 & \text{for } n = 0 \\
1 & \text{for } n = 1 \\
F_{n-1} + F_{n-2} & \text{for } n = 0 
\end{cases}
\]
The next digits of the sequence are called Fibonacci numbers. As stated before, including the 0 to the elements of the sequence is agreed on – some authors define two first digits of the sequence as: \( F_1 = F_2 = 1 \). In this article, the 0 as a first digit version is used.

Using the formal notation one can create an infinite sequence of numbers, where the first twenty digits are presented in the Table 1.

**Table 1.** The first twenty digits of Fibonacci sequence.

| \( F_0 \) | \( F_1 \) | \( F_2 \) | \( F_3 \) | \( F_4 \) | \( F_5 \) | \( F_6 \) | \( F_7 \) | \( F_8 \) | \( F_9 \) | \( F_{10} \) | \( F_{11} \) | \( F_{12} \) | \( F_{13} \) | \( F_{14} \) | \( F_{15} \) | \( F_{16} \) | \( F_{17} \) | \( F_{18} \) | \( F_{19} \) |
| 0 | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 | 144 | 233 | 377 | 610 | 987 | 1597 | 2584 | 4181 |

The issue associated with Fibonacci sequence is the golden ratio, also known as the golden division or the harmonic division (Shneider, 2014). The golden ratio is the division of the section into two parts so as to the length ratio of the longer one to the shorter one was the same as the whole section to the longer part. In other words the length of the longer part of the section has to be the geometric average of the whole section and its shorter part. The algebraic way in which it can be expressed is as follows:

\[
\frac{a + b}{a} = \frac{a}{b} = \varphi
\]

The ratio, mentioned before, is called the golden number and is described as the twenty first letter of the Greek alphabet - \( \varphi \). It is possible to determine the value of the golden number as:

\[
\varphi = \frac{1 + \sqrt{5}}{2} = 1.6180339887...
\]

Therefore, the golden ratio is often given in elaborations as 1.618. Other important values used in the golden ratio are approximations of appropriate limits of the sequence, which are shown in Table 2 (Nowakowski and Borowski, 2005).

**Table 2.** Other important values used in the golden ratio.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Limit</th>
<th>Value (approximately)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{\varphi} )</td>
<td>( \lim_{n \to \infty} \frac{k_n}{k_{n+1}} )</td>
<td>0.618</td>
</tr>
<tr>
<td>( \frac{1}{\varphi^2} )</td>
<td>( \lim_{n \to \infty} \frac{k_n}{k_{n+2}} )</td>
<td>0.382</td>
</tr>
<tr>
<td>( \varphi^2 )</td>
<td>( \lim_{n \to \infty} \frac{k_{n+2}}{k_n} )</td>
<td>2.618</td>
</tr>
</tbody>
</table>
3. FIBONACCI NUMBERS ON STOCK MARKETS

The usage of Fibonacci sequence on stock market is connected to coefficients, that are used to determine appropriate formations associated with the golden ratio. Table 3 lists values of the most important Fibonacci numbers used in the technical analysis (Nowakowski and Borowski, 2005).

Table 3. Fibonacci numbers used in the technical analysis.

<table>
<thead>
<tr>
<th>Exponent</th>
<th>Fibonacci Coefficient (rounded to the third decimal place)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0.236 (23.6%)</td>
</tr>
<tr>
<td>-2</td>
<td>0.382 (38.2%)</td>
</tr>
<tr>
<td>-1.44</td>
<td>0.5 (50%)</td>
</tr>
<tr>
<td>-1</td>
<td>0.618 (61.8%)</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.786 (78.6%)</td>
</tr>
<tr>
<td>1</td>
<td>1.618 (161.8%)</td>
</tr>
<tr>
<td>2</td>
<td>2.618 (261.8%)</td>
</tr>
<tr>
<td>3</td>
<td>4.236 (423.6%)</td>
</tr>
</tbody>
</table>

In table 3 there is an exponent that equals -1.44, which produces Fibonacci coefficient equal 0.5. It is due to the practical and traditional standpoint, allowing for the appropriate level of visualisation of given results.

