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SHORT COMMUNICATION

Components for differential cross-section with taking into account the polarization effects in elastic electron-deuteron scattering

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ABSTRACT

The components for differential cross-section was considered with taking into account the polarization effects in elastic electron-deuteron scattering. These characteristics are analyzed for angle of electron scattering θ = 70º and different momentums. The angular-momentum dependence of values components τ_{2M} for differential cross-section have been evaluated in 3D format. The wave function of the deuteron in the coordinate representation for the Reid93 potential was applied to numerical calculations.

***Keywords*:** deuteron, cross-section, electron-deuteron scattering, deuteron wave function

1. INTRODUCTION

The elastic electron-deuteron scattering remains interesting for researchers. For example, in the works [1-5] the results of theoretical and experimental studies of structural functions, deuteron form factors and polarizations are presented.

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By analysis the reaction of elastic electron-deuteron scattering one can study the structure of the deuteron, the distribution of the nucleon and charge densities in deuteron [1, 6, 7]. This elastic process was studied theoretically used various models and approaches for nucleon-nucleon interaction. In this paper, the components for differential cross-section with taking into account the polarization effects in elastic electron-deuteron scattering [8] are considered. The deuteron wave functions in the coordinate representation for the Reid93 potential are used to calculate these components.

2. COMPONENTS FOR DIFFERENTIAL CROSS-SECTION

The final polarization can be measured by scattering the recoil deuteron on a second nuclear target. The differential cross-section for the second scattering written as [8]

$$\left(\frac{d\sigma}{d\Omega}\right)_2 = \left(\frac{d\sigma_0}{d\Omega}\right)_2 \Big\{1+\langle T_{20}\rangle_f \langle T_{20}\rangle_2 + \\ +2\left[\langle iT_{11}\rangle_f \langle iT_{11}\rangle_2 - \langle iT_{21}\rangle_f \langle iT_{21}\rangle_2\right]\cos\phi_2 + 2\langle T_{22}\rangle_f \langle T_{22}\rangle_2 \cos(2\phi_2)\Big\}. \quad (1)$$

The values T_{ij} are expressed within the approach for the spin-density matrix ρ for the final state, which was obtained for an arbitrary initial state

$$\langle T_{JM}\rangle_f = \frac{\mathrm{Tr}(MM^{\dagger}T_{JM})}{\mathrm{Tr}(MM^{\dagger})}. \quad (2)$$

Here $\left(\frac{d\sigma_0}{d\Omega}\right)_2$ – the differential cross-section for the nuclear scattering of an unpolarized incoming deuteron beam; the values $\langle T_{JM}\rangle_2$ will be equivalent as (2) for scattering of unpolarized deuteron on the second target; ϕ_2 – the polar angles with respect to direction of motion of the deuteron after the first scattering.

The calculations of the polarization or of the cross section for elastic electron-deuteron scattering will be expressed by matrices $\Omega = MM^{\dagger}$ and $\Omega' = M^{\dagger}M$. The differential cross-section for the second scattering of the recoil deuteron on a nuclear target written in final form according to [8]

$$\left(\frac{d\sigma}{d\Omega}\right)_2 = \left(\frac{d\sigma_0}{d\Omega}\right)_2 \frac{1}{\mathrm{Tr}\Omega}\Big\{\mathrm{Tr}\Omega + \langle T_{20}\rangle_2 \mathrm{Tr}(\Omega T_{20}) - \\ -2\langle T_{21}\rangle_2 \mathrm{Tr}(\Omega T_{21})\cos\phi_2 + 2\langle T_{22}\rangle_2 \mathrm{Tr}(\Omega T_{22})\cos(2\phi_2)\Big\}, \quad (3)$$

where

$$\mathrm{Tr}\Omega = 3G_C^2 + \frac{8}{3}\eta^2 G_Q^2 + 2\left[1+2tg^2\left(\frac{\theta}{2}\right)\right]\eta G_M^2, \quad (4)$$

$$\mathrm{Tr}\left(\Omega T_{20}\right)=\frac{\sqrt{2}}{3}(1-3\cos\theta)\eta^2 G_Q^2+\frac{3\cos^2\theta-2\cos\theta+3}{2\sqrt{2}(1+\cos\theta)}\eta G_M^2+$$
$$+\sqrt{2}(1-3\cos\theta)\eta G_C G_Q-6\sqrt{2}\sin\left(\frac{\theta}{2}\right)\eta^{3/2}G_Q G_M, \tag{5}$$

