



# World Scientific News

An International Scientific Journal

WSN 129 (2019) 193-210

EISSN 2392-2192

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## Viscous dissipation and Joule heating effects on MHD flow of a Thermo-Solutal stratified nanofluid over an exponentially stretching sheet with radiation and heat generation/absorption

Thiagarajan Murugesan<sup>1</sup> and M. Dinesh Kumar<sup>2</sup>

Department of Mathematics, PSG College of Arts & Science, Coimbatore, India

<sup>1,2</sup>E-mail address: [thiyagu2665@gmail.com](mailto:thiyagu2665@gmail.com) , [dineshmdkc.111@gmail.com](mailto:dineshmdkc.111@gmail.com)

### ABSTRACT

The present study is focused on the influence of viscous and joules dissipation on magnetohydrodynamic flow of a double-stratified nanofluid past an exponentially stretching sheet in the presence of thermal radiation and heat generation/absorption effects. The governing nonlinear partial differential equations are transformed into a system of nonlinear ordinary differential equations using similarity variables and then solved numerically for these equations is procured by employing the Nachtsheim-Swigert shooting technique scheme together with Runge-Kutta Fourth-Order method. A parametric analysis has been carried out to investigate the impacts of physical parameters taken in the problem. The pattern worked for the nanofluid transport equations unified the impacts of thermophoresis and Brownian motion. Numerical results are obtained for the velocity, temperature and concentration in the boundary layer region is studied in elaborate. The impact of physical interest such as local skin friction coefficient, local nusselt number, and local Sherwood number are also investigated numerically and are tabulated. It is remarked that Schmidt number and thermal stratification parameter decreases the dimensionless temperature, during the solutal stratification parameter decreases the dimensionless concentration.

**Keywords:** Nanofluid, exponentially stretching sheet, double stratification, heat generation/absorption, viscous-ohmic dissipation

**2010 Mathematical Subject Classification:** 76W05, 76D10, 85A25, 76A02

## **1. INTRODUCTION**

Nanofluid technology, a modern interdisciplinary field of good importance where thermal engineering, Nanoscience, and nanotechnology has evolved massively over the past years. The numerous benefits of nanofluids over conventional solid-liquid suspensions, the following are worth mentioning Choi [1]. Although endeavor has been made to explain the physical reasons for such augmentation in nanofluids. There are many daily life applications in which nanofluids could be appropriate for, such as in the automotive production. Numerical works goal at fundamental understanding are intermittent in the literature, and the papers published recently such as Ali et al. [2], Li et al. [3], Choi [4], and Sreelakshmy et al. [5].

The study of the magnetohydrodynamic or hydrodynamics (MHD) is the macroscopic concept of electrically conducting fluids, providing a potent and practical theoretical framework for describing both astrophysical plasmas and laboratory. Due to these applications, Chakrabarti and Gupta [6] was investigated heat transfer over a nonlinearly stretching sheet with magnetohydrodynamic impact. Ishak et al. [7] was studied the MHD flow and heat transfer outside a stretching cylinder by numerical solution. Two dimensional magnetohydrodynamic boundary layer of stagnation-point flow past a flat plate in a nanofluid analyzed by Thiagarajan and Selvaraj [8]. El-Mistikawy [9] was obtained the solution for the hydrodynamic flow and heat transfer over a linearly stretching sheet with influence of induced magnetic field.

Nanoparticles move through the molecules of the base fluid and sometimes collide with each other by means of Brownian motion (Brown [10] and Einstein [11]). The various characteristics of Brownian movement have been discussed by several authors (Gupta and Kumar [12], Shukla and Dhir [13], Dinarvand et al. [14], and Mami and Bouaziz [15]). Also thermophoresis are interesting and most important. Both Brownian motion and thermophoresis has been analyzed by Babu and Sandeep [16], Makinde and Animasaun [17], Mahmoodi [18], and Astanina et al. [19].

A very small noticed has been given to the channel flows driven due to the stretching surface. The numerical solution of magnetohydrodynamic flow and heat transfer with viscous dissipation over a vertically stretching sheet with buoyancy effects was discussed by Abel et al. [20]. Thereafter, different aspects of the problem have been analyzed by many researchers such as Hayat [21], Chandrasekar [22], and Rout and Mishra [23]. The analysis of heat generation/absorption in motion fluids is important in numerical problems dealing with chemical reactions and those correlated with dissociating fluids. Thermo-micropolar fluid flow along a vertical permeable plate with uniform surface heat flux in the including of heat generation has been discussed numerically by Rahman and Eltayeb [24]. Recent decades, Aziz et al. [25] discussed the hydromagnetic flow of a nanofluid with heat generation/absorption by a rotating disk using numerical study.

Ohmic dissipation is attracting significance production interest for incessant sterilization of solid-liquid blends. Khalaf and Sastry [26] studied the Ohmic heating effects of fluid viscosity with both rate of solid and liquid. The recent works for both viscous dissipation and ohmic dissipation can be seen in references Alam and Hossain [27], Mutuku et al. [28], & Thumma and Mishra [29].

Stratification is the tier of a fluid process to temperature differences and variations in concentration or the presence of different fluids with diverse densities. Thermal and Solutal stratifications are effective in many applications like oceans, thermal stratifications of reservoirs, rivers, and industrial processing.

Ibrahim and Makinde [30] examined the influence of double stratifications on boundary layer flow and heat transfer of nanofluid past a vertical plate. This has headed to reinforced surveys in mass and heat transfer in thermally stratified and solutal stratified (Singh [31], Hayat et al. [32], and Daniel et al. [33]).

To the best of the author’s knowledge, no attempt has been made to examine analyzes the effects of viscous and joule dissipation on magnetohydrodynamic flow of a Thermo-Solutal stratified nanofluid over an exponentially stretching sheet. Also, radiation effect, thermophoresis, Brownian motion, and suction are taken into account. Solutions are presented in graphically and table format for velocity, temperature, concentration field, skin friction coefficient, rate of heat transfer and Sherwood number for different values of various parameters. The result shows the superiority of the proposed approach.

