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NUMERICAL MODELLING OF SUB-GLACIAL FLOW IN A RETENTION RESERVOIR

MODELOWANIE NUMERYCZNE PRZEPŁYWÓW PODLODOWYCH W ZBIORNIKACH RETENCYJNYCH

Abstract

This paper deals with applying a numerical finite element model of water dynamics to sub-ice flow in a retention reservoir. On the basis of earlier experiences, the AdH model is chosen for this task. The previous usage of this model is outlined along with the physical basics used by this algorithm. The application procedure is then described and finally a few simulation results obtained for different conditions are shown. The results comparison and analysis indicate that the chosen model is capable of performing the required tasks.

Keywords: *AdH, numerical modeling, sub-ice flow, retention reservoir*

Streszczenie

Artykuł omawia zagadnienie zastosowania modelu numerycznego bazującego na metodzie elementów skończonych do symulacji dynamiki przepływu wody w pokrytym lodem zbiorniku retencyjnym. Do tego zadania na podstawie wcześniejszych doświadczeń został wybrany model AdH. W artykule zawarte są podstawowe równania, na których oparty jest algorytm modelu; przedstawiono też krótko jego wcześniejsze zastosowania do modelowania przepływu w zbiornikach retencyjnych. Następnie zaprezentowana została procedura dostosowania modelu do wyznaczonego mu zadania, zaś na koniec pokazano kilka wyników, których porównanie i analiza wykazują, że model AdH nadaje się do wykonania postawionych mu w tej pracy zadań.

Słowa kluczowe:

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1. Introduction

Water quality issues are among the most important fields of research nowadays. They require an interdisciplinary approach merging environmental sciences tools with methods of physics, chemistry, biology, and even social sciences. The aim of the research presented in this paper was to explore one of the physical aspects of this subject, namely the dynamics of the sub-glacial flow of water in retention reservoirs. The dynamics of the water flow provides one of the basic means of transport of various substances influencing water quality. This may be either the direct transport of or sedimentary particles, or just one of driving forces for particles embedded in living forms.

Recognising these phenomena is vital especially for reservoirs that supply water for human use. However, even in other dammed lakes, it is still important as good water quality is also demanded there either to maintain biodiversity or to provide higher recreational value.

The formation and disappearance of an ice layer on the lake surface is a thermodynamic process that involves many factors originating from both water and air. The temperature of both the water and the air, the chemical composition of substances dissolved in the water, the speed of the current, the air humidity, the wind conditions, precipitation, and overnight cloud cover all have an impact on this phenomenon.

In the Polish climate, one should expect the surface of almost every lake to be frozen each winter for a period ranging from a few weeks to a few months. Fig. 1 [10] shows a histogram of the seasonal ice cover duration for the lake of Goczałkowice over 40 years. As one can see, the mean and expected times of the sub-glacial regime is close to one fourth of the total time.

Fig. 2 presents the evolution of the ice cover thickness on the Goczałkowice lake for the winter season of 1963/64. This is analogous to other years during which measurements were conducted. It appears that ice cover of up to half a meter in thickness may form on Polish retention reservoirs.

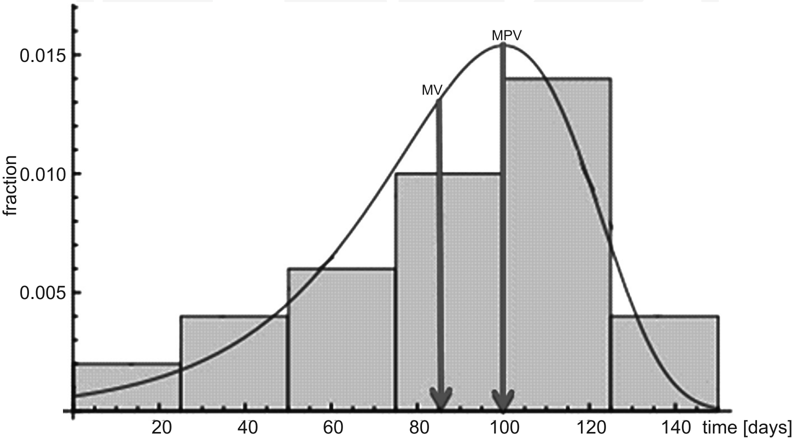

Fig. 1. Duration of sub-glacial regime in days with mean value and most probable value indicated (data from the Goczałkowice lake)