$$\mathrm{Tr}\left(\Omega T_{21}\right)=\frac{2}{\sqrt{3}}\sin\theta\eta^2 G_Q^2+\sqrt{3}tg\left(\frac{\theta}{2}\right)(1-\cos\theta)\eta G_M^2+$$
$$+2\sqrt{3}\sin\theta\eta G_C G_Q-2\sqrt{3}\cos\theta\sec\left(\frac{\theta}{2}\right)\eta^{3/2}G_Q G_M, \tag{6}$$

$$\mathrm{Tr}\left(\Omega T_{22}\right)=-\frac{1}{\sqrt{3}}(1+\cos\theta)\eta^2 G_Q^2-\frac{\sqrt{3}}{4}(3-\cos\theta)\eta G_M^2-$$
$$-\sqrt{3}(1+\cos\theta)\eta G_C G_Q-2\sqrt{3}\sin\left(\frac{\theta}{2}\right)\eta^{3/2}G_Q G_M. \tag{7}$$

In terms of $\mathrm{Tr}\Omega$, $\mathrm{Tr}\left(\Omega T_{20}\right)$, $\mathrm{Tr}\left(\Omega T_{21}\right)$, $\mathrm{Tr}\left(\Omega T_{22}\right)$ components $\left\langle T_{JM}\right\rangle_f$ can be labeled as

$$\tau_{20}=\left\langle T_{20}\right\rangle_f=\frac{\mathrm{Tr}\left(\Omega T_{20}\right)}{\mathrm{Tr}\Omega};\qquad \tau_{21}=\left\langle T_{21}\right\rangle_f=\frac{\mathrm{Tr}\left(\Omega T_{21}\right)}{\mathrm{Tr}\Omega};\qquad \tau_{22}=\left\langle T_{22}\right\rangle_f=\frac{\mathrm{Tr}\left(\Omega T_{22}\right)}{\mathrm{Tr}\Omega}. \tag{8}$$

The values τ_{2M} (M=0;1;2) are similar to deuteron tensor t_{2j} polarizations [1, 9, 10]. Moreover, the structure is similar in size to the factor $S(p,\theta_e)=A(p)+B(p)\mathrm{tg}^2\left(\frac{\theta_e}{2}\right)$, where *A* and *B* – the structural functions.

3. DEUTERON FORM FACTORS

Components (4)-(7) for the differential cross section (3) consist of charge $G_C(p)$, quadrupole $G_Q(p)$ and magnetic $G_M(p)$ deuteron form factors, which containing information on the electromagnetic properties of deuteron [1, 11]:

$$G_C=G_{EN}D_C;\ \ G_Q=G_{EN}D_Q;\ \ G_M=\frac{m_d}{2m_p}\left(G_{MN}D_M+G_{EN}D_E\right), \tag{9}$$

where isoscalar electric and magnetic form factors

$$G_{EN}=G_{Ep}+G_{En};\qquad G_{MN}=G_{Mp}+G_{Mn}.$$

The components of the form factors D_i are determined by the formulas

$$D_C = S_0^{(1)} + S_0^{(2)}; \qquad D_Q = \frac{3}{\sqrt{2}\eta}\left(S_2^{(1)} - \frac{1}{\sqrt{8}}S_2^{(2)}\right); \tag{10}$$

$$D_M = 2\left(S_0^{(1)} - \frac{1}{2}S_0^{(2)} + \frac{1}{\sqrt{2}}S_2^{(1)} + \frac{1}{2}S_2^{(2)}\right); \qquad D_E = \frac{3}{2}\left(S_0^{(2)} + S_2^{(2)}\right) \tag{11}$$

where elementary spherical $S_0^{(i)}$ and quadrupole $S_2^{(i)}$ form factors [12, 13]

$$S_0^{(1)} = \int_0^\infty u^2 j_0 dr; \qquad S_0^{(2)} = \int_0^\infty w^2 j_0 dr; \qquad S_2^{(1)} = \int_0^\infty uwj_2 dr; \qquad S_2^{(2)} = \int_0^\infty w^2 j_2 dr. \tag{12}$$

Here *u(r)* and *w(r)* – the radial wave functions of the deuteron in the coordinate representation; j_0, j_2 - the spherical Bessel functions of zero and second order from the argument $pr/2$; $G_{Ep}$and G_{En} – proton and neutron isoscalar electric form factors; G_{Mp} and G_{Mn} – proton and neutron isoscalar magnetic form factors.

The calculations use the original dipole approximation (DFF) [14] for the proton and neutron form factors:

$$G_{Ep} = F_N; \qquad G_{En} = 0; \qquad G_{Mp} = \mu_p G_{Ep}; \; G_{Mn} = \mu_n G_{Ep}, \tag{13}$$

where the nucleon form factor is written in the form of a dipole

$$F_N(p^2) = \left(1 + \frac{p^2}{18.235\, fm^{-2}}\right)^{-2}, \tag{14}$$

where μ_p = 2.7928 and μ_n = -1.9130 – the magnetic proton and neutron moments in nuclear magnetons.