## 2. MATHEMATICAL FORMULATION

We consider the steady, nonlinear, electrically conducting, radiative incompressible nanofluid flow over an exponentially stretching sheet with double stratification, viscous and Ohmic dissipations (joule heating) in the presence of heat generation/absorption effects. A uniform magnetic field of strength  $B_0$  is applied normally to the sheet. The induced magnetic field is neglected under the inference of a small magnetic Reynolds number. The stretching sheet velocity is  $u_w = U_0 e^{x/L}$ .

The ambient temperature and dimensionless concentration at the wall and for away are regarded to be exponentially stratified for  $T_w = T_0 + b e^{x/2L}$ ,  $C_w = C_0 + a e^{x/2L}$ ,  $T_\infty = T_0 + c e^{x/2L}$ , and  $C_\infty = C_0 + d e^{x/2L}$ , respectively where  $a, b, c$ , and  $d$ . Using the above inference with the boundary layer estimations, the governing equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\mu}{\rho} \frac{1}{K^*} u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_f} \left( \frac{\partial q_{rad}}{\partial y} \right) + \frac{\sigma B_0^2 u^2}{(\rho C_p)_f} + \frac{\mu}{(\rho C_p)_f} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{Q^* (T - T_\infty)}{(\rho C_p)_f} + \frac{(\rho C)_p}{(\rho C)_f} \left[ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right] \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

where  $u$  and  $v$  are the velocity components together with the  $x$  and  $y$  directions, respectively;  $T$  and  $T_\infty$  are the local temperature and temperature of the ambient fluid from the sheet,  $C$  is the nanoparticle volume fraction,  $K^*$  is the permeability of the porous medium,  $\rho$  and  $\mu$  are the density of the fluid and the dynamic viscosity of the fluid,  $(\rho C_p)_p$  and  $(\rho C_p)_f$  are the heat capacity of the nanoparticle and fluid,  $D_B$  is the Brownian motion coefficient,  $D_T$  is the thermophoresis coefficient,  $\alpha = k_f / (\rho C_p)_f$  is the thermal diffusivity,  $\sigma$  is the electrical conductivity,  $Q^*$  is the temperature-dependent of the rate of heat generation when  $Q^* > 0$  and heat absorption  $Q^* < 0$  receptively and  $q_{rad}$  is the radiative flux.

Using Rosseland approximation for radiation (Hossain et al. [34]), we get

$$q_{rad} = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \tag{5}$$

where,  $\sigma^*$  is the Stefan-Boltzmann constant and  $k^*$  is the absorption coefficient of the fluid. Applying the expansion into the Taylor series, therefore  $T^4$  about  $T_\infty$  and ignoring higher order terms.

We obtain

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

Hence, the Equations (3) is modified to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k^* (\rho C_p)_f} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2 u^2}{(\rho C_p)_f} + \frac{\mu}{(\rho C_p)_f} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{Q^* (T - T_\infty)}{(\rho C_p)_f} + \frac{(\rho C_p)_p}{(\rho C_p)_f} \left[ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right] \tag{7}$$

The boundary conditions can be presented as

$$y = 0: \quad u = u_w = U_0 e^{x/L}, v = -V_w, T = T_w = T_0 + b e^{x/2L}, C = C_w = C_0 + a e^{x/2L}$$

$$y \rightarrow \infty: \quad u \rightarrow 0, v \rightarrow 0, T = T_\infty = T_0 + c e^{x/2L}, C = C_\infty = C_0 + d e^{x/2L} \tag{8}$$

By presenting the following similarity relations

$$\left. \begin{aligned} \eta &= \left( \frac{U_0}{2\nu_f L} \right)^{\frac{1}{2}} y e^{x/2L}, & u &= U_0 f'(\eta) e^{x/L}, \\ v &= - \left( \frac{\nu_f U_0}{2L} \right)^{\frac{1}{2}} e^{x/2L} \{ f(\eta) + \eta f'(\eta) \}, \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, & g(\eta) &= \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \right\} \quad (9)$$

So that Equation (1) is identically satisfied, and Equations (2), (4), and (7) are then obtained as follows

$$f''' + f f'' - 2f'^2 - M f' - K f' = 0 \quad (10)$$

$$\frac{1}{Pr} (1 + Rd)\theta'' + Ec \{ f''^2 + Mf'^2 \} - f' \{ S_t + \theta \} + \theta' f + \theta' \{ Nb g' + Nt\theta' \} + Q_H \theta = 0 \quad (11)$$

$$g'' + Sc \{ f g' - f' g - S_p f' \} + \left( \frac{Nt}{Nb} \right) \theta'' = 0 \quad (12)$$

with boundary conditions

$$\begin{aligned} f(0) = fw, \quad f'(0) = 1, \quad \theta(0) = 1 - S_p, \quad g(0) = 1 - S_t \quad \text{at } \eta = 0 \\ f'(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad g(\infty) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \end{aligned} \quad (13)$$

where  $M = \frac{2\sigma B_0^2 L}{\rho_f U_0}$  is the magnetic interaction parameter,  $K = \frac{\nu_f L}{K^* U_0 e^{x/L}}$  is the porosity parameter,  $Rd = \frac{16\sigma^* T_\infty^3}{3k^* k_{nf}}$  is the thermal radiation parameter,  $Ec = \frac{U_0^2 e^{2x/L}}{(c_p)(T_w - T_\infty)}$  is the viscous dissipation (Eckert number),  $S_t = \frac{c}{b}$  and  $S_p = \frac{d}{a}$  are the thermal stratification parameter and solutal stratification parameter, respectively;  $Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu}$  and  $Nt = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty}$  are the Brownian motion parameter and thermophoresis parameter,  $Sc = \frac{\nu}{D_B}$  is the Schmidt number,

$f_w = -V_w / \left( \sqrt{\frac{\alpha \nu}{2L}} \right)$  for  $f_w > 0$  corresponds to suction and  $Q_H = \frac{2Q^*L}{U_0(\rho C_p)_f}$  for  $Q_H > 0$  corresponds to heat generation and for  $Q_H < 0$  corresponds to heat absorption.