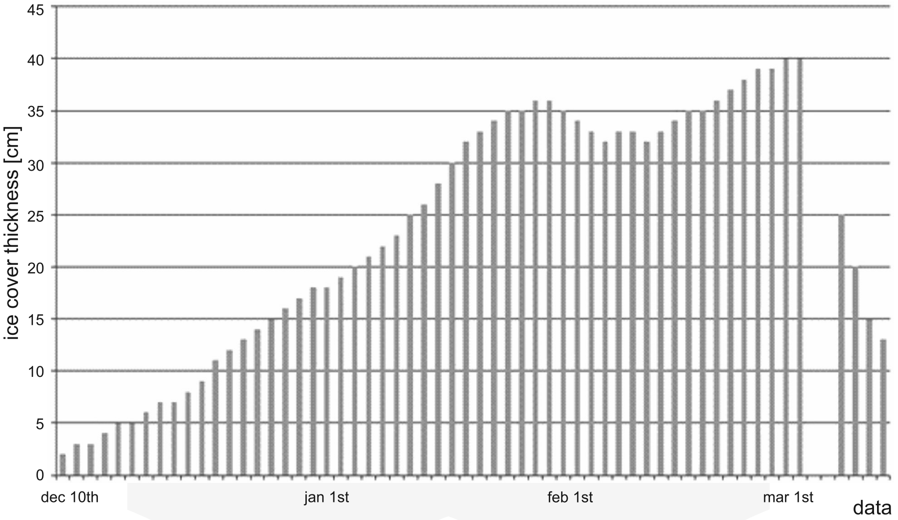

Fig. 2. Ice cover thickness evolution during one winter period

It is obvious that measuring the parameters of the sub-glacial flow in a real lake is very difficult. Obtaining a thorough insight into this phenomenon is even harder than doing it for free-surface flow. Taking into account that the latter is considered practically infeasible, we have to admit that the real life data will always be incomplete considering the large scale flow fields under the ice layer. However, some partial observational information is accessible either through observations of the ice formation, melting and breaking up, the ice type and structure, or through direct measurements using holes drilled in the ice cover.

Keeping the above in mind, the standard solution seems to be to use numerical models. However, there are only a few trustworthy models worldwide that are capable of properly describing the flow dynamics in retention lakes even working in the free surface conditions Among the modelling tools familiar to the authors, the only one also capable of performing calculations for pressure flow in ice-covered reservoirs is the AdH (adaptive hydraulics model) created by The US Army Corps of Engineers [2, 11] and available in the SMS (Surface water Modeling Solution) package distributed by the AQUAVEO company [1]. It should be noted that the model is targeted towards rivers; therefore, its application to storage reservoirs is not direct. No applications for an ice-covered lake or reservoir have been published so far.

This is why this model, which has been applied before to free surface water dynamics in a few different retention lakes, has been chosen for this preliminary study. The main goals were to set up the model so it could calculate sub-glacial flow fields giving physically justified results and to test its sensitivity to changes of a few vital parameters to verify whether it runs stably. Achieving these goals would indicate that the given model may be used for the desired simulations. Please note that the actual observational or experimental verification of the results is not needed at such an early stage. This will be the next step to perform.

2. Theoretical background and its implementation

The AdH model belongs to the group of two-dimensional depth averaged (also known as '2.5-dimensional') finite element method modelling tools. The computational domain for such a calculation is divided into elements of a simple shape. For AdH, the elements are triangular with vertices located at the corners. This makes it different from most models that prefer quadrilateral elements.

The basic input for this model includes:

1) bed shape (bathymetry);
2) bed friction parameters (they may be expressed as the roughness coefficients or as the equivalent bed roughness height);
3) initial water surface level;
4) water parameters (density, viscosity – the latter may be depth dependent);
5) inflows and outflows (constant, time dependent or surface level dependent).

The above may be extended by various case-specific conditions including, but not limited to, wind conditions, atmospheric pressure changes, Coriolis effect, fixed ceilings (e.g. bridges), and – not least significantly – the parameters of the ice layer: its range, density, thickness and roughness.

The main result of the model run is a two-dimensional planar field of the mean average horizontal velocity of water. Vertical movement and other vertically dependent phenomena are neglected. This field may be static or time-dependent according to the conditions imposed upon the domain.

2.1. Governing equations

In order to obtain the solution, the AdH model uses conservation equations for mass (continuity equation) and momentum. The following subsections present the appropriate formulas [2, 11].

2.1.1. Continuity equations

The following basic equations are in use for determining the cross-section and discharge in any given element junction:

$$A = \int_B H\ dB \tag{1}$$

$$Q = \int_B U \cdot H dB \tag{2}$$

where:

A – cross-section,
Q – discharge,
U – vertically averaged velocity of the flow,
H – local water depth,
B – section width.

The mass conservation equation for any given element of the domain:

$$\frac{\partial H}{\partial t} + \frac{\partial (Hv_x)}{\partial x} + \frac{\partial (Hv_y)}{\partial y} = 0 \tag{3}$$

where:

v_x, v_y – velocity components in x and y directions respectively,
t – time.