So far, there are three main ways the Fibonacci sequence is used to analyse the financial markets (Nowakowski and Borowski, 2005):

1. Time methods (passage of time)
2. Price methods (price changes)
3. Time-price methods (passage of time and price changes)

Depending of the chosen method, the lines that create Fibonacci objects will have a different structure. Fibonacci objects include levels, zones, range, arc and radius. However, the most often used tools of the technical analysis based on the Fibonacci sequence include retracement, fan and projection (Gaucan, 2011).

In the next part of this article the most popular Fibonacci objects are shown and characterised, which investors can easily use in planning their investments.
Figure 1. Fibonacci Retracements.

Figure 2. Fibonacci Fans.
The Figure 1 shows the example of Fibonacci retracements on S&P500 market’s historical data. The percent rate correction on the chart of the price of the financial instrument is called the Fibonacci retracement, which can be external or internal. Internal retracements are corrections of previous trend on the financial instrument of the price chart for less than 100%, whereas external retracements are the analogical corrections, but beyond 100%.

The main problem in using the Fibonacci retracements is that at the very beginning of the correction the investor cannot predict its range – for example either 38.2% or 61.8%. Nevertheless, it is possible to get the specific levels of support/resistance, which is worth taking into consideration when making a decision (Fischer and Fischer, 2003).

Figure 2 shows the usage of Fibonacci range on the same market, as the Figure 1. The Fibonacci range consist of three lines corresponding to the certain levels of retracement: 0.382; 0.500 and 0.618 (Gaucan, 2011).

The Fibonacci fan sets the levels of support and resistance for the price and shows how they expand (move) overtime. Investing with the usage of the Fibonacci fan, the investor may expect that the prices will move between the lines. If the price stays on the line of the fan, the occurrence of support is confirmed. But, if it becomes broken, it is a signal, that the price is ready to move to the next line of the fan. Similarly to the other way around – if the price is under the line of the fan, the resistance occurs. After leaving the fan – the last broken line starts to mark the support/resistance of the new trend (Gaucan, 2011).

Figure 3, on the other hand, shows the Fibonacci projection, which is using the Fibonacci numbers to allow the prediction of potential levels of resistance in the upward trend and supports in the downward trend and the range of movement.

Figure 3. Fibonacci Projection.
In the Figure 3, the blue line shows the increasing wave and its correction, which set the base to calculate the potential range of increase. The interesting fact is that in the next step, the next phase of increases (after the level of correction) reached the level of 100%. If that level would be exceeded, then another Fibonacci retracement (161.8%) should be added and the investor should expect the continuation of increases corresponding to this added retracement.

4. PRICE FORMATIONS BASED ON FIBONACCI SEQUENCE

The Fibonacci sequence and its numbers have a lot of capabilities and bring many tools to the technical analysis of the financial market. Apart from methods, which were already mentioned in the previous part of the article, the Fibonacci numbers allow for the analysis of price formations, known as harmonic formations. They help to precisely predict the price movement and turning points, often appearing on the market.

To calculate the formations, methods based on the Fibonacci numbers and geometry should be used. With them, the range and the price movement of a specific financial instrument and its range of turning points, can be easily measured. There are different types of these structures on the market. The particular characteristic of these formation are the exactly defined proportions, which are expressed by the Fibonacci numbers.

The simplest are the three-wave ABCD formations, in the Elliott wave principle known as the ABC Correction. The more complex formation XABCD, consists of 4 waves connected to each other with strictly defined coefficients. Certain patterns can be distinguished with this approach, namely: the Gartley’s, the Bat, the Butterfly and the Crab. Some investors claim that these formations are more efficient than the classical ones, because there is less freedom in interpretation of the information resulting from the chart. Harmonic formations use mathematic differences between the price and time, therefore, they belong to the price-time method and in turn, they are equally efficient on every market and interval. Including the harmonic formation to the investor’s workshop highly expands the investing possibilities.