The local skin friction coefficient  $C_f$ , rate of heat transfer  $Nu_x$ , and Sherwood number  $Sh_x$  are defined by the relations,

$$C_f = \frac{2 \tau_w|_{y=0}}{\rho U_0^2 e^{2x/L}}, \quad Nu_x = \frac{-x q_w}{k(T_w - T_\infty)}, \quad Sh = \frac{-x q_m}{D_B(C_w - C_\infty)} \quad (14)$$

Using similarity transformations Eq. (9), we get

$$\left. \begin{aligned} C_f \left( \frac{Re_x}{2} \right)^{\frac{1}{2}} &= f''(0), \\ Nu_x Re_x^{-\frac{1}{2}} \left( \frac{2L}{x} \right)^{\frac{1}{2}} &= -\frac{1}{1-St} \theta'(0), \\ Sh Re_x^{-\frac{1}{2}} \left( \frac{2L}{x} \right)^{\frac{1}{2}} &= -\frac{1}{1-S_p} g'(0) \end{aligned} \right\} \quad (15)$$

where  $Re_x = U_0 x / \nu e^{-x/L}$  is the local Reynolds number.

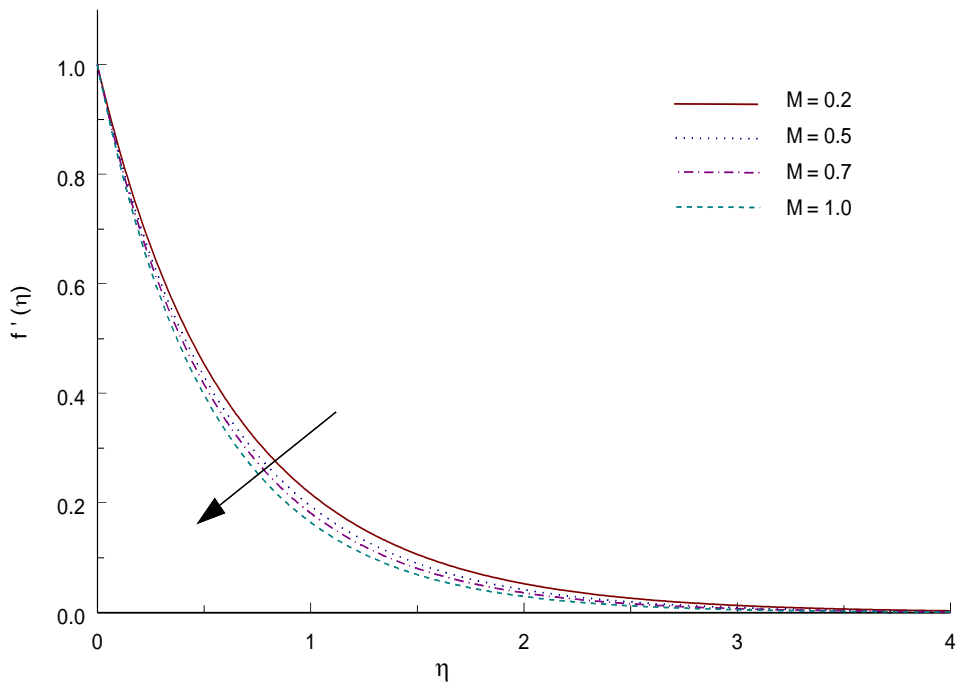
### 3. NUMERICAL SOLUTION OF THE PROBLEM

The numerical solution to system of nonlinear ordinary differential equations (10), (11), and (12) with the conditions (13) have been solved using Nachesheim-Swigert shooting iteration technique with employing the Runge-Kutta fourth-order method. The most significant thing to be regarded here is that the initial guesses for values of  $f''(0)$ ,  $\theta'(0)$  and  $g'(0)$  to begin the shooting procedure are to be made. The success of the procedure depends very much on how these guesses are. In this process we have to selected a appropriate finite value of  $\eta \rightarrow \infty$ , say  $\eta_\infty$ . The convergency is taken upto  $10^{-5}$ . Numerical results of dimensionless velocity, temperature, and concentration profiles are found and are given graphically.

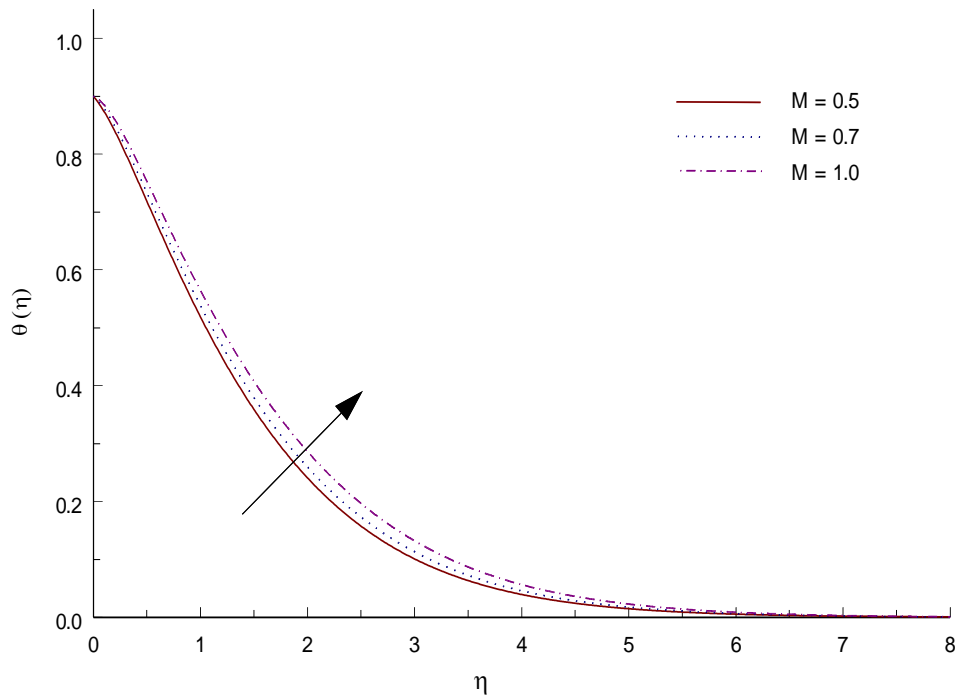
### 4. RESULTS AND DISCUSSION

Let us discuss the behavior of several parameters on the velocity, temperature and concentration profiles. In Fig. 1 depicts to examined the impacts of the magnetic interaction parameter  $M$  on the dimensionless velocity profile  $f'(\eta)$ , with enhancing values of  $M$ . It shows that velocity was reduces, because for such decrement in  $f'(\eta)$  was described by Lorenz