2.1.2. Momentum equations

The AdH model uses two-dimensional momentum conservation equations. The equation for the x direction is as follows:

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}-c_f\cdot v_y+g\frac{\partial \varsigma}{\partial x}+g\cdot v_x\frac{\sqrt{v_x^2+v_y^2}}{C^2\cdot H}-v_x\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}\right)+\frac{1}{\rho}\frac{\partial p}{\partial x}=F_x \quad (4)$$

where:

c_f – Coriolis factor,
g – gravitational acceleration,
ς – bed level,
C – roughness coefficient,
p – external pressure,
ρ – water density
F_x – specific force acting on an element from its neighbours in the x direction.

The equation for the y direction is analogous.

This basic equation may be both simplified and supplemented if necessary. For standard calculations of water flow in retention lakes, the external pressure, treated as atmospheric, is considered to be constant and the Coriolis effect is neglected. This allows omitting appropriate terms and simplifying the equation. However, for the ice-covered regime the external pressure comes not only from the atmosphere but also from the ice layer and it cannot be treated as constant. Moreover, additional water/ice 'ceiling' friction should also be taken into account [2–4, 8].

2.1.3. Specific sub-glacial flow terms

When solving a sub-glacial flow issue, one must take into account both pressure and friction changes due to the ice layer. The formula for the additional pressure, assuming full coverage, no holes and no 'hanging' ice is:

$$P_{ice}=\rho_{ice}g t_{ice} \quad (5)$$

where:

ρ_{ice} – ice density,
t_{ice} – ice layer thickness.

For the friction impact of the ice ceiling on the flowing water, a few assumptions are made, these arise from the classical viscosity theory (see also Fig. 3):

- the ice and bed regions have the same maximum velocity, which is located at the junction of the velocity profiles;
- since the vertical velocity gradient at the maximum velocity height is zero, the shear stress at the maximum velocity height is also zero;
- since no energy can be exchanged across a horizontal plane of zero shear stress, the two velocity profiles are hydraulically independent. Therefore, the total shear is equal to the

sum of ice and bed shear stresses, this can be evaluated independently. Each of these profiles has its own height ('water depth').

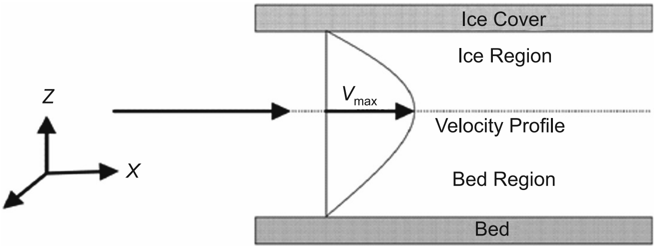

Fig. 3. Sub-glacial flow regime scheme

As a result of the above, the calculations for the 'ice' and 'bed' profiles may be conducted independently as long as the level of the line dividing these two is known.

According to [2] and [3], the total shear is equal to the sum of partial shears, for the *x* direction it is:

$$\tau_x = \tau_{BED.x} + \tau_{ICE.x} \tag{6}$$

where:

$\tau_{BED.x}$ – shear in the *x* direction resulting from the bed friction,
$\tau_{ICE.x}$ – shear in the *x* direction resulting from the ice friction.

The formula for the y direction is analogous.

The level of division between two profiles (the maximum velocity level) is located as follows:

$$z_{mv} = \frac{1}{R^2 + 1} H \tag{7}$$

where:

R – dimensionless factor denoting the relative difference between the ice and bed roughnesses.

2.2. Implementation issues

The AdH model was applied to a very complex system of a retention reservoir. It is a multi-step process starting from digitising the bathymetric map, then going through the process of building the computational mesh, imposing the conditions and finally, running and refining the model. The general guidelines for such an application are described in [6] and [7]. Although these studies took into consideration the older numerical model, FESWMS [5], nearly all the issues addressed in these papers have their counterparts in the preparation of the AdH model for a simulation run.

The most important point to mention here is the need to follow the features of the reservoir bathymetry with the guiding lines used for building the computational mesh. The size of the elements should be reduced in potentially vulnerable regions including areas close to inflows and outflows as well as in regions in which the bed slope changes rapidly.

Fortunately, there are two guidelines that don’t apply to AdH modelling that were important for older models used for water dynamic simulations in lakes (like FESWMS or RMA). Firstly, since AdH uses only triangular elements, there is no need to consider maintaining a good quadrilaterals to triangles ratio. Secondly, the central elements located in the flat part of the reservoir bed may be significantly bigger than before, as AdH has limited capabilities for the automatic splitting of computational elements that generate higher approximation errors.

Preparing a mesh for ice-covered flow computations ensures that the research faces a few new challenges.

