DETERMINATION OF CAPACITY AND RULES OF THE VARIABILITY OF MAXIMUM FORCE USING NONLINEAR MATHEMATICAL MODELS: A CASE STUDY


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Abstract The aim of this study is to determine the capacity and the variability of maximum force rules measured 1 RM for eight muscle groups (back-hip extensors, legs extensors, arm extensors, back extensors, shoulder and arms extensors, shoulder joint flexors, hip and knee extensors, trunk flexors). The determination was performed on the experimental results of the top basketball center player using repeated measurements and nonlinear mathematical models methods. Changes in maximum force were induced with 8 months of weight lifting training and analysed with nonlinear regression analysis within 95% confidence interval. The results indicate that from all the models applied only the Asymptotic Regression, Michaelis-Menten and Gompertz Growth models had satisfactory performance and provided solid solutions to the given problem. This means that the models developed in this study properly and reliably determine the capacity and predicted changes in the maximum force (1 RM) for all eight monitored muscle groups.

Key words additive model, interactive model, nonlinear models, 1 repetition maximum prediction, resistance training

Introduction For proper training programming and selection of athletes, it is necessary to know the capacity and the variability of performances. To determine the capacity and variability of athletes’ performances, two models (theory) are commonly used, additive and interactive (Tucker, Collins, 2011). With the additive model, capacity and volatility are determined by the sum of the genetic and training variance. This model assumes that both parts of the total variability are additive, which may not be entirely valid. One of the extremes of the additive model is the Galton
model (Galton, 1869, quoted in Ericsson, Nandagopal, Roring, 2009) by which training contributes to developing of
the performances, but capacity (upper limit of development) is determined solely by a genetic component. The other
extreme is the Ericsson model (Ericsson, Krampe, 1996; Ericsson, Nandagopal, Roring 2009) where capacity is
determined exclusively by a training component. An interactive model in addition to the genetic and training component
introduces the component that indicates the interaction between genetic and simulation components, as well
(Schneider, 1997; Vaeyens, Güllich, Warr, Philippaerts, 2009). This model implies that the volatility of performance
and thus the capacities and the top sports achievements, in addition to these two components, are determined by
result of interaction between genetic and training component. In this paper, the problem of determining the capacity
and variability of maximum force in particular muscle groups has been resolved in an elite basketball center player.

The problem of modern training is the fact that average values, obtained from practice and scientific research,
are used as reference values for training programming in order to produce elite athletes and champions. In contrast,
in this paper a methodology is presented, that allows for assessment of the development dynamics of maximum
force, quantification of training goals and the selection of the each individual based on the estimated potential
rather than their current state. This methodology also allows for individual programming and control of the training
work as well as training effects and changes by the use of modern hardware and software equipment. Proposed
methodology can be used in any sport in order to produce elite athletes and champions.

It was presumed that the problem could be solved using the method of repeated measurements and
nonlinear mathematical models (Motulsky, Christopouls, 2003; Schumaker, Solieman, Chen, 2010; Watts, Bates,
2007). The choice of mathematical models was based on assumption that a basketball player, after the initial
measurements, over time, with individually programmed training would rapidly improve performance under the
influence of all three components (Tucker, Collins, 2011), However with prolonged training, improvements would still
continue, but at a somewhat slower pace. This slowdown should become increasingly obvious as the basketball
player approaches the limits of his capacity. Once the influence all three components are exhaustand, it would not
be possible to improve performance, regardless of the type and time of the future training application (Milosevic,
Milosevic, 2013a; Milosevic, Milosevic, 2013b; Milosevic, 2010; Tucker, Collins, 2011). That moment and level of
force achieved represent a measurement of the participant’s capacity in the maximum force (1 RM) of the monitored
muscle groups (Milosevic, Milosevic, 2013a, b; Milosevic, Blagojevic, Pilipovic, Tosic, 2000; Milosevic, Dopsaj,
Blagojevic, Mudric, 2012).

In according to the hypothesis, using the method of repeated measurements and nonlinear mathematical
models, the aim of this study was to determine the capacity and the rules of the variability of 8 muscle groups
(back-waist extensors, legs extensors, arm extensors, back extensors, shoulder and arms extensors, shoulder joint
flexors, hip and knee extensors, trunk flexors) maximum force that are relevant for achieving optimal basketball
performance (Milosevic, Milosevic, 2013a, b; Milosevic, 2010; Motulsky, Christopouls, 2003; Schumaker Solieman,

Materials and Methods

Participants

Experiment data that were used for developing a methodology for assessing the variability rules and limit value
(capacity) of maximum force were obtained from a basketball player (center position) (BH = 2.12 m; BW = 125 kg
before treatment, 118 kg after treatment; Age = 22 years). The participant was a member of Serbian National
team in a full multi-year training process. Participant gave his informed consent to the procedures of the study. The conditions of the study were approved by the university’s ethics committee.

