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Thermodynamical analysis for a variable generalized Chaplygin gas

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ABSTRACT

Considering the work of Panigrahi (2015) and Panigrahi and Chatterjee (2016), in this paper we study some properties of variable generalized Chaplygin gas (VGCG) model, exotic matter used in some cosmological theories whose equation of state is $P = -B / \rho^\alpha$ where $B = B_0 V^{-n/3}$. We obtain an analytical expression for Joule-Thomson coefficient and we have deduced the inversion temperature for variable and variable generalized Chaplygin gas. We investigate the thermodynamical behavior for VGCG model in different process.

Keywords: Variable generalized Chaplygin gas, Variable Chaplygin gas, Exotic matter, Joule-Thomson coefficient, Inversion temperature

1. INTRODUCTION

The evidences of the cosmic acceleration of the universe [1-3] has caused a primordial challenge in the fundamental theories of physics and cosmology. This acceleration is frequently attributed to a hypothetical energy called dark energy and the Chaplygin type of gas cosmology is one of the most reasonable explanations of this recent phenomena. The form

of the Chaplygin gas equation of state is the following $P = -\frac{B}{\rho^\alpha}$ where P is the pressure of the fluid, ρ is the energy density of the fluid and B is a constant.

The thermodynamical behaviour of the Chaplygin gas model was studied by Santos et al. [4], Guo and Zhang [5], Myung [6], Panigrahi [7] and Malaver [8,9]. Santos et. al. [4] have studied the stability in Chaplygin gas model and determined that the variable generalized Chaplygin gas (VGCG) is thermodynamically stable during any expansion process. Guo and Zhang [5] proposed a new generalized Chaplygin gas model that includes the original Chaplygin gas (CG) model as a special case and found that the background evolution for the model is equivalent to that for a coupled dark energy model with dark matter.

Myung [6] obtains a new general equation of state that describes the Chaplygin gas completely and confirms that the CG model could show a unified picture of dark energy and energy which cools down through the universe expansion without any critical point. Panigrahi [7] concludes that the volume increases when temperature falls during adiabatic expansions in the variable Chaplygin gas (VCG) model, which also is observed in an ideal gas [10]. Malaver [8,9] respectively, found that the thermodynamic efficiency of Carnot cycle for CG model only depends on the limits of maximum and minimal temperature as in the case of the ideal gas and the photons gas and that the adiabatic compressibility for this model only will depend on the pressure and the thermal capacity at constant pressure C_p is always positive. More recently, Panigrahi and Chatterjee [11] have shown that VGCG model satisfies the third law of thermodynamics. Ökcü and Aydinler [12] studied the Joule-Thomson effects for charged AdS black holes and obtained inversion temperatures and curves.

In this paper an expression is deduced for the Joule-Thomson coefficient μ_{JT} from the thermal equation of state of the VGCG model. With the equation for μ_{JT} we obtain the condition for the inversion temperature in this model. We also studied and derived expressions as functions of temperature, pressure and volume for different thermodynamical processes.

The article is organized as follows: in Section 2, it presents the definition of the Joule-Thomson coefficient; in Section 3, we show the deduction for the Joule-Thomson coefficient and the condition of the inversion temperature for a Chaplygin gas; in Section 4, we have derived thermal equations for the VGCG model for some thermodynamical processes; in Section 5, we conclude.

2. THE JOULE-THOMSON COEFFICIENT

In this section, we present the coefficient of Joule-Thomson expansion [12,13]. The Joule-Thomson expansion is a throttled process where a gas at a high pressure passes through a porous valve to a region with low pressure in a thermally insulated tube under conditions of constant enthalpy. The change of the temperature with the pressure is given by

$$\mu_{JT} = \left(\frac{\partial T}{\partial P} \right)_H \tag{1}$$

The quantity (1) is called Joule-Thomson coefficient. We can express $(\partial H/\partial P)_H$ in function of heat capacity at constant pressure C_p and $(\partial H/\partial P)_T$. As the enthalpy is a state function that depends of pressure and temperature, we can write:

$$H = H(P, T) \quad \text{and}$$

$$dH = \left(\frac{\partial H}{\partial P}\right)_T dP + \left(\frac{\partial H}{\partial T}\right)_P dT \quad (2)$$

Following Dickerson [13], the Joule-Thomson coefficient μ_{JT} can be obtained deriving (2) with respect to pressure under conditions of constant enthalpy:

$$\left(\frac{\partial H}{\partial P}\right)_H = \left(\frac{\partial H}{\partial P}\right)_T + \left(\frac{\partial H}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_H \quad (3)$$