While allowed in principle, inflows and outflows being covering by ice tends to cause numerical instabilities. It is good practice to leave all the inflows and outflows in the ice-free surface regime, creating artificial run paths (or run channels) if necessary. Usually, it is enough to start the ice lid at the distance of several elements far from such a boundary.

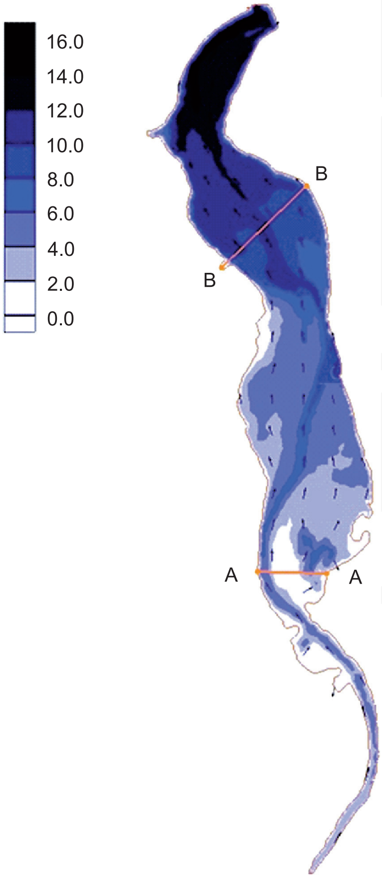

Fig. 4. Depth map with two cross-sections

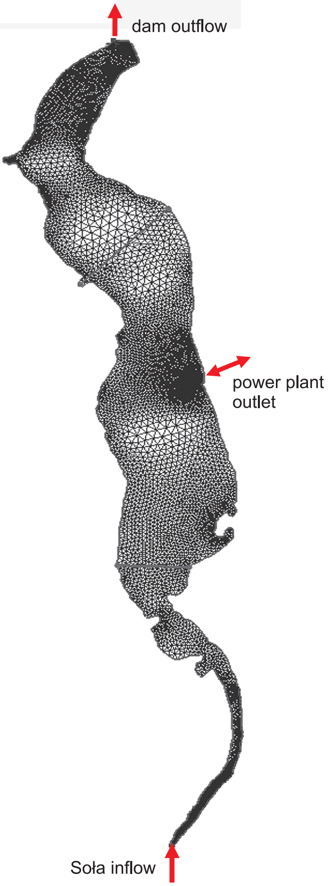

Fig. 5. Computational mesh with inflows and outflows

One should also consider making denser fragments of the mesh in the regions where the ice cover edge appears. It is also better not to overlap these regions with zones where the slope of the bed changes rapidly.

For the test runs as well as for the stability and sensitivity tests of the AdH model the Porąbka retention reservoir has been chosen The main reason for this choice was the fact that this model has been used several times before to simulate various open-surface flow scenarios in the lakes of the Soła Cascade and the results of the model runs were stable and physically justified, for example [12]. Moreover, this lake is interesting from the fluid dynamics point of view due to the fact that it is the lower reservoir of a pumped storage hydropower plant. The bathymetric map of the lake is shown in Fig. 4. There are two cross-sections introduced there – one in the upper part of the lake (A-A) and one in the lower part (B-B). These cross-sections will be used as controls for some results presented in the next section of this paper.

Figure 5 presents the computational mesh. It contains 13,230 triangular elements and 7,098 nodes. Inflow and outflow boundary condition lines are denoted by the arrows. The water surface level is set to 320 m a.s.l.

3. Achieved results

This section outlines some representative results of the performed simulations of sub-glacial flow in the Porąbka lake. The boundary conditions for the model are as follows: a constant water surface level at the lower boundary, a constant inflow discharge for the upper boundary. The calculations were conducted for a water surface level of 320 m a.s.l. (which is the normal water level for this reservoir) and for two values of the main inflow (the Soła river): 5 m^3/s which is close to the most common inflow, and 25 m^3/s which is close to the yearly average inflow. Out of different roughness coefficient values for the ice cover, the calculations were conducted for the values of 0.005 and 0.06 $s/m^{1/3}$ – these were chosen as the low and high values respectively. One should be aware here that for models like AdH, the values of the roughness coefficient required for a stable and calibrated run may significantly differ from the values of the Manning coefficient of a given material and it should not be named as one to avoid confusion. For a few of the simulations, the influence of the power plant is added either by pumping additional water to the lake or taking water from it. The results are then compared to those resulting from simulations of open surface flow to check whether the model behaviour is physically justified.

Figures 6A to 6E show the velocity maps generated by the model for a small main inflow of $5m^3/s$. Results obtained for sub-glacial pressure flow are juxtaposed with the image resulting from a free surface flow of the same discharge. It can be seen that increasing either the ice thickness or its roughness leads to damping of the velocity field. Side currents and whirlpools that are present for the free surface scenario tend to disappear when ice cover is introduced. For the sub-glacial flow cases, the main current tends to move towards the places where the depths are greater. Stagnant areas (purple colour on the maps) appear along both banks of the lake.

