TECHNICAL TRANSACTIONS | CZASOPISMO TECHNICZNE
ARCHITECTURE | ARCHITEKTURA
10-A/2015

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CYLINDRICAL ANAMORPHIC IMAGES –A DIGITAL METHOD OF GENERATION

CYFROWA METODA GENEROWANIA ANAMORFICZNYCH OBRAZÓW WALCOWYCH

Abstract

The aim of this paper is to present a practical construction for some cylindrical anamorphic images. The method is based on the analytic properties of reflective anamorphic image construction – topics which have been discussed in previous papers. This time, the authors deal with the analytical analysis of a transformation that is applied in order to obtain an anamorphic image, and provide an innovative digital notation of the reflective transformation discussed. The analytical model described here allows us to generate a cylindrical anamorphic image of any object that is represented in the form of a set of parametric equations. Some example anamorphic images together with their counter-images reflected in the surface of a cylindrical mirror will be presented here. The method of construction described enables the development of any design project of an anamorphic image in the urban planning environment and within the interiors of public spaces.

Keywords: transformation, anamorphic image, visualization of an anamorphic image, reflective cylinder

Streszczenie

Niniejszy artykuł przedstawia praktyczną metodę konstruowania obrazów anamorficznych w oparciu o własności analityczne dla anamorf refleksyjnych. Praca jest kontynuacją zagadnień związanych z określeniem geometrycznych zasad powstawania obrazów anamorficznych na bazie obrazu rzeczywistego, natomiast prezentuje ona analityczne przekształcenie oraz innowacyjny cyfrowy zapis takich obrazów. Tak więc opracowany model analityczny pozwala generować obrazy anamorficzne dowolnych projektowanych obiektów zapisanych w formie równań parametrycznych. Przykładowe anamorfy zaprezentowano wraz z ich obrazami restytuowanymi za pomocą prototypowego zwierciadła walcowego. Powyższe rozwiązania dają możliwość precyzyjnego projektowania obrazów anamorficznych w zurbanizowanej przestrzeni miejskiej oraz architektonicznych wnętrzach przestrzeni publicznej.

Słowa kluczowe: przekształcenie, restytucja, anamorfa, walec refleksyjny

DOI: 10.4467/2353737XCT.15.157.4194

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Symbols

A_p – designed point;
A_{anm} – anamorphic image of a designed point;
x_{anm} – abscissa of an anamorphic image;
y_{anm} – ordinate of an anamorphic image;
R – radius of the reflective cylinder;
x_p – abscissa of a designed point;
y_p – ordinate of a designed point;
ζ – reflection angle.

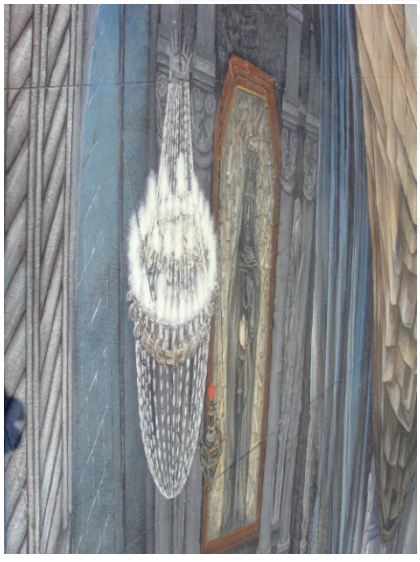
1. Introduction

Anamorphic, perspective images can be described as those with the possibility of being perceived through a double vision process: 1) through direct vision on a scene or a picture and 2) when the drawings produce a certain illusion when they are reflected in the surface of a mirror. This specific illusion is usually hidden behind the dual meaning of an image. At a very first glance at an anamorphic image we can usually see a geometric drawing which makes a good aesthetic impression due to its geometrical composition that is ruled by the principles of reflective transformation. However, there is another layer hidden inside the image that has the beauty of a planar geometry and a sense hidden behind the complicated shapes that we can see directly in the picture. This secondary level of the anamorphic image becomes readable when they are observed from a specifically taken stationary point or viewpoint, or when the picture is reflected in a specifically adopted mirroring surface.