**Testing and training**

The development of maximal force was performed through the development of the following neuromuscular qualities: (i) the maximal rate of the force development, (ii) synchronization and speed of recruitment of motor units, (iii) the maximal force of certain motor unit groups level, (iv) overall muscle density, (v) intramuscular coordination, (vi) intermuscular coordination. Testing, programming and control of the development of maximal force as well as chosen neuromuscular qualities was done using a new modeling approach (to be explained in detail later), standardized procedures and certified hardware and software system (VAC Bioengineering). At the beginning of the training process (one initial and 8 transition tests) the maximal force of the following muscle groups by using the one repetition maximum (1 RM) test method: power clean (an integral indicator of back-hip extensors), bench press (arm extensors), half-squat (legs extensors), behind-the-neck press (shoulder and arms extensors), dead lift (back extensors), “good morning” (back extensors), pull-over (shoulder joint flexors), sit-ups (trunk flexors) and step-up (step test - hip and knee extensors) was tested. These neuromuscular qualities, as well as weight and weight lifting speed were tested at the beginning of each month (beginning of the training cycle) for each muscle group (VAC Bioengineering). Testing results were used for calculating status, models of the variability of maximum force, potential value of maximum lifted weight, training time in which a potential maximum were achieved, increase of weights for monitored muscle groups (Tables 1 and 2). After each test (every month) a new training program (VAC Bioengineering) was designed. Training sessions were conducted twice a week (Mondays and Fridays) and took one hour each. At each training session, 6 muscle groups were treated (3 exercise pairs), in five sets with 1–5 repetitions, with 3 minutes rest period (Milosevic, Milosevic, Nemec, Zivotic, Radjo, 2014a; Milosevic, Milosevic, 2013a, b; Milosevic, 2010). Free weights were used as a basic means of force development. During the month, three weeks were used for training, and the last week was for test and rest (Bosquet, Montpetit, Mujika, 2007; Milosevic et al., 2014a; Milosevic, Milosevic, 2013a; Milosevic, 2010). Monthly training sessions were designed in such a way that on Mondays in the first week of training, the speed of recruitment for motor units would be developed, while on Fridays the rate of force development would be developed (Aagaard et al., 2002; Milosevic, Dzoljic, Milosevic, Jourkesh, Behm, 2014b; Milosevic, Milosevic, 2013a, b; Milosevic, 2010). To develop motor unit speed of recruitment for the chosen muscle groups a weight of 70% of 1 RM was used. This weight was lifted at the maximum lifting speed for the particular weight chosen according to the particular participant for 5 sets of 5 repetitions each (Blagojevic, Milosevic, Aleksic, Papadimitriou, Dopsaj, 1998; Milosevic et al., 2014; Milosevic, Milosevic, 2013a; Milosevic, 2010; Milosevic, Stefanovic, Dopsaj, Blagojevic, 1998a; Milosevic, Cirkovic, Mihajlovic, Blagojevic, Dopsaj, 1998b; Milosevic et al., 2002). For the development of the maximal rate of force development the weight of 80% of 1 RM was used in 5 sets of 5 reps each, and for muscle density the weight of 90% of 1 RM, in 5 sets, each comprised of 3 repetitions (Blagojevic et al., 1998; Furandžijev, Abadžijev, 2003; Milosevic et al., 2014a, b; Milosevic, Milosevic, 2013a; Milosevic, 2010; Milosevic et al., 1998a, b, 2000, 2002). Lifting was performed at maximum speed in both cases. During the second week of training the maximal force of certain motor units groups would be developed on Mondays, while on Fridays intramuscular coordination in combination with motor unit synchronization would be developed (Milosevic et al., 2014a, b; Milosevic, Milosevic, 2013a, b; Milosevic, Mudric, Mudric, Milosevic, 2012; Milosevic, 2010). The development of the maximal force of certain motor units groups, intramuscular coordination...
and intermuscular coordination was accomplished by varying the resistance of weights and lifting speed (maximal, submaximal and large (80% of maximal). The weights of 30, 40, 50, 70, 75, 80, 85, 90, 95, 100, 130 and 150% of 1 RM were employed, and exercises carried out in 5 sets of 1–5 repetitions each (Milosevic et al., 2014; Milosevic, Milosevic, 2013a, b; Milosevic et al., 2012; Milosevic, 2010). During the third week of training the muscle density would be developed on Mondays, whereas on Fridays intermuscular coordination would be developed (Furandžijev, Abadžijev, 2003; Milosevic et al., 2014a, b; Milosevic, Milosevic, 2013a, b; Milosevic et al., 2000, 2012; Milosevic, 2010). Motor unit synchronization development was based on exercises employing the weight of 95% and 100% of 1 RM, in 5 sets of 1 to 2 repetitions each, done at the maximum speed of lifting of the particular weight for the particular participant (Milosevic et al., 2014; Milosevic, Milosevic, 2013a; Milosevic, 2010). The intramuscular coordinations in combination with the motor units synchronization was done by combining weight lifting 70, 75, 80, 85, 90, 95 and 100% of 1 RM. The following exercise pairs were performed on Mondays: power clean and sit-ups, half-squat and bench press, dead lift and torso rotation (Barbell with disc weights – 40 kg) (Milosevic et al., 2014a, b; Milosevic, Milosevic, 2013a, b; Milosevic, 2010). The following exercise pairs were performed on Fridays: power clean and sit-ups, step-up and behind-the-neck press, “good morning” and pull-over (Milosevic et al., 2014; Milosevic, Milosevic, 2013a, b; Milosevic, 2010). Training session was designed in such way that one performs all the sets and all the repetitions of the first pair at the beginning, then the second pair and at the end the third pair (Milosevic et al., 2014; Milosevic, Milosevic, 2013a, b; Milosevic, 2010). Five sets of 1 to 5 repetitions each were performed for each muscle group. Each of the five sets for each muscle group was done with each of the different weights mentioned above. Immediately following the last repetition of the fifth set, 2 additional series of repetitions were performed for each muscle group with a weight of 60% of 1 RM with a five second break between repetitions. Each series was continued until the point at which the participant was unable to lift the weight (Milosevic et al., 2014; Milosevic, Milosevic, 2013a; Milosevic, 2010). The lifting speed was maximal. The total amount of work was 48 training sessions (48 hours of training work of which the pure time of weight lifting was from 10 to 12 minutes per session, and on the eight-month level from 8 up to 9.6 hours).