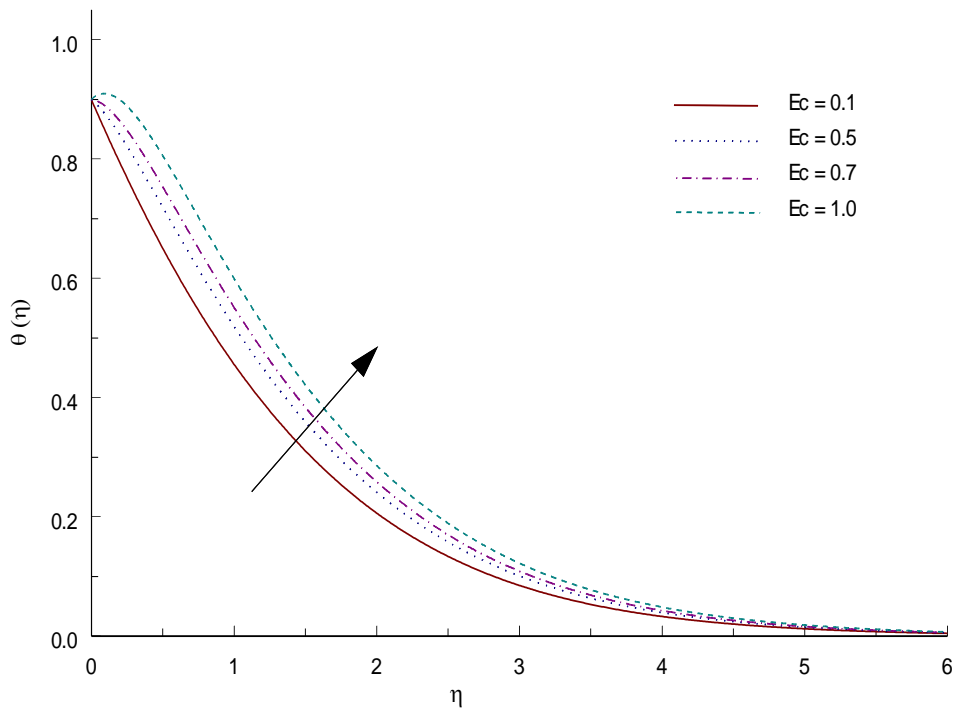
force. Fig. 2 presents the temperature for different values of  $M$ , in the presence of  $Pr = 1.7$ . The temperature distribution of the nanofluid increases with enhance in  $M$ .



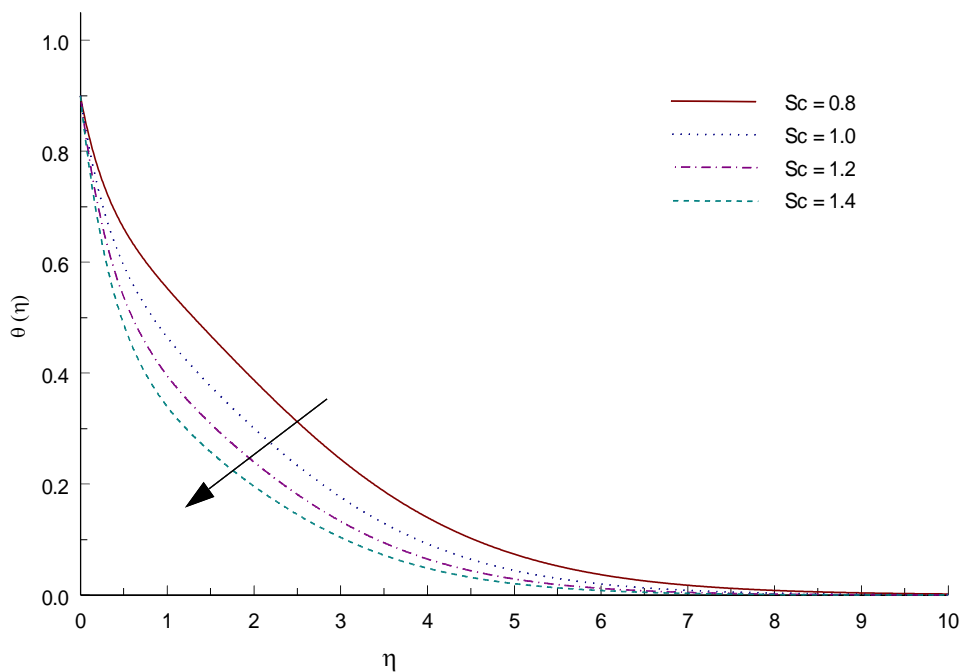
**Fig. 1.** Effect of magnetic interaction parameter  $M$  on velocity profiles.