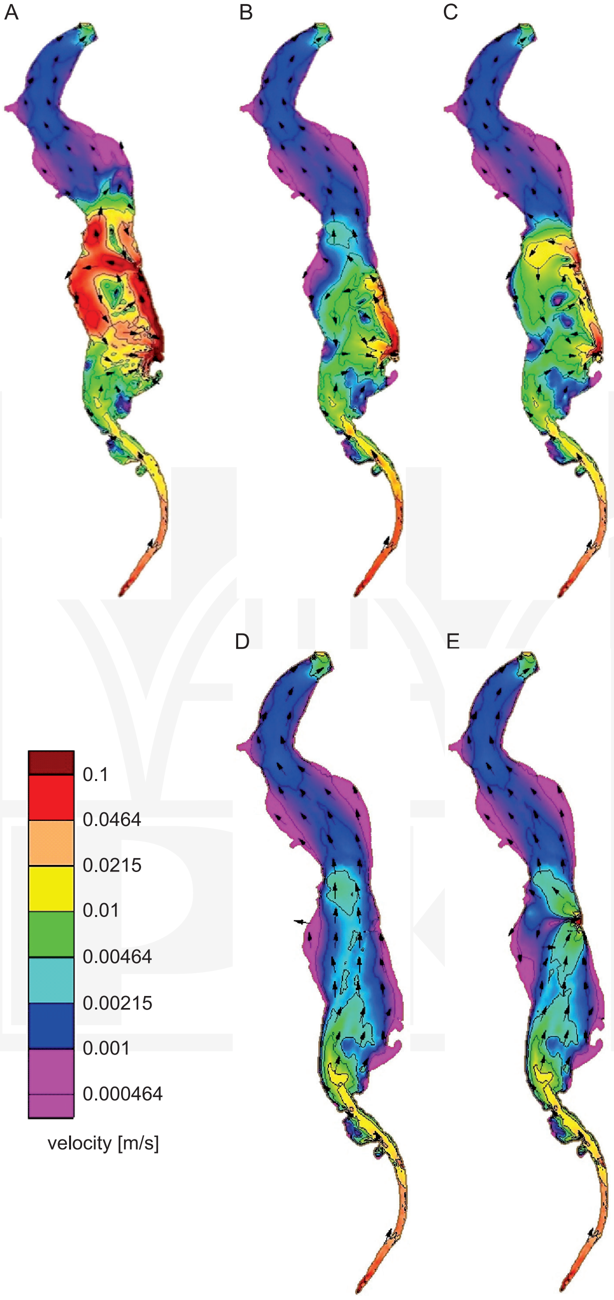

Fig. 6. Flow velocity map comparison for 5 m^3/s main inflow: A – ice-free surface flow; B – ice cover 10 cm, roughness 0.005; C – ice cover 10 cm, roughness 0.06; D – ice cover 50 cm, roughness 0.005; E – ice cover 50 cm, roughness 0.06

Fig. 7 and Fig. 8 present the depth profile of the two cross-sections mentioned in the previous section. Fig. 9 and Fig. 10 show appropriate distributions of the specific discharge (depth times velocity) along these profiles for two different ice layer thicknesses. It can be seen that the thicker ice makes more water follow the depths of the bed.

The achieved results are consistent and they fit the theoretical expectations.

Fig. 11A to 11E are maps analogous to those above, but they are made for a higher main inflow – 25 m^3/s. The effects previously visible also appear in this case; however, the damping is slightly weaker especially for the thinner ice cover. This is because the momentum of the current is on average, four times higher in this case and as such, it requires appropriately larger forces in order to be changed.