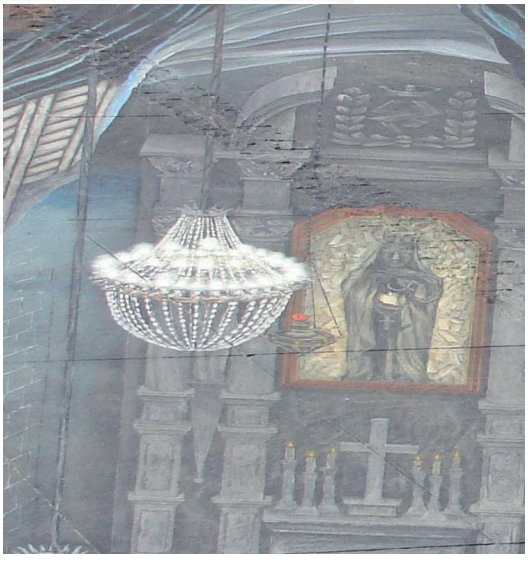
The art of creating anamorphic images survives and today returns. They appear in public areas either as the planar, colourful and surprising compositions or as being brought to a 3-D space with the aid of special mirrors helping to see them in three dimensions (Ill. 1, Ill. 2 and Ill. 3). In a series of publications on anamorphic image creation ([6–8]), Zdziarski provides a geometrical analysis of particular types of anamorphic transformations together with their classification and possible methods of visualization. The previous papers describe the highly complicated principles for creating an anamorphic image so that a view of a real life object is obtained when reflected in a mirror or when observed from a specific viewpoint. The metric parameters of the reflective cylindrical anamorphic images have also been analysed. A special taxonometry has been introduced in order to classify all possible of various types of axonometric images and to arrange them into specific sub-groups. This classification and special nomenclature has been provided in previous publications. In consequence, it has become possible to create much more advanced anamorphic images and to introduce simplified methods for creation and "reading" the images in a 3-D space. It is also possible to create "deformation nets" which help in drawing the anamorphic images and understanding the ways they should be constructed.

Additionally, some analysis has been done in order to describe the planar constructions of anamorphic images, the constructions aided with a device called a "pantograph". An optical principle has been applied in order to execute this type of transformation.

Ill. 1. Planar anamorphic image in the closed municipal area of the Main Square in Wieliczka, PL-Ryszard Paprocki (photo by M. Jonak)



Ill. 2. Detail of an anamorphic image as observed from a randomly chosen view-point (photo by M. Jonak)



Ill. 3. Detail restitution seen from specified view point (photo by M. Jonak)

An innovative idea of how architects and designers can introduce anamorphic images into their design projects in urban planning and/or within the interiors of the public spaces has been brought about by A. Zdziarski ([6–8]). A wide range of the papers written by A. Zdziarski may well serve the architects and the artists who want to use the guidelines presented in his work today.