**Data Analysis**

To determine the capacity values and the variability maximum force of 8 muscle group rules the following nonlinear regression models were used (Ivancevic, Ivancevic, 2006; Motulsky, Christopoulus, 2003; Watts, Bates, 2007): (i) Asymptotic Regression (concave), (ii). Michaelis-Menten, (iii). Gompertz Growth, (iv) Logistic Growth, (v) Loglogistic Growth, (vi) Weibull Growth, (vii) 1-parameter Sigmoid, (viii) 2-parameter Sigmoid 1, (ix) 2-parameter Sigmoid 2. For each model the flow of nonlinear regression was determined, the maximum force capacity of the subject was estimated by increasing the 1 RM (parameter Theta 1), time point at which the concavity of the function/ curve changes sign, ie. changes from plus to minus (parameter Theta 2 or in the case of Michaelis-Menten it would be 50% of Theta 1), 95% confidence interval, the standard errors of the model parameters evaluation and summary statistics from which mean square error (MSE) was chosen between two or more regressions. The lower value of MSE, bolded in Table 1, suggests that the model according to this criterion had better performance, and was elected to evaluate the capacity of the subject and the training time, for which the capacity value can be achieved.
Results

Table 1 shows nonlinear mathematical models of the variability of maximum force for all screened/tested muscle groups and exercises.

Table 1. Models of the variability of maximum force measured using one-repetition maximum (1 RM) test