**Fig. 2.** Effect of magnetic interaction parameter  $M$  on temperature distribution.



**Fig. 3.** Effect of eckert number  $Ec$  on temperature distribution.

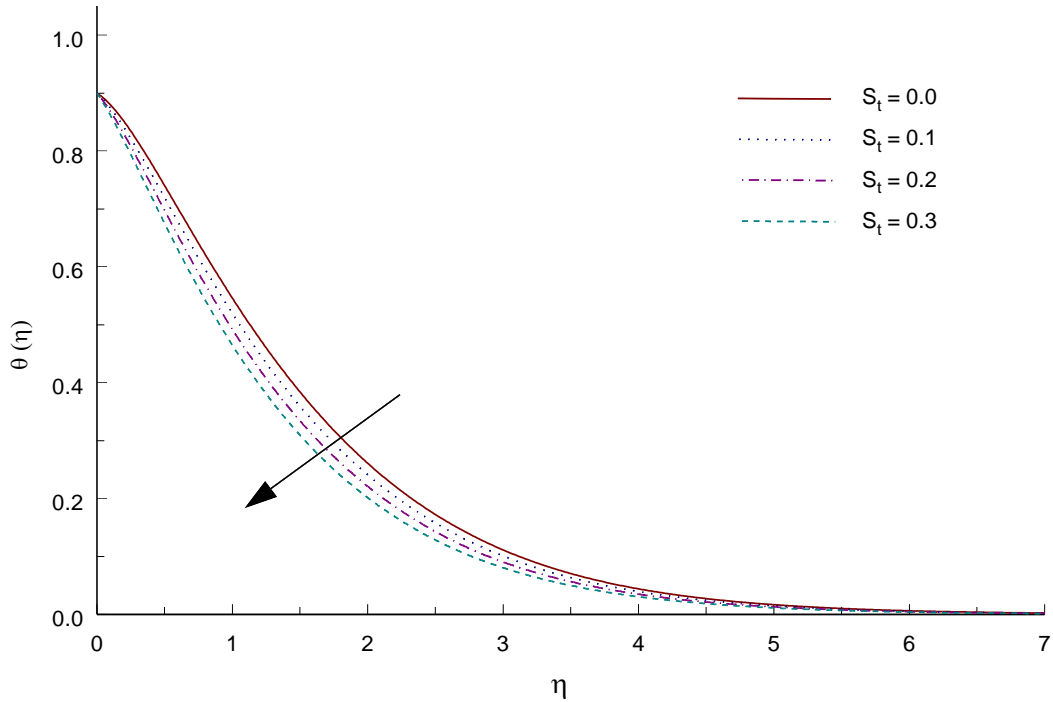


**Fig. 4.** Effect of schmidt number  $Sc$  on temperature distribution.

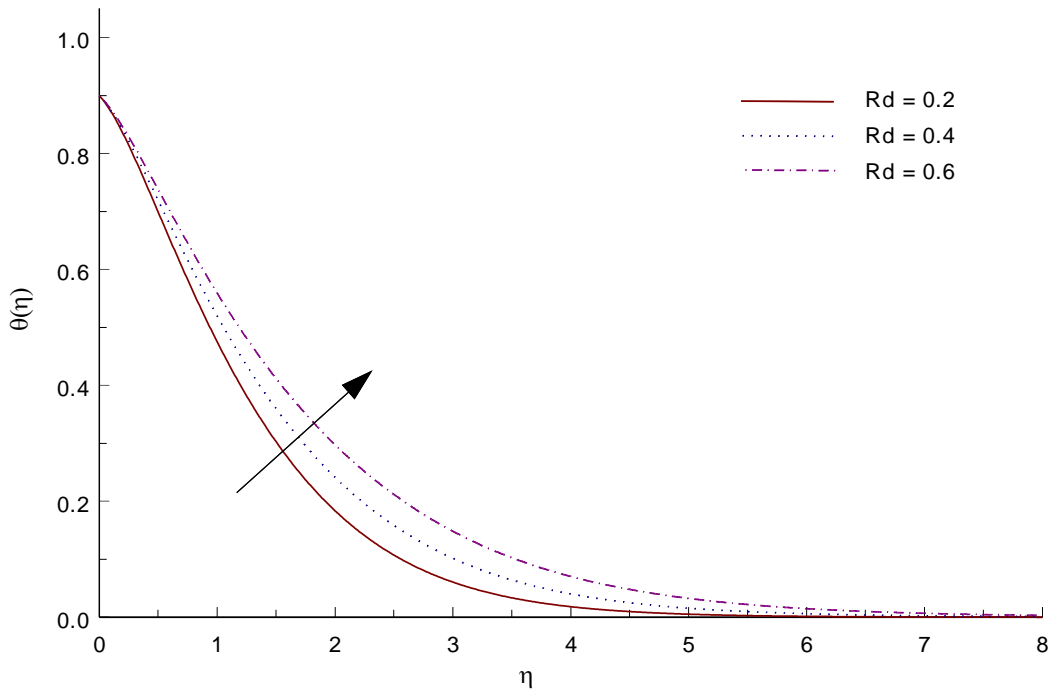
Fig. 3 shows that the influence of  $Ec$  on temperature field. It reflects that the  $\theta(\eta)$  enhancing with an increase in viscous dissipation  $Ec$ . It generates heat due to drag amidst the



fluid particles. Effect of thermal stratification parameter  $Sc$  on the temperature field in a flow is displayed in Fig. 4. It will be reduced with the increasing value of  $Sc$ . Fig. 5 displays the temperature profile for different values of  $S_t$ . It is observed that the temperature of the fluid is a