2. Description of a projection method used to create anamorphic images

2.1. Projection variables (Figure 1)

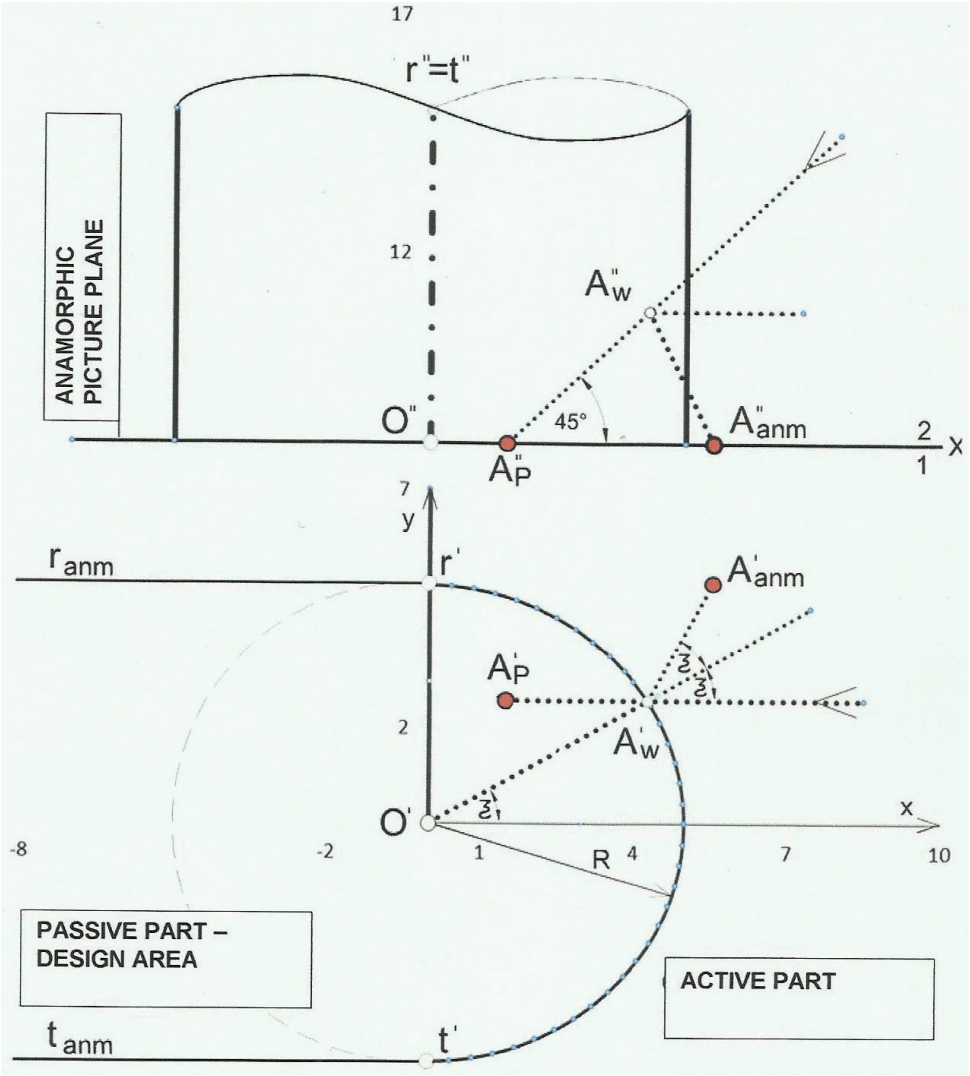
The elements of projection system create:
- an anamorphic picture plane which is a horizontal plane of the cylinder's base,
- a reflective cylinder of revolution with a set-up diameter and vertical axis,
- direction of a projection (LOS) which has a slope of 45° in reference to the horizontal picture plane.

2.2. Description of a projection method

As has been typically described in many of the available textbooks [6–8], the described type of projection is realized practically based on the principles of a central projection (theory of seeing vs. perspective projection). In the case described here, the authors have adopted the position of an observer at infinity and thus they assumed that a parallel projection will represent the projection method. The parallel bunch of lines of sight (LOS) that are inclined at 45° in reference to the anamorphic picture plane represent the direction of projection. This assumption means the digital notation of all equations be simplified and we can easily design numerous anamorphic images.

The adopted direction of projection (LOS) determines two of its parts on the cylinder's surface: a) an **active part** in terms of optical conditions (this is the part facing the observer); the active part of a cone will be limited with two generators r and t; and b) a **passive part** (the remaining part of the cylinder). The semi-lines r_{anm} and t_{anm} are the images from two generators r and t. These two lines also divide the anamorphic picture plane into two areas: a) a **passive part** – the part where we can place a design pattern; and b) an **active part** where an anamorphic image of a designed object will be projected. The bordering polyline between the two parts of the anamorphic picture plane will be made of two semi-lines r_{anm} and t_{anm} and a semicircle of the cylinder's base (in ill. 4 the line shown with a thick continuous line represents the bordering line). Thus, in the anamorphic picture plane we can distinguish two areas with differing terminations: a design area where we can place the transformed image, and an area where the anamorphic image will be active. The design area is necessary to realize the digital input of the transformation.

Illustration 4 shows the principle of construction of the anamorphic image A'_{anm} of the designed point A_p that belongs to the passive part of the design area. The image A'_{anm} lies in the active part of the anamorphic picture plane.