Substituting $(\partial H/\partial P)_H=0$, $C_p = (\partial H/\partial T)_P$ and $\mu_{JT}=(\partial T/\partial P)_H$ in (3)

$$0 = \left(\frac{\partial H}{\partial P}\right)_T + C_p \mu_{JT} \quad (4)$$

Rearranging the equation (4),

$$\left(\frac{\partial H}{\partial P}\right)_T = -C_p \mu_{JT} \quad (5)$$

and μ_{JT} can be written as

$$\mu_{JT} = -\frac{1}{C_p} \left(\frac{\partial H}{\partial P}\right)_T \quad (6)$$

With the definition of enthalpy H it is possible calculate $(\partial H/\partial P)_T$. H is given by

$$H = U + PV \quad (7)$$

With (7) we obtain

$$\left(\frac{\partial H}{\partial P}\right)_T = \left(\frac{\partial U}{\partial P}\right)_T + \left(\frac{\partial(PV)}{\partial P}\right)_T \quad (8)$$

$\left(\frac{\partial(PV)}{\partial P}\right)_T$ can express as

$$\left(\frac{\partial(PV)}{\partial P}\right)_T = \left(\frac{\partial(PV)}{\partial V}\right)_T \left(\frac{\partial V}{\partial P}\right)_T \quad (9)$$

and

$$\left(\frac{\partial U}{\partial P}\right)_T = \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial P}\right)_T \quad (10)$$

Replacing (9) and (10) in (8), μ_{JT} can be written as follow

$$\mu_{JT} = -\frac{1}{C_P} \left(\frac{\partial H}{\partial P}\right)_T = -\frac{1}{C_P} \left[\left(\frac{\partial U}{\partial V}\right)_T + \left(\frac{\partial(PV)}{\partial V}\right)_T \right] \left(\frac{\partial V}{\partial P}\right)_T \quad (11)$$

At the inversion temperature $(\partial H/\partial P)_T = 0$ and $\mu_{JT} = 0$. Since $(\partial H/\partial P)_T = 0$ for an ideal gas, according to the eq. (6), the Joule-Thomson coefficient for an ideal gas is always zero [13].

3. THE INVERSION TEMPERATURE IN AN CHAPLYGIN GAS

In this section, we have determined the inversion temperature of Joule-Thomson expansion for the VCG and VGCG models. For the VCG model, the thermal equation of state for the pressure [7] is given by

$$P = -\left(\frac{B_0 N}{2}\right)^{1/2} V^{-n/6} \left(1 - \frac{T^2}{\tau^2}\right)^{1/2} \quad (12)$$

where B_0 is a positive universal constant, n is a constant, $N = \frac{6-n}{3}$ and τ is a universal constant with dimension of temperature.

For the internal energy as function of T and V

$$U = \left(\frac{2B_0}{N}\right)^{1/2} V^{N/2} \left(1 - \frac{T^2}{\tau^2}\right)^{-1/2} \quad (13)$$

With the eq. (12), we obtain

$$\left(\frac{\partial P}{\partial V}\right)_T = \frac{n}{6} \left(\frac{B_0 N}{2}\right)^{1/2} V^{-\frac{n}{6}-1} \left(1 - \frac{T^2}{\tau^2}\right)^{1/2} \quad (14)$$

That implies that

$$\left(\frac{\partial V}{\partial P}\right)_T = \frac{1}{\frac{n}{6} \left(\frac{B_0 N}{2}\right)^{1/2} V^{-\frac{n}{6}-1} \left(1 - \frac{T^2}{\tau^2}\right)^{1/2}} \quad (15)$$

From the eq. (13) we have deduced

$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{N}{2} \left(\frac{2B_0}{N}\right)^{1/2} V^{\frac{N}{2}-1} \left(1 - \frac{T^2}{\tau^2}\right)^{\frac{1}{2}} \quad (16)$$

and again for (12)

$$\left(\frac{\partial(PV)}{\partial V}\right)_T = -\left(\frac{6-n}{n}\right) \left(\frac{B_0 N}{2}\right)^{1/2} V^{\frac{6-n}{6}-1} \left(1 - \frac{T^2}{\tau^2}\right)^{1/2} \quad (17)$$