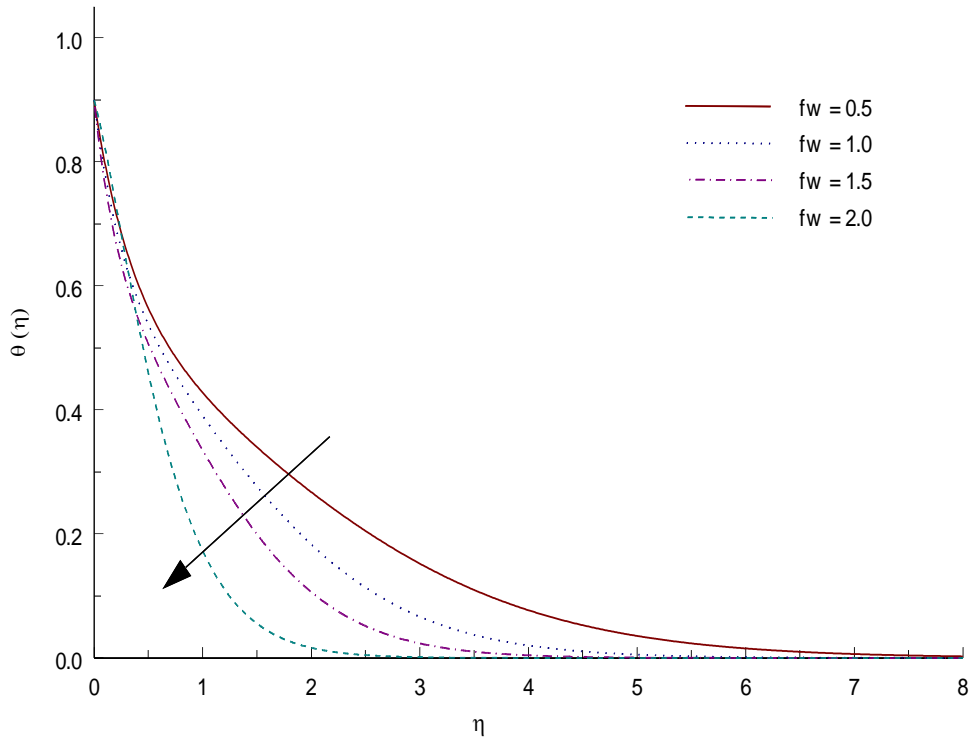


**Fig. 5.** Effect of thermal stratification  $S_t$  on temperature distribution.

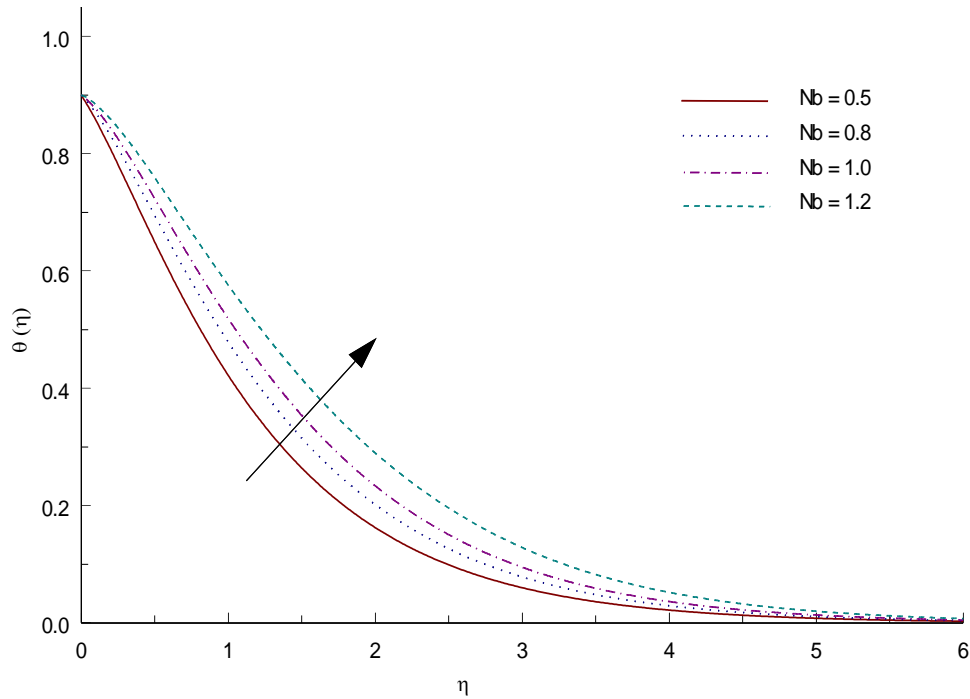


**Fig. 6.** Effect of thermal radiation  $Rd$  on temperature distribution.

decrement of enhance in thermal stratification parameter. The influence of thermal radiation on  $\theta(\eta)$  is portrayed in Fig. 6. An enhance in values of  $Rd$  raises the heat flux from nonlinear