Ill. 4. Two-view orthographic projection of principles of anamorphic transformation

The base for creation of any anamorphic image is the well-known optical law of two angles equality: the angle of incidence equals the angle of reflection in geometric optics (in ill.4: angle ζ). Point A_p has been randomly specified in the anamorphic picture plane. From point A_p a projector parallel to the direction of the LOS has been drawn. The point of intersection A_w is the point of incidence of the LOS passing through A_p and lies on the active part of a cylinder. Next, a surface normal to the cylinder at point A_w has been drawn. According to the geometric optic we can determine the angle ζ between the incident ray and a normal n and construct the reflected ray making the same angle with n. A_{anm} will be an anamorphic image of point A_p.

2.3. Analytic approach and problem solution

The digital method for solving the problem is based on determining the relations existing between the designed geometry and the anamorphic geometry, i.e. between two images: one belonging to the passive part and the other belonging to the active part of the anamorphic picture plane. The problem has been solved with the use of theorems well known from trigonometry[1]. The analysis of the geometrical transformation presented in ill. 4 allows us to determine the corresponding relation existing between the coordinates of the designed point $A'_P(x_p, y_p)$ and the coordinates of its anamorphic image $A'_{\text{anm}}(x_{\text{anm}}, y_{\text{anm}})$. The equations can be formulated as follows:

$$x_{\text{anm}} = \text{R} * \cos(\zeta) + [\text{R} * \cos(\zeta) - x_p] * \cos(2 * \zeta) \tag{1}$$

$$y_{\text{anm}} = y_p + [\text{R} * \cos(\zeta) - x_p] * \sin(2 * \zeta) \tag{2}$$

where:

x_{anm} – abscissa of the anamorphic image of the designed point;
y_{anm} – ordinate of the anamorphic image of the designed point;
R – radius of the cylinder of revolution;
x_p – abscissa of the designed point;
y_p – ordinate of the designed point;
$\zeta = \arcsin(y_p/\text{R})$.

The goal of this paper is to present the application elaborated in MSExcel[2] by M. Jonak. This individual application helps to creating an anamorphic image of a designed object using a default function in MS Excel – the function which is called a "dotted diagram with smoothing option". It is possible to precisely define the contours of a diagram while the line weights and colours required may be used for visualization.

Illustration 5 presents a screenshot taken from MSExcel in which a set of parametric equations has been formulated to calculate the coordinates of points on a semi-circle with radius r_1. An anamorphic image of the circle has been calculated according to the equations (1). In ill. 1 we can see a pair of corresponding points A_p and A_{anm} that have been highlighted red: one point belongs to a semi-circle in the design part of the plane and the other represents its anamorphic image and belongs to the active part of the plane.

Now, if we take a look at the reflective surface of a cylinder in such a way that the LOS makes an angle of 45° with the anamorphic picture plane, we will see the image of an ellipse with two axes [r1 i r1/√2] – its axis parallel to the x axis will become foreshortened with the parameter √2 (√2 is the diagonal length in a square with side length 1). The foreshortening factor results from the fact that the Line of Sight (LOS) makes 45° with the anamorphic picture plane.

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[1] I.N. Bronsztejn, K.A. Siemiendiajew, *MATEMATYKA Poradnik encyklopedyczny*, Wydawnictwo Naukowe PWN, Warszawa 2000 (Część druga – III. *Geometria i IV. Trygonometria*).

[2] MS Excel, 2008.

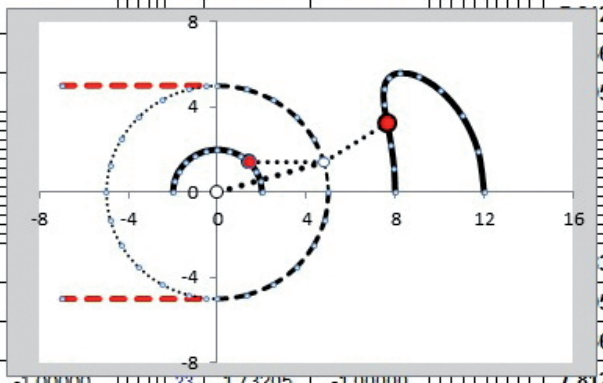
DESCRIPTION

BASE of REFLECTIVE CYLINDER: CIRCLE, RADIUS R, CENTER (0,0)

ANAMORPHIC IMAGE A(xanm,yanm) of A POINT A(xp,yp)

ζ PARAMETER

	R=	5,00						
	r1=	2,00	RADIUS OF THE TRANSFORMED CIRCLE				[1]