Substituting (15), (16) and (17) in (11) and rearranging terms we have

$$\mu_{JT} = \frac{2}{(N-2)C_p} \left[\frac{1 - \frac{N}{2} \left(1 - \frac{T^2}{\tau^2}\right)}{1 - \frac{T^2}{\tau^2}} \right] V \quad (18)$$

The expression for Joule-Thomson coefficient (18) is an explicit function of the temperature and the volume. In the inversion temperature T_i , $\mu_{JT} = 0$ that is

$$T_i = \tau \left(\frac{N-2}{N}\right)^{1/2} \quad (19)$$

With the objective of determining T_i in VGCG model, the thermal equation of state [11] is

$$P = - \left(B_0 V^{-\frac{n}{3}} \right)^{\frac{1}{1+\alpha}} \left(\frac{N}{1+\alpha} \right)^{\frac{\alpha}{1+\alpha}} \left(1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right)^{\frac{\alpha}{1+\alpha}} \quad (20)$$

and for the internal energy

$$U = V \left[\frac{B_0 (1+\alpha) V^{-\frac{n}{3}}}{N} \right]^{\frac{1}{1+\alpha}} \left[1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right] \quad (21)$$

With (20) and (21) we obtain the following expressions:

$$\left(\frac{\partial V}{\partial P} \right)_T = \frac{3(1+\alpha)}{n B_0^{\frac{1}{1+\alpha}} V^{-\frac{n}{3(1+\alpha)} - 1}} \left(\frac{1+\alpha}{N} \right)^{\frac{\alpha}{1+\alpha}} \frac{1}{\left[1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right]^{\frac{\alpha}{1+\alpha}}} \quad (22)$$

$$\left(\frac{\partial U}{\partial V} \right)_T = \left[\frac{3(1+\alpha) - n}{3(1+\alpha)} \right] V^{\frac{3(1+\alpha) - n}{3(1+\alpha)} - 1} \frac{\left[\frac{B_0 (1+\alpha)}{N} \right]^{\frac{1}{1+\alpha}}}{\left[1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right]^{\frac{1}{1+\alpha}}} \quad (23)$$

$$\left(\frac{\partial(PV)}{\partial V} \right)_T = - \left[\frac{3(1+\alpha) - n}{3(1+\alpha)} \right] V^{\frac{3(1+\alpha) - n}{3(1+\alpha)} - 1} B_0^{\frac{1}{1+\alpha}} \left(\frac{N}{1+\alpha} \right)^{\frac{\alpha}{1+\alpha}} \left(1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right)^{\frac{\alpha}{1+\alpha}} \quad (24)$$

Again, replacing (22), (23) and (24) in eq. (11), we obtain

$$\mu_{JT} = -\frac{(1+\alpha)}{((1+\alpha)-N)C_p} \left[\frac{1}{1-\left(\frac{T}{\tau}\right)^{\frac{1+\alpha}{\alpha}}} - \frac{N}{(1+\alpha)} \right] V \quad (25)$$

and the inversion temperature T_i is given by

$$T_i = \tau \left(\frac{N - (1+\alpha)}{N} \right)^{\alpha/(1+\alpha)} \quad (26)$$

For $\alpha = 1$ in the equations (20) and (21) we have the expressions for P and U of work of Panigrahi [7] with the VCG model and the values of μ_{JT} and T_i of (18) and (19).

4. PROCESSES IN A CHAPLYGIN GAS

We will consider now some thermodynamical processes in the VGCG model and we have derived useful expressions for the study of these processes.

A. Reversible Isothermal Processes

Following Dickerson [13], in an reversible isothermal process, there is no temperature change and $P_{ext} = P_{gas} = P$. The thermodynamical variable internal energy $U(V,T)$ can be expressed by

$$dU = \left(\frac{\partial U}{\partial V} \right)_T dV + \left(\frac{\partial U}{\partial T} \right)_V dT \quad (27)$$

Then $dT = 0$ and dU takes the form

$$dU = \left(\frac{\partial U}{\partial V} \right)_T dV \quad (28)$$

With (23) and considering $N = (3(1+\alpha)-n)/3$ in according to Panigrahi and Chatterjee [11], the eq. (28) can be integrated and we obtain

$$\Delta U = \left[\frac{B_0(1+\alpha)}{N} \right]^{\frac{1}{1+\alpha}} \left[1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right]^{\frac{1}{1+\alpha}} \left(V_2^{\frac{N}{1+\alpha}} - V_1^{\frac{N}{1+\alpha}} \right) \quad (29)$$