<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Power clean (kg)</strong></td>
<td>Asymptotic Regression PC = 113.613 – 31.1808 × exp(-0.344942 × Month)</td>
<td>5.21000</td>
</tr>
<tr>
<td></td>
<td>Michaelis-Menten PC = 114.173 × Month / (0.279141 + Month)</td>
<td>6.13529</td>
</tr>
<tr>
<td></td>
<td>Gompertz Growth PC= 113.37 × exp[-exp(-1.17504 – 0.372801 × Month)]</td>
<td>7.67756</td>
</tr>
<tr>
<td><strong>Half-squat (kg)</strong></td>
<td>Michaelis-Menten HS = 314.086 × Month / (1.16544 + Month)</td>
<td>29.0581</td>
</tr>
<tr>
<td></td>
<td>Gompertz Growth HS = 284.896 × exp[-exp(-0.0688132 – 0.427014 × Month)]</td>
<td>34.7278</td>
</tr>
<tr>
<td><strong>Bench press (kg)</strong></td>
<td>Michaelis-Menten BP = 127.836 × Month / (0.499298 + Month)</td>
<td>26.4913</td>
</tr>
<tr>
<td></td>
<td>Gompertz Growth BP = 147.821 × exp[-exp(-0.574319 – 0.157755 × Month)]</td>
<td>6.93020</td>
</tr>
<tr>
<td><strong>Dead lift (kg)</strong></td>
<td>Michaelis-Menten DL = 207.815 × Month / (0.480778 + Month)</td>
<td>9.89002</td>
</tr>
<tr>
<td></td>
<td>Gompertz Growth DL = 195.921 × exp[-exp(-0.526638 – 0.614665 × Month)]</td>
<td>21.6435</td>
</tr>
<tr>
<td><strong>Behind-the-neck press (kg)</strong></td>
<td>Michaelis-Menten BNP = 85.9301 × Month / (1.41143 + Month)</td>
<td>10.6047</td>
</tr>
<tr>
<td></td>
<td>Gompertz Growth BNP = 85.9848 × exp[-exp(-0.0395771 – 0.255884 × Month)]</td>
<td>1.78360</td>
</tr>
<tr>
<td><strong>Pull-over (kg)</strong></td>
<td>Michaelis-Menten PL = 125.457 × Month / (1.10068 + Month)</td>
<td>67.5278</td>
</tr>
<tr>
<td></td>
<td>Gompertz Growth PL= 174.404 × exp[-exp(0.0439944 – 0.123983 × Month)]</td>
<td>14.0954</td>
</tr>
<tr>
<td><strong>Step-up (kg)</strong></td>
<td>Michaelis-Menten ST = 151.403 × Month / (1.20534 + Month)</td>
<td>18.9145</td>
</tr>
<tr>
<td></td>
<td>Gompertz Growth ST= 125.248 × exp[-exp(0.760158 – 1.08348 × Month)]</td>
<td>35.3003</td>
</tr>
<tr>
<td><strong>Sit-ups (kg)</strong></td>
<td>Michaelis-Menten SU= 60.4435 × Month / (1.74436 + Month)</td>
<td>4.28548</td>
</tr>
<tr>
<td></td>
<td>Gompertz Growth SU= 49.0218 × exp[-exp(0.370396 – 0.597066 × Month)]</td>
<td>8.21129</td>
</tr>
</tbody>
</table>

Based on the model evaluation it could be said that they have satisfactory validity and reliability; PC – power clean, HS – half-squat, BP – bench press, DL – dead lift, BNP – behind-the-neck press, PL – pull-over, ST – step-up, SU – sit-ups

Figures 1–8 shows nonlinear mathematical models of the variability of maximum force for all screened/tested muscle groups and exercises.

Table 2 contains the following results for all participant muscle groups and exercises: one-repetition maximum (1 RM) measured initially and after 8 monthly training cycles. Estimated potential one-repetition maximum (1 RM) and the number of months needed to reach the value. Table 2 also shows the differences between estimated potential values and the values measured initially and after 8 monthly training cycles for all muscle groups and exercises. Finally, Table 2 shows the reached increase after 8 monthly training cycles for all muscle groups and exercises.
Figure 1. Asymptotic Regression model of the variability of maximum force of back-waist extensors

Figure 2. Michaelis-Menten model of the variability of maximum force of legs extensors

Figure 3. Gompertz Growth model of the variability of maximum force of arm extensors
Figure 4. Michaelis-Menten model of the variability of maximum force of back extensors

Figure 5. Gompertz Growth model of the variability of maximum force of shoulder and arms extensors

Figure 6. Gompertz Growth model of the variability of maximum force of shoulder joint flexors
Therefore, the paper presents the results only for Asymptotic Regression model, Michaelis-Menten model and Gompertz Growth model (Tables 1–2 and Figures 1–8) as they have satisfactory reliability and validity. Applying the selected models (Motulsky, Christopouls, 2003; Watts, Bates, 2007) enabled the assessment of capacity and determination of the variability of 8 muscle groups’ maximum force rules for the participant (Tables 1–2 and Figures 1–8). To assess the capacity and the variability maximum force rules with the power clean, the best performance was associated the Asymptotic Regression model (Table 1 and Figure 1). In the half-squat, dead lift, step-up and sit-ups the best performance was achieved with the Michaelis-Menten model (Table 1 and Figures 2, 4, 8). While with the bench press, behind-the-neck press and pull-over, the best performance was found with the Gompertz Growth model (Table 1 and Figures 3, 5, 6, 7).
Table 2. State, anticipated capacity and lifted weight changes of 1 RM