Investors aim to look for the opportunity to finalise the transactions, which have a lot of probability to be successful. Therefore, specific price layouts, which fulfil the conditions of the minimal exposure of the capital (the level of the investment risk) and also the high level of potential risk need to be found out.

The popularity and a high formation investment effectiveness based on Fibonacci numbers is due to the fact that the method, unlike other, is able to fulfil these conditions. It is worth mentioning that the harmonic setups appear in every time scale – minute-, hour-, daily-, up to monthly-wise.

4.1. AB=CD

AB=CD formation is the simplest price formation based on the Fibonacci numbers. For the first time it was described by H.M. Gartley in 1935.

The feature of this formation is its place in the definition of the measured movement. AB=CD formation is either downward- (Bearish) or upward-oriented (Bullish), which is presented in the Figure 4.
The scheme of movement in this formation consists of moving from A to B, then after a short correction from B to C the formation ends with the next move – from C to D. The price range of CD is the same as in AB. The equality of the AB and CD (AB=CD) section gives the name of the formation. Point C (the turning point) has to be put in such a place so as to BC could be described by the product AB and numbers 0.618 or 0.786. In turn, CD should end on the retracement 1.618 or 1.272 of the BC section. The range of CD is, however, the product of BC range and numbers 1.618 (with product 0.618 and AB) or 1.272 (with product 0.786 and AB).

The expected change of trend occurs in the D point. The investor at this point is interested in buying or selling, depending on the position. If it is accompanied by the occurrence of the candlestick formation meaning the reversal of the trend confirmed by the rotation or the analysis of time, it is clear for the investor to define the turning point with the minimal level of risk and finalise the transaction (Nison, 1991). Sometimes, the share price will not be precisely showing the relation AB=CD, it most likely will be approximate to 1-2 points. What may happen is, the pattern will be exact, the lean will be similar as well as the duration of the specific price movements (The Trader’s Journal, 2006).

Therefore, the general properties of AB=CD formation can be described as follows:

\[
AB = CD \\
BC = 0,618 \times AB \quad \text{or} \quad BC/AB = 0,786 \\
CD/BC = 1,618 \quad CD/BC = 1,272
\]

Even though this structure is ideal and is rather often seen, in practice formations such as ABCD will meet other assumptions. The perfect symmetry indicating the equality between AB and CD will occur in approximately 40% of cases. In the remaining 60%, there will be other variations of this pattern (Pesavento, 1997).
It is worth remembering, that price patterns are based on probability. There is no certainty that they will appear every time. It is not possible to look for these relations, therefore, the investors should properly maintain the capital, investments and the knowledge about the price patterns use as a support in decision-making process. It is important to finalise the transaction at a given market point, whether it will be the transaction that closes the current position or the one that opens the opposite position to the current setup, the investor should look for the confirmation from a few different price relation, like external and internal retracements, projection analysis and expansion in different scales of time.

As it is seen in the Figure 5, the AB=CD formation can also be used when it comes to the polish indexes like WIG20. In this case, the downward trend appeared in 02.05.2016 and according to AB=CD formation the price of the index was decreasing up to 01.06.2016, then after a small increasing correction in 08.06.2016 it began to decrease again. After this day, the reversal trend in 06.07.2016 is visible. Interestingly, the proportion does not completely accord to the Fibonacci number. The retracement equals 10%, so it is not high, but the changes on the market confirm the functioning of this formation.

Figure 5. Bullish AB=CD on WIG20
4. 2. Gartley Pattern

The ideal Gartley’s pattern is based on the use of the number $\phi$ to predict the price formation. Its name comes from H.M. Gartley (Gartley, 1935; Teseo, 2001), who was the first one to use it and what is interesting, he did not describe it with any values. According to Gartley’s suggestion the downward, correction ended in point B should equal 61.8% of the range of XA.