4. CALCULATIONS AND CONCLUSIONS

The deuteron wave functions in the coordinate representation for the Reid93 potential are used to calculate components (4)-(8) for differential cross-section with taking into account the polarization effects.

The deuteron wave function is represented as simple sums [15]

$$\begin{cases} u(r) = r^{3/2}\sum_{i=1}^{N} A_i \exp(-a_i r^3), \\ w(r) = r\sum_{i=1}^{N} B_i \exp(-b_i r^3). \end{cases} \tag{15}$$

All coefficients in (15) for this potential are given in paper [15]. DWF for this potential do not contain an unnecessary knot at the origin of coordinate. Therefore, it can be used for correct numerical calculations. The results of the obtained components (4)-(8) are shown in Figures 1 and 2.

Here $\theta = 70º$ – the angle of electron scattering. The momentum interval is up to 7 fm^{-1}.

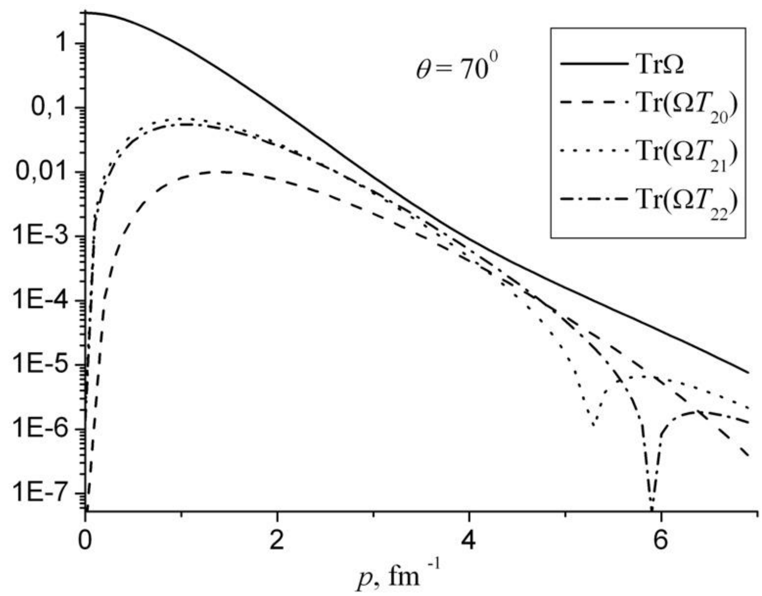

Figure 1. Components (4)-(7) for the differential cross-section

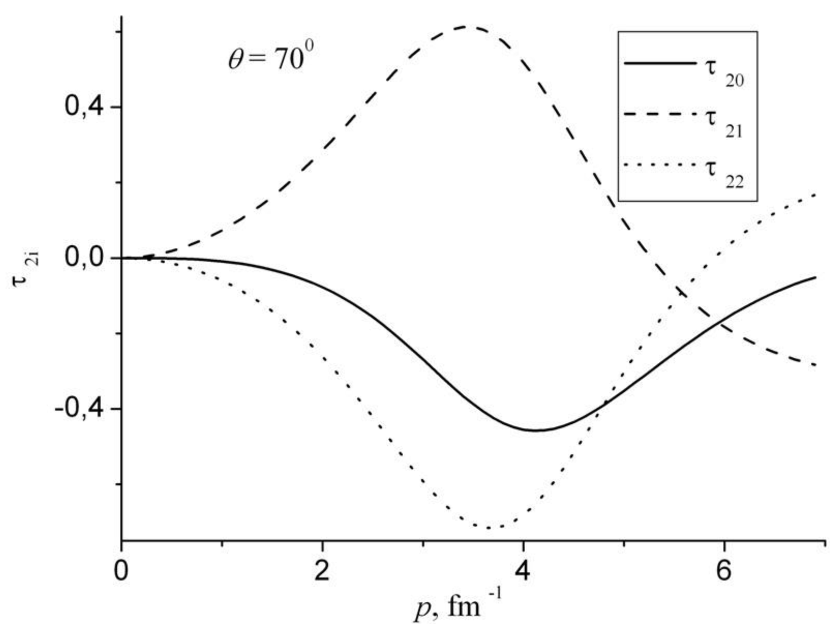

Figure 2. Components (8) for the differential cross-section

The results of numerical calculations of components τ_{2M} (8) are presented also in 3D format in Figures 3-5. The component $\tau_{21}(p,\theta)$ is characterized by a hump (peak), and for the component $\tau_{22}(p,\theta)$ there is a pit.