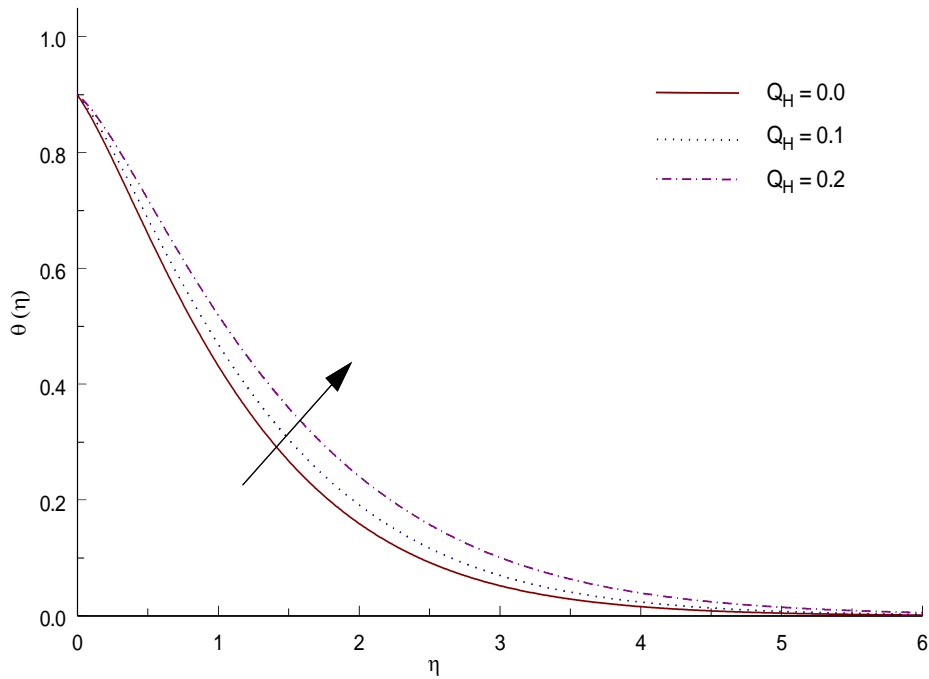


**Fig. 7.** Effect of suction parameter  $fw$  on temperature distribution.

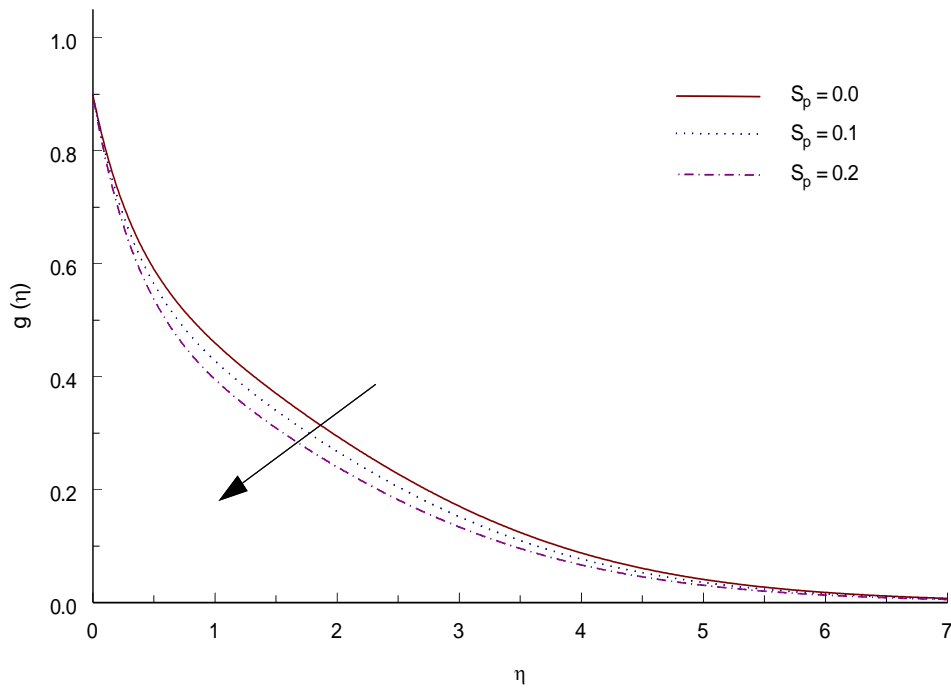


**Fig. 8.** Effect of brownian motion  $Nb$  on temperature distribution.

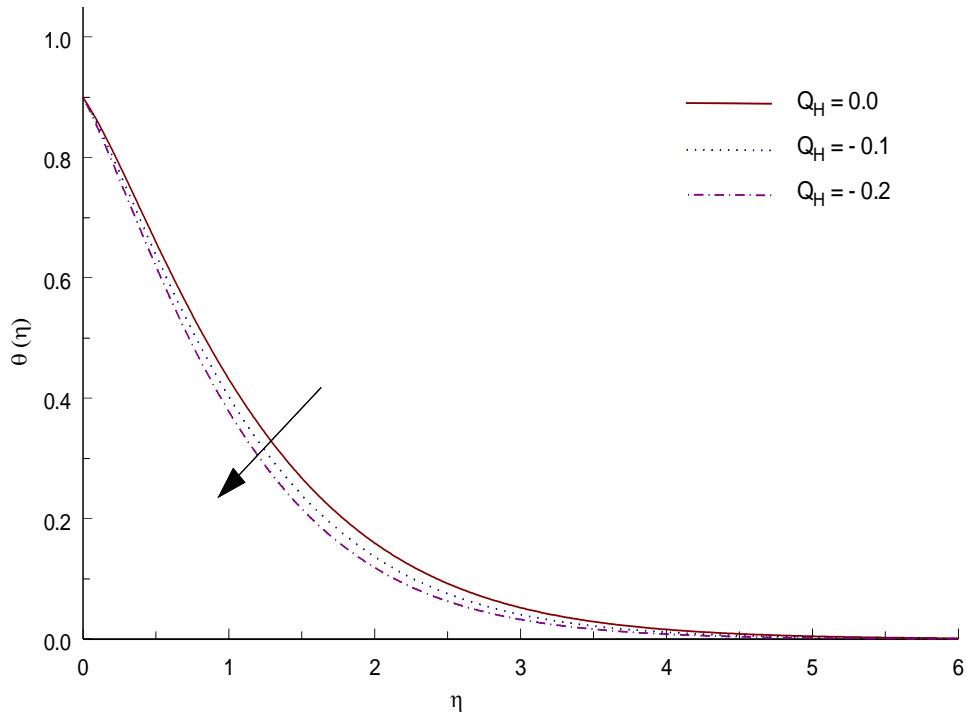
temperature. Fig. 7 reveals the impact of suction parameter  $f_w$  on temperature of the nanofluid. It is noted that the enhanced values of  $f_w$ , the dimensionless temperature reduced. Also, thermal boundary layer thickness and surface temperature is decrement.



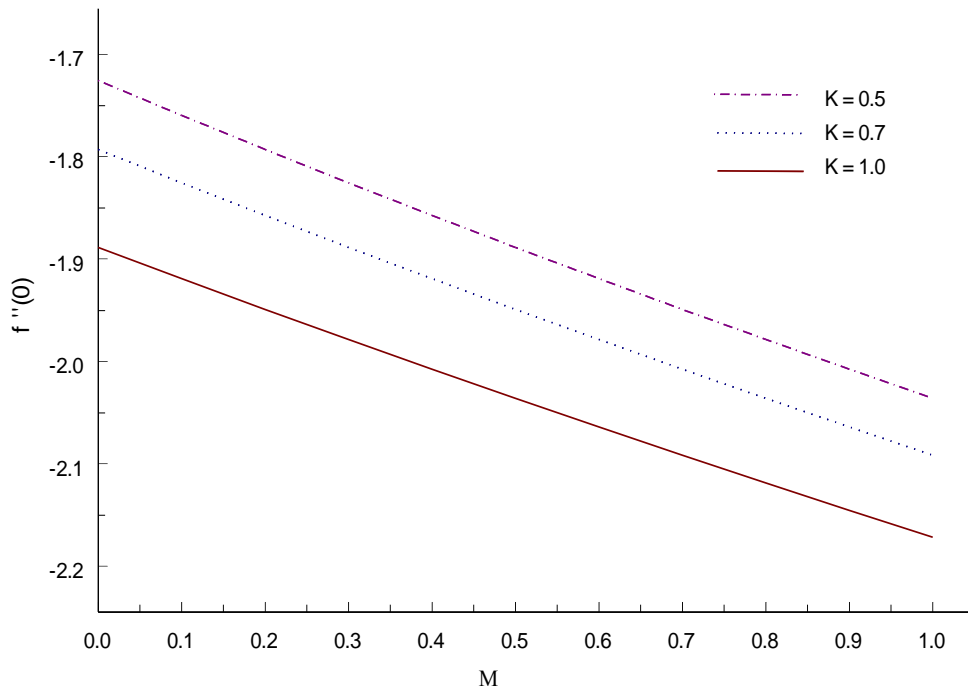
**Fig. 9.** Effect of heat generation  $Q_H$  on temperature distribution



**Fig. 10.** Effect of solutal stratification  $S_p$  on concentration profiles



**Fig. 11.** Effect of heat absorption  $Q_H$  on temperature distribution



**Fig. 12.** Effects  $M$  and  $K$  on local skin friction coefficient

Fig. 8 shows the influence of Brownian motion parameter  $Nb$  on temperature field on several parameters taken values are  $M = 0.5, K = 0.1, Rd = 0.2, Q_H = 0.2, fw = 0.5$  and

$Pr = 1.7$ . It is possible to see that the temperature graph enhanced and increase in thermal boundary thickness of  $Nb$  values are increases. Fig. 9 depicts that the temperature field increases, when heat generation  $Q_H$  increases. Both increases of  $Q_H$  and thermal boundary layer thickness.