<table>
<thead>
<tr>
<th>Variables / muscle Groups and exercises</th>
<th>Power clean</th>
<th>Sit-ups</th>
<th>Half-squat</th>
<th>Bench Press</th>
<th>Dead lift</th>
<th>Behind the neck press</th>
<th>Pullover</th>
<th>Step-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial state (kg)</td>
<td>90</td>
<td>20</td>
<td>150</td>
<td>90</td>
<td>140</td>
<td>40</td>
<td>70</td>
<td>60</td>
</tr>
<tr>
<td>Estimated capacity (kg)</td>
<td>115(^1)</td>
<td>53.3(^2)</td>
<td>288.2(^2)</td>
<td>137.5(^3)</td>
<td>200.4(^2)</td>
<td>83.1(^1)</td>
<td>144(^2)</td>
<td>139(^2)</td>
</tr>
<tr>
<td>In time (month)</td>
<td>11</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Starting level based on capacity (%)</td>
<td>78</td>
<td>37.5</td>
<td>52</td>
<td>65.4</td>
<td>69.8</td>
<td>48.1</td>
<td>48.6</td>
<td>43.2</td>
</tr>
<tr>
<td>State after 8 monthly training cycles (kg)</td>
<td>115</td>
<td>50</td>
<td>280</td>
<td>120</td>
<td>200</td>
<td>75</td>
<td>110</td>
<td>130</td>
</tr>
<tr>
<td>Increase after 8 monthly training cycles (kg)</td>
<td>25</td>
<td>30</td>
<td>130</td>
<td>30</td>
<td>60</td>
<td>35</td>
<td>40</td>
<td>70</td>
</tr>
<tr>
<td>Reached level based on capacity (%)</td>
<td>100</td>
<td>93.8</td>
<td>97.1</td>
<td>97.2</td>
<td>99.8</td>
<td>90.2</td>
<td>76.3</td>
<td>93.5</td>
</tr>
</tbody>
</table>

Prediction by \(^1\) Asymptotic Regression model, \(^2\) Michaelis-Menten model, \(^3\) Gompertz Growth model.

The participant reduced his body weight 6% from 125 to 118 kg. At the same time, he increased the 1 RM from 25 kg to 130 kg (Table 2). The capacity values were calculated for each muscle group, as well as the time for which it could be reached (Table 2). After 48 training sessions the participant increased from 76.3 to 100% of his capacity. Results (Table 2) showed that the basketball player needed 11 to 13 months of training for each muscle group to achieve the necessary capacity values.

Discussion

Applying regression models (Ivancevic, Ivancevic, 2006; Motulsky, Christopoulos, 2003; Watts, Bates, 2007) to determine the capacity and time to achieve sports performance illustrated the problem of short series. This problem led to an iterative process in the evaluation of the function parameters (iv-ix) that did not fulfill the convergence criteria. The applied regression models (Table 1 and Figures 1–8) provided a proper maximal force change prediction for all muscle groups in the period of 1–13 months. The capacity and variability data on monitored performances allowed the improvement of early selection, programming and training control in basketball. The early selection of future champions or top basketball players would be done by comparing the capacity value of each individual with the champion basketball requirements and their performance level (Milosevic, Milosevic, 2013a, b; Milosevic et al., 2012, 2014a, b; Milosevic, 2010; Tucker, Collins, 2011). From this relationship the ability to reach the requirements of the game and champions performance could be predicted. The knowledge of the athlete's capacity and current state (Table 2) would allow the prediction of potential capacity and future performance changes (Table 1). With this predictive power, it should be possible to directly quantify the training objectives, training effects, changes and training work required for one or more training sessions within one or more months (Aagaard et al., 2002; Furandžijev, Abadžijev, 2003; Milosevic, Milosevic, 2013a, b; Milosevic, 2010; Milosevic et al., 2002, 2012, 2014a, b).

Conclusions

Based on the results the mathematical model developed in this study demonstrated satisfactory validity and reliability for providing an accurate estimation of the maximum force capacity (1 RM) for eight muscle groups relevant for achieving top results in basketball. In addition to determine capacities, models provided a reliable maximal force in the prediction of training process changes. To assess the capacity and the maximum force by weight lifting rules
variability (1 RM) for the power clean, the best performance was with the Asymptotic Regression model. With the half-squat, dead lift, step-up, and sit-ups the best performances was achieved with the Michaelis-Menten model. While for the bench press, behind-the-neck press and pull-over the best performance was associated with the Gompertz Growth model. It can be concluded that the capacity and rules of variability of maximum force can be determined, separately for each individual basketball player, using nonlinear mathematical models. For future research of this type it is recommended to use a series with a larger number of observations.

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