After the future calculation in a 3D format of the deuteron tensor polarizations t_{2i} from the paper [15] and comparing them with the obtained values of the components τ_{2M} (Figures 3-5) for the differential cross-section in this paper, it is possible to better explain and illustrate the laws of elastic electron-deuteron scattering.

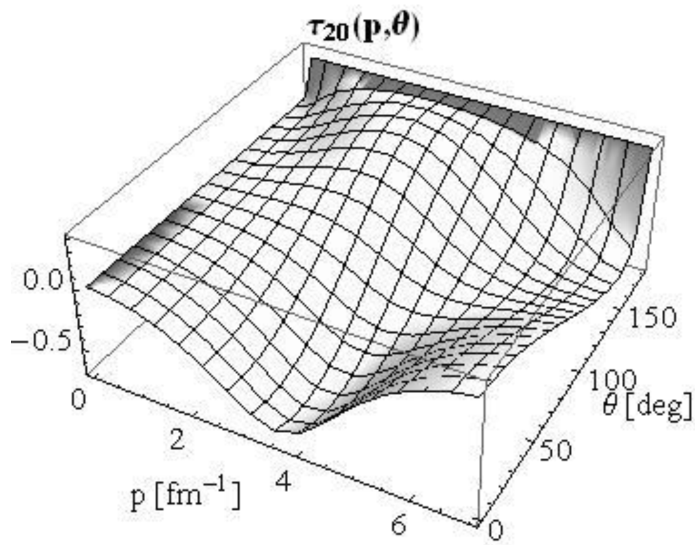

Figure 3. The angular-momentum dependence of component $\tau_{20}(p,\theta)$

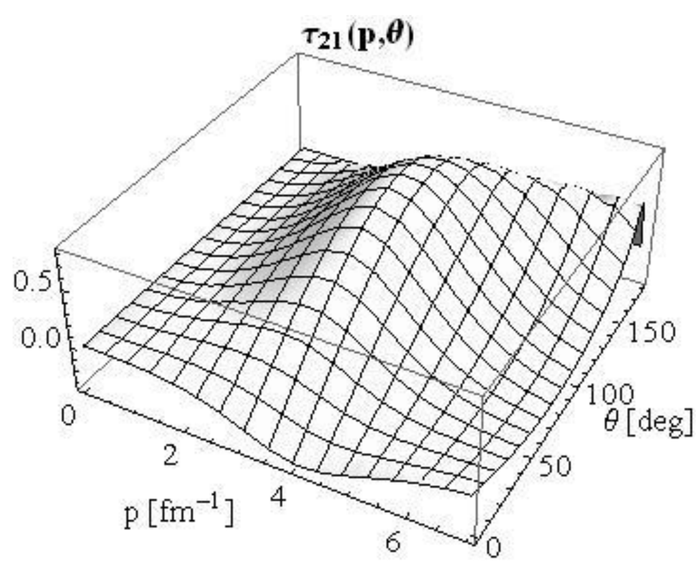

Figure 4. The angular-momentum dependence of component $\tau_{21}(p,\theta)$

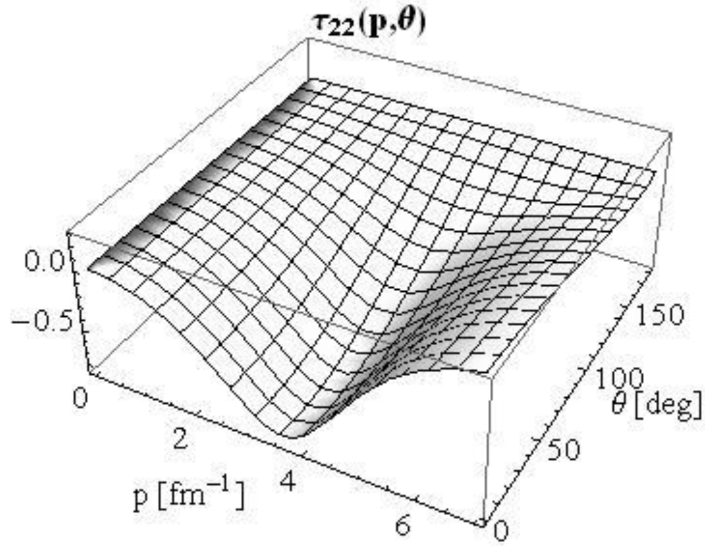

Figure 5. The angular-momentum dependence of component $\tau_{22}(p,\theta)$

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