After the change of the downward trend to the upward one in B, the price of the asset increases to point C. The ideal Gartley’s pattern is visible, when the range of movement between B and C will be put in the interval between 61.8% and 78.6% of the range of movement AB. After changing the tendency in point C, the decrease of the price of the asset may be observed up until the D point. The range of CD movement should belong to the interval from 127% to 161.8% of BC movement. In the D point, the investors get the signal to take the long position. The minimal range of increase DT is 61.8% of the CD range. The complete range of movement for AD is 78.6% of XA and has a key meaning in recognition of the Gartley’s ideal pattern. If in point T, the trend line defined by A and C breaks, then the formation of changing the higher degree trend appears (Hartle, 2000). The example of the ideal Gartley’s pattern in the upward trend is presented in the Figure 6.

![Figure 6. Ideal Bullish Gartley.](image-url)

The modified Gartley’s pattern is used in technical analysis in investing practice. This version can be seen on Figure 7. The general properties of modified form of Gartley’s formation are as follows:

\[
\begin{align*}
AB/XA &= 0.618 \\
BC/AB &= 0.618 \text{ or } 0.786 \text{ (in investing practice: } 0.382 - 0.886) \\
CD/BC &= 1.272 \text{ or } 1.618 \\
AD/XA &= 0.786
\end{align*}
\]
In case of Gartley’s formation the investor should take notice for a few characteristic features. First of all, in investing practice, BC/AB in most cases is in the interval between 0.382 and 0.886 instead of 0.618 and 0.786. Moreover, the external BC retracement cannot exceed 1.618. Then, there are conditions preventing this formation:

- The peak of point D cannot exceed point X,
- The BC wave cannot exceed point A (it can, though, be double peaked or troughed from point A),
- The AB wave cannot exceed point X.

The Gartley’s pattern can be divided into few segments in the range of the chart indications. Point X is the highest and the lowest point in the structure and is the starting point of the formation. This point can be found on the higher scale trend or on the main peaks and troughs. It can be the peak or the trough of the higher degree trend. Point X then, will be the price, which will be observed by every investor using the chart analysis. When point X forms and the XA wave begins, on this stage it is impossible to highlight its end. Some clues to the length of the movement might be arcs or bars with the high range of variation and closings in the upper range of daily range of prices in the direction of the trend. In this case, this movement may take a while to get to the point of correction.

When XA movement stops, the next step is the observation of the AB wave that is forming. It is the first reaction to the initial wave from point X. The key numbers, that can be observed here are Fibonacci retracements to which the market amends and the amount of bars in the chart, which form the wave data and the lean.

The attempt of finding the Gartley’s formation within the index prices in WIG20 is shown in the Figure 8.
Figure 8 shows Bullish Gartley Pattern, which is seen on the Figure of the WIG20 index. The interesting fact is that, the moment in which AB=CD formation ends (Figure 5) is the starting point for Bullish Gartley Pattern (06.07.2016). This whole formation is between 06.07.2016 and 14.09.2016. According to predictions, in continued downward trend that began in point C (08.09.2016) the reversal appears in point D and then the upward trend is visible, even though the correction comes quickly. In the described formation, BC/AB is 0.591, fitting in the agreed range in the investing practice.

What this formation offers to investors is the entry to the market with a specific level of retracements with low or specific level of risk. Statistically this formation works in 70% of cases. The ideal Garley’s pattern has variations, which are identified as the Bat, the Crab and the Butterfly patterns, which will be briefly characterized in the next part of the article.