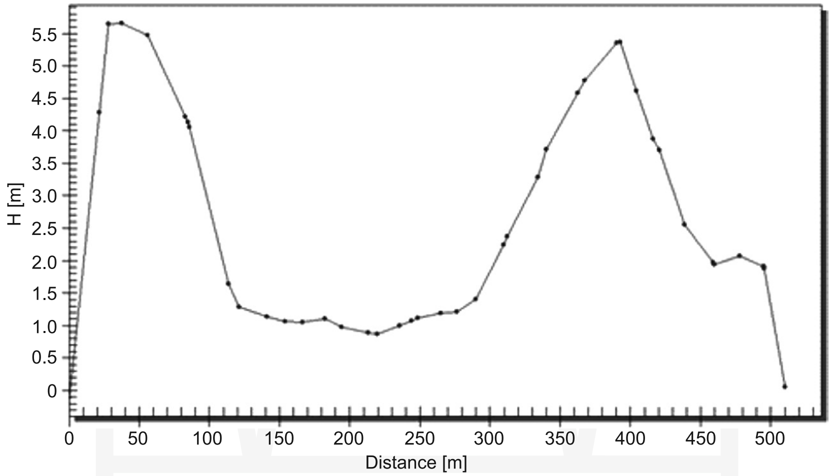

Fig. 7. Depth in profile A-A

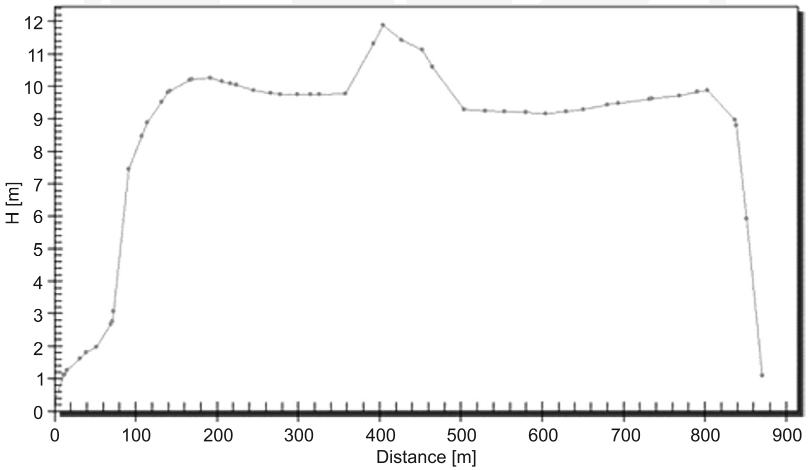

Fig. 8. Depth in profile B-B

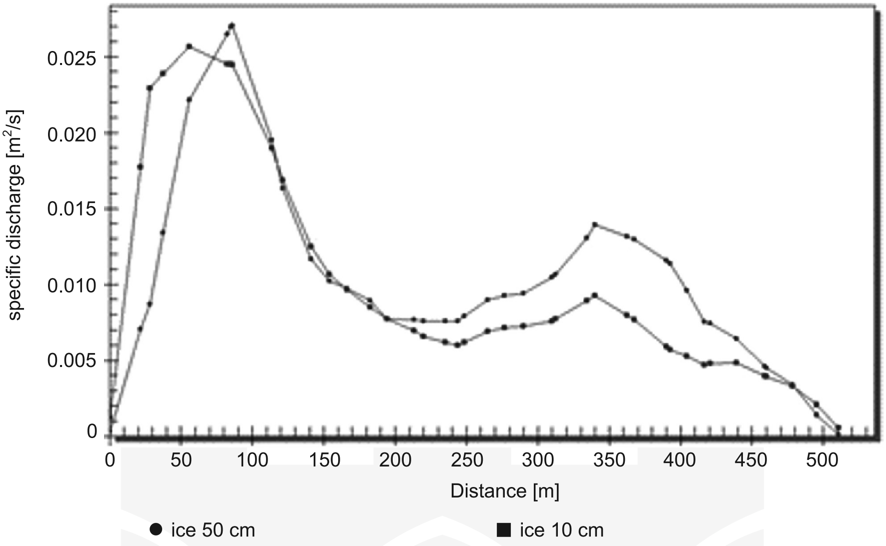

Fig. 9. Specific discharge along the A-A profile

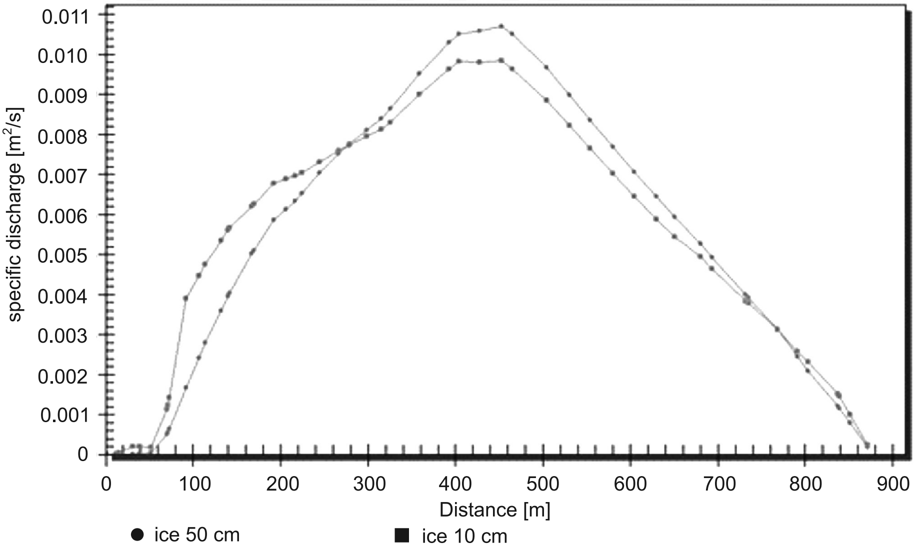

Fig. 10. Specific discharge along the B-B profile

Finally, the impact of the ice cover on the flow pattern caused by the hydropower plant was taken into account (Figs. 12–17). Contrary to the previous example where stationary time-independent flow fields were calculated, here the calculations took into account a dynamic case – the daily cycle of the pumped storage power plant involving both taking the water from the lake and letting it flow back there. Two time frames were taken both for