Fig. 10 shows the effect of solutal stratification parameter  $S_p$  on the concentration field in different values of  $Nb = Nt = 0.8$ ,  $K = 0.1$ ,  $Rd = 0.2$ ,  $fw = 0.5$ , and  $Sc = 0.1$ . It is observed that when  $S_p$  increases, the concentration field and mass boundary layer thickness reduced. Fig. 11 shows the heat absorption on temperature field. It is seen that decreases of  $\theta(\eta)$  with increasing values of  $Q_H$ . Fig. 12 gives the skin-friction  $-f''(0)$  against magnetic interaction parameter  $M$  for several values of porosity parameters  $K$ . The effect of  $M$  is to reduced the skin-friction, also its decreases with  $K$  increase.

Table 1 is the comparison values of  $-f''(0)$  with various values in compared with previously published papers. It is clearly remark that authors results are very good agreement with that of Magyari and Keller [35].

Table 2 and 3 portrayed the rate of heat (nusselt number) and mass (Sherwood number) transfer. Non-dimensional rate of heat transfer decreases in magnitude for increasing radiation parameter and Eckert number. Non-dimensional rate of mass transfer increases in magnitude for increasing  $Sc$ ,  $S_p$  and  $Nb$  and reversed with increase of  $S_i$ .

**Table 1.** Comparison of results for  $-f''(0)$  to previously published data values at  $M = fw = Rd = Ec = K = 0$

Values of	Magyari and Keller [35]	Present Study
$-f''(0)$	1.281808	1.28181
$f(\infty)$	0.905639	0.90565

**Table 2.** Nusselt number for  $K = 0.1$ ,  $Pr = 1.7$ ,  $Nb = Nt = 0.8$ ,  $fw = 0.5$  and different values of  $R_d$ ,  $Ec$ ,  $M$ ,  $Q_H$  and  $S_i$

$Rd$	$Ec$	$M$	$Q_H$	$S_i$	$-\theta'(0)$	$-\theta'(0)/(1-S_i)$
0.2	0.5	0.5	0.2	0.1	0.20907	0.23223
0.4					0.20660	0.22955
0.6					0.19572	0.21746
0.4	0.1	0.5	0.2	0.1	0.54684	0.60759
	0.5				0.20660	0.22955
	0.7				0.03461	0.03845
	1.0				-0.22616	-0.25218

0.4	0.5	0.2	0.2	0.1	0.35068	0.38964
		0.5			0.20660	0.22955
		0.7			0.15104	0.16782
		1.0			0.07112	0.07902
0.4	0.5	0.5	-0.2	0.1	0.46420	0.51577
			-0.1		0.41114	0.45681
			0.0		0.35282	0.39201
			0.1		0.28665	0.31849
			0.2		0.20660	0.22955
0.4	0.5	0.5	0.2	0.0	0.14227	0.14227
				0.1	0.20660	0.22955
				0.2	0.27362	0.34202
				0.3	0.34325	0.48035

**Table 3.** Sherwood number for  $K = 0.1$ ,  $Pr = 1.7$ ,  $Nt = 0.8$ ,  $fw = 0.5$  and different values of  $Sc$ ,  $S_p$ ,  $S_t$  and  $Nb$

$Sc$	$S_p$	$S_t$	$Nb$	$-g'(0)$	$-g'(0)/(1-S_p)$
0.8	0.1	0.1	0.8	0.82772	0.91968
1.0				1.04344	1.15936
1.2				1.24169	1.37964
1.4				1.42718	1.58574
1.1	0.0	0.1	0.8	1.04892	1.04892
	0.1			1.14441	1.27155
	0.2			1.24066	1.55082
1.1	0.1	0.0	0.2	1.32723	1.47468
		0.1		1.14441	1.27155
		0.2		0.95911	1.06566
		0.3		0.77125	0.85693
1.1	0.1	0.1	0.5	1.15763	1.28624
			0.8	1.16512	1.29456
			1.0	1.18681	1.31866
			1.2	1.21953	1.35502

## 5. CONCLUSIONS

In this paper, the influence of viscous dissipation and joules heating (Ohmic dissipation) on magnetohydrodynamic flow of a double stratified nanofluid past an exponentially stretching sheet has been studied. Radiative nanofluid and heat generation/absorption impacts are taken into account. The numerical results are portrayed for several parameters like thermal radiation parameter, Eckert number, magnetic interaction parameter, heat generation/absorption, schmidt number and suction parameter. Moreover, the use of a double stratification of thermal and

solubility boundary conditions of this survey makes a novel. These doubly stratification in the presence of nanofluid has been more interest in various thermal applications.

The important results of this examination are listed as follows:

- The influence of magnetic interaction parameter  $M$  increases, dimensionless velocity profile reduces and dimensionless temperature field enhances.
- It is noted that Schmidt number and thermal stratification parameter decreases the dimensionless temperature, while the solubility stratification parameter decreases the dimensionless concentration.
- The increase in the thermal stratification parameter  $S_t$  impacted in lower temperature fields, but rise in the concentration of the nanofluid.
- An enhance in thermal radiation parameter decreases for nusselt number.
- Rate of heat transfer raises with an increase of both Schmidt number  $Sc$  and Brownian motion  $Nb$ .
- The thickness of thermal boundary layer increases with an increase in the values of  $Nt = Nb$ .
- An increase in  $M, Ec, K, Nb$  yields an increase in dimensionless temperature, whereas the reverse is noted with increases in  $f_w$  and  $S_t$ .

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