4.3. Bat, Crab and Butterfly as variations of Gartley

The Bat formation was first published in 2001 by Scott Carney. This formation, similarly to Gartley’s, is an internal structure, therefore the AD range does not exceed the XA wave, but ends of the 0.786 or 0.886 retracement. AB range, in this case, has to be lower than 61.8% of XA wave. Usually, AB wave can be described through the retracement in the 38.2%
to 50% interval of XA wave. The range of AD movement equals 88.6% of the range of XA movement. The range of CD wave varies between 161.8% to 268.1% of the range of BC wave. Equality in length between AB and CD are rarely seen. The Bat formation is presented in the Figure 9.

**Figure 9.** Bat Pattern.

The features of the Bat formation:

- AB correction should be at 38.2%, 44.7%, 48.6% or 50% of the previous XA movement,
- BC wave should be at 38.2%, 44.7%, 50%, 61.8%, 78.6% or 88.6% of the length of AB wave,
- CD wave should be at 161.8%, 190.2%, 200%, 205.8%, 223.6%, 261.8% of the previous BC movement and should stop on the 88.6% retracement of XA wave.

The next Gartley’s formation is the Crab. The Crab formation was first presented in 2000 by Scott Carney. It is one of the most precise pattern out of all harmonic formations. The critical aspect of this formation is the potential point of reverse in point D created on the 1.618 retracement of XA wave and on the external retracement of BC wave resulting from the usage of certain coefficients: 2.236, 2.617, 3.14, 3.618. Point B is set in such place, that the AB wave overwrites the XA wave in the interval from 38.2% to 61.8%. This formation is presented in the Figure 10.

The features of the Crab formation:

- AB correction should be at 38.2%, 44.7%, 48.6%, 50% or 61.8% of the previous XA movement,
- BC wave should be at 38.2%, 44.7%, 48.6%, 50%, 61.8%, 78.6% or 88.6% of the length to the AB wave,
CD wave should be at 223.6%, 261.8%, 314%, 361.8% of the previous BC movement and should stop on the 161.8% retracement of XA wave.

The last variation of Gartley’s is the Butterfly pattern. This formation was discovered by Bryce Gilmore and Larry Pesavento (Teseo, 2001) and is another form of Gartley’s formations. The difference is that the three-wave movement is longer than the initial measurement section. The condition that has to be fulfilled to create the ideal Butterfly formation is to create the turning point B in such place, that the AB range was equal to the product of XA range and the number 0.786. In Scott Carney’s modified version the AB/XA relation occurs in the 0.5-0.786 interval. This formation is presented in the Figure 11.

Figure 10. Crab Pattern.

Figure 11. Butterfly Pattern.
5. CONCLUSIONS

The Fibonacci sequence and numbers based on it create a lot of application opportunities in different branches of life. Financial markets are no different, because the sequence and numbers are able to be used in the technical analysis. Tools based on the Fibonacci numbers work and are reflected in the actual investment reality. Popularisation of the technical analysis and the Fibonacci sequence allows investors to use the arc of tools supporting their decision-making process.

Price patterns, which were analysed in this article can be effectively used on the financial markets. These tools allow for the correct observations of trends, which steer the changes of financial instrument charts. Thanks to that, the price patterns may be used to forecast future financial instrument price changes.

Price formations based on the Fibonacci sequence have a wide range of application and can be successfully used on polish stock market. In this article two formation were observed in polish investing reality (AB=CD and Gartley’s), over few months on WIG20 index markings. It means, that the price patterns can be used not only for the technical analysis of share prices, but to whole indexes, which in turn proves their versatility.

The important fact is that these price patterns observed on WIG20 occurred one after another, meaning, that the chart analysis should be looked in a complex way analysing changes that appear on markings. It confirms the fact that the technical analysis should not be used as the only form of supporting the investment decision-making process. It is incredibly important addition to the fundamental or behavioural analysis and using all tools properly will allow for the appropriate maintaining of finances.

It is worth remembering that the technical analysis based on the Fibonacci numbers is connected only to a certain level of probability in specific events to occur. However, this branch is extremely interesting and is worth exploring further because the given results can not only surprise but bring more tools, that will allow for even more precise measurements in technical analysis.

References


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