For the first law of the thermodynamics

$$dU = dQ + dW \quad (30)$$

Substituting (29) in (30), we have for dq

$$dq = \left[P + \left(\frac{\partial U}{\partial V} \right)_T \right] dV \quad (31)$$

With the well known thermodynamical relation

$$\left(\frac{\partial U}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V - P \quad (32)$$

The eq. (31) can be written as

$$dq = \left[T \left(\frac{\partial P}{\partial T} \right)_V \right] dV \quad (33)$$

and with the expression (20) for P for the VGCG model, we have

$$T \left(\frac{\partial P}{\partial T} \right)_V = \left(B_0^{\frac{n}{3}} \right)^{\frac{1}{1+\alpha}} \left(\frac{N}{1+\alpha} \right)^{\frac{\alpha}{1+\alpha}} \left[1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right]^{\frac{\alpha}{\alpha+1}-1} \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \quad (34)$$

for replacement of (34) in (33) and integrating we obtain

$$q = \left(\frac{N}{1+\alpha}\right)^{\frac{\alpha}{\alpha+1}} \frac{B_0^{\frac{1}{1+\alpha}}(1+\alpha)}{N} \left(\frac{T}{\tau}\right)^{\frac{\alpha+1}{\alpha}} \frac{\left(V_2^{\frac{N}{1+\alpha}} - V_1^{\frac{N}{1+\alpha}}\right)}{\left[1 - \left(\frac{T}{\tau}\right)^{\frac{\alpha+1}{\alpha}}\right]^{\frac{1}{\alpha+1}}} \quad (35)$$

We have used the convention of Wark and Richards [14] that defines the work during a reversible process as

$$W = -\int PdV \quad (36)$$

The work for an isothermal reversible process in an VGCG model is given by

$$W = \left[\frac{(1+\alpha)B_0}{N}\right]^{\frac{1}{1+\alpha}} \left[1 - \left(\frac{T}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right]^{\frac{\alpha}{1+\alpha}} \left(V_2^{\frac{N}{1+\alpha}} - V_1^{\frac{N}{1+\alpha}}\right) \quad (37)$$

Again, with $\alpha = 1$ in (29), (35) and (37), we can recover the expressions for ΔU , q and W deduced for Malaver [8] with the VCG model.

B. Reversible Adiabatic Processes

An adiabatic process is one in which there is no heat flow in or out of the system [13]. In this case $Q = 0$ and for the eq. (30)

$$dU = dW = C_v dT = -PdV \quad (38)$$

C_v is the thermal heat capacity at the constant volume

With (27), (32) and (38) we obtain

$$T \left(\frac{\partial P}{\partial T}\right)_V dV + C_v dT = 0 \quad (39)$$

In the VGCG model [11]

$$C_V = \frac{\left[\frac{B_0(1+\alpha)V^N}{N} \right]^{\frac{1}{1+\alpha}} \left(\frac{T}{\tau^{1+\alpha}} \right)^{\frac{1}{\alpha}}}{\alpha \left\{ 1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right\}^{\frac{2+\alpha}{1+\alpha}}} \quad (40)$$

Replacing eq. (34) and eq. (40) in (39) we have

$$\frac{N}{1+\alpha} \frac{dV}{V} + \frac{dT}{\alpha T \left[1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right]} = 0 \quad (41)$$

Integrating

$$N \ln \frac{V_2}{V_1} = \ln \left[\frac{\left(\frac{\tau}{T_2} \right)^{\frac{1+\alpha}{\alpha}} - 1}{\left(\frac{\tau}{T_1} \right)^{\frac{1+\alpha}{\alpha}} - 1} \right] \quad (42)$$

Then for a reversible adiabatic process in an VGCG model we obtain

$$\frac{V}{\left[1 - \left(\frac{\tau}{T} \right)^{\frac{1+\alpha}{\alpha}} \right]^{\frac{1}{N}}} = \text{const} \quad (43)$$

We can deduce also an expression for an adiabatic process in terms of P and T . With the eq. (20),

$$P_1 = - \left(B_0 V_1^{-\frac{n}{3}} \right)^{\frac{1}{1+\alpha}} \left(\frac{N}{1+\alpha} \right)^{\frac{\alpha}{1+\alpha}} \left(1 - \left(\frac{T_1}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right)^{\frac{\alpha}{1+\alpha}} \quad (44)$$

and

$$P_2 = -\left(B_0 V_2^{-\frac{n}{3}}\right)^{\frac{1}{1+\alpha}} \left(\frac{N}{1+\alpha}\right)^{\frac{\alpha}{1+\alpha}} \left(1 - \left(\frac{T_2}{\tau}\right)^{\frac{1+\alpha}{\alpha}}\right)^{\frac{\alpha}{1+\alpha}} \quad (45)$$