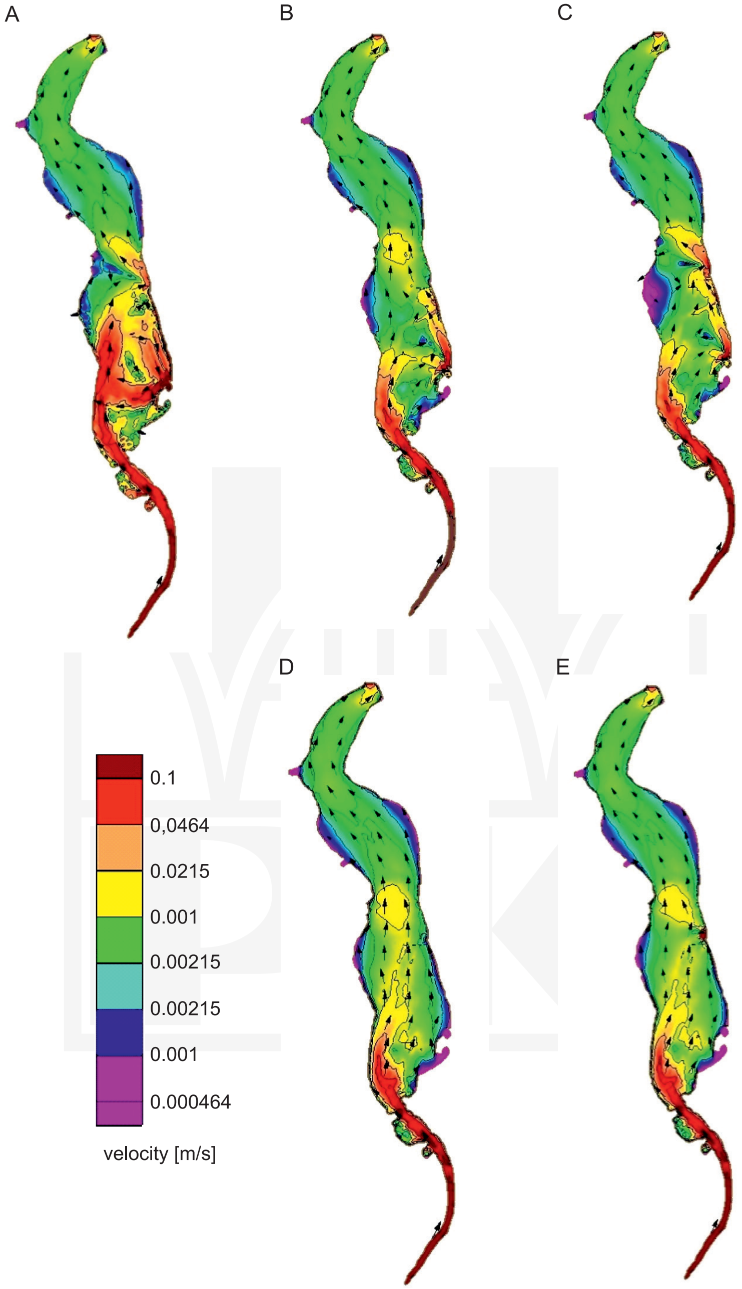

Fig. 11. Flow velocity map comparison for 25 m^3/s main inflow: A – ice-free surface; B –ice cover 10 cm, roughness 0.005; C – ice cover 10 cm, roughness 0.06; D – ice cover 50 cm, roughness 0.005; E – ice cover 50 cm, roughness 0.06

the free surface and ice-covered flow: one for pumping 12 m³/s from the lake and another for injecting additional 14 m³/s to the lake. Both frames were set near the end of appropriate periods of the power plant working cycle. There are also to differential maps added (Figs. 14 and 17) to focus on the differences The blue regions on the differential maps are the regions where the velocity of the sub-glacial flow is lower than the ice-free surface flow. If the sub-

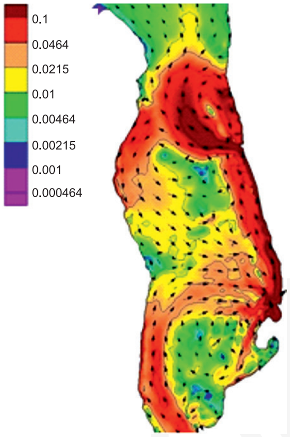

Fig. 12. Planar velocity field for 20 m³/s main inflow, the power plant dumps water into the reservoir at 14 m³/s, free surface