Dividing (44) and (45) we have

$$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^{-\frac{n}{3(1+\alpha)}} \left(\frac{1 - \left(\frac{T_1}{\tau}\right)^{\frac{1+\alpha}{\alpha}}}{1 - \left(\frac{T_2}{\tau}\right)^{\frac{1+\alpha}{\alpha}}}\right)^{\frac{\alpha}{1+\alpha}} \quad (46)$$

By substituting of (42) in (46), we obtain an equivalent expression in function of P and T

$$\frac{P_1 T_1^{\frac{N-(1+\alpha)}{N}}}{\left[\left(\frac{T_1}{\tau}\right)^{\frac{1+\alpha}{\alpha}} - 1\right]^{\frac{N-1}{N}}} = \frac{P_2 T_2^{\frac{N-(1+\alpha)}{N}}}{\left[\left(\frac{T_2}{\tau}\right)^{\frac{1+\alpha}{\alpha}} - 1\right]^{\frac{N-1}{N}}} \quad (47)$$

The eq. (47) implies that

$$\frac{PT^{\frac{N-(1+\alpha)}{N}}}{\left[\left(\frac{T}{\tau}\right)^{\frac{1+\alpha}{\alpha}} - 1\right]^{\frac{N-1}{N}}} = \text{const} \quad (48)$$

when $\alpha = 1$ we obtain the expressions for adiabatic process in the VCG model [8,9].

C. Reversible Isochoric Processes

The isochoric process is one that is carried out to constant volume. For this case $dV = 0$ and $dW = 0$. For dU we have

$$dU = dQ = \left(\frac{\partial U}{\partial T} \right)_V dT \tag{49}$$

But we know that

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V \tag{50}$$

Replacing eq.(40) in (49) and integrating we obtain

$$Q = \Delta U = \left[\frac{B_0(1+\alpha)V^N}{N \left(1 - \left(\frac{T_1}{\tau} \right)^{\frac{\alpha+1}{\alpha}} \right) \left(1 - \left(\frac{T_2}{\tau} \right)^{\frac{\alpha+1}{\alpha}} \right)} \right]^{\frac{1}{1+\alpha}} \left[\left(1 - \left(\frac{T_1}{\tau} \right)^{\frac{\alpha+1}{\alpha}} \right)^{\frac{1}{1+\alpha}} - \left(1 - \left(\frac{T_2}{\tau} \right)^{\frac{\alpha+1}{\alpha}} \right)^{\frac{1}{1+\alpha}} \right] \tag{51}$$

with the expression for C_V of VCG model [7]

$$C_V = \left(\frac{2B_0}{N} \right)^{\frac{1}{2}} V^{\frac{N}{2}} \frac{T}{\tau^2} \frac{1}{\left(1 - \frac{T^2}{\tau^2} \right)^{\frac{3}{2}}} \tag{52}$$

we have deduced

$$Q = \Delta U = \left(\frac{2B_0}{N} \right)^{\frac{1}{2}} V^{N/2} \left(\frac{1}{\left(1 - \frac{T_2^2}{\tau^2} \right)^{\frac{1}{2}}} - \frac{1}{1 - \left(\frac{T_1^2}{\tau^2} \right)^{\frac{1}{2}}} \right) \tag{53}$$

that is equivalent the case when $\alpha = 1$ in the eq. (51)

5. CONCLUSIONS

In this paper, we studied some thermodynamical aspects of VGCG model. We deduced an equation for Joule-Thomson coefficient for this model and we have derived expressions for W , ΔU and Q in some thermodynamical process. We have determined the inversion temperature of Joule-Thomson expansion which is function of parameter α . In all the studied cases, we obtain the VCG model when $\alpha = 1$.

As in the case of the ideal gas, we have used the thermodynamic relation (32) in order to obtain expressions for reversible adiabatic process for VGCG model as function of temperature, volume, pressure and the parameter α .

The study of the different models of Chaplygin gas allows enrich the courses of thermodynamics, which contributes to a better comprehension of the thermal phenomena. The equations that describe the behavior of the Chaplygin gas are tractable mathematically and offer a wide comprehension of the accelerated universe expansion and of the basic ideas of the modern cosmology.

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