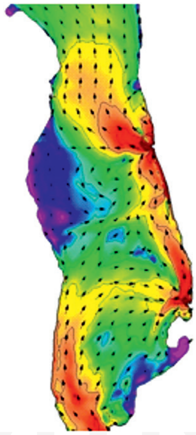

Fig. 13. Planar velocity field for 20 m³/s main inflow, the power plant dumps water into the reservoir at 14 m³/s, ice cover

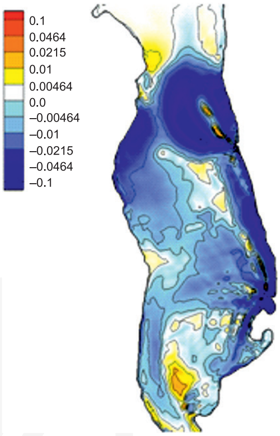

Fig. 14. Differential velocity map made from those shown in Figs. 12 and 13

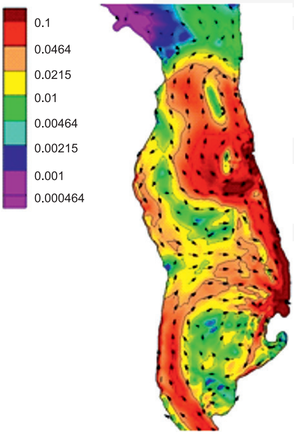

Fig. 15. Planar velocity field for 20 m³/s main inflow, the power plant takes water from the reservoir at 12 m³/s, free surface

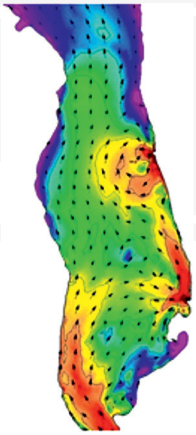

Fig. 16. Planar velocity field for 20 m³/s main inflow, the power plant takes water from the reservoir at 12 m³/s, ice cover

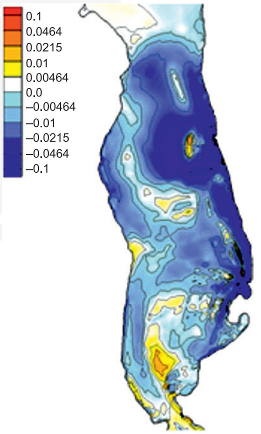

Fig. 17. Differential velocity map made from those shown in Figs. 15 and 16

-glacial velocity is higher, the corresponding color is yellow or orange. It is noteworthy that yellow regions occur in two cases: firstly, in deeper parts of the lake, which are preferred by the sub-ice flow; secondly, in the regions that for ice-free surface flow are stagnant due to the fact that they lay between whirlpools.

Again, the impact of the ice cover is clearly visible and it follows the physical intuition. Ice dampens the weaker currents and reduces the number and size of whirlpools. It is noteworthy that while for the free surface situation the power plant dictates the regime the lake is in, when the reservoir is ice-covered, the range of the power plant influence is much shorter; in the latter case, what governs the flow pattern is rather the bed bathymetry.

4. Conclusions

The obtained results indicate that by using the AdH model, it is possible to simulate sub-glacial water flow in retention reservoirs. The obtained results are stable and physically justified. Imposing an ice cover upon the lake causes additional friction forces that modify the pattern of the water current. The thicker and the more rough the cover is, the higher is its impact on the water dynamics, and the model behaves accordingly. The presence of the ice layer causes more areas of the lake to be stagnant – the currents tend to straighten and whirlpools tend to disappear.

Identifying the sub-ice stagnant zones is important for the sake of water quality. During winters when the ice prevents the water-air oxygen exchange as well as cutting off the sunshine needed for photosynthesis, it is only the current that can bring oxygen into the reservoir. In the stagnant parts, there may then develop an oxygen deficit which may lead to the formation of ‘dead zones’ in which harmful anaerobic processes may occur. If such zones are correctly identified, we may try to prevent noxious processes by, for example, making air holes in appropriate places.

Another point to consider is the impact of the pumped storage power plant. During the thaw period, water in the upper reservoir is warmer than in the lower reservoir. This is because of shallower depths, continuous water movement and the lack of any ice cover. Injecting a high volume of significantly warmer water beneath the ice cover of the lower reservoir has a firm impact on the pattern of the ice thawing in the lake and on the biological processes that are temperature sensitive. Recognising hydrodynamic patterns of these phenomena is then important from both of these perspectives.

By having a suitable verified tool, we may then proceed towards understanding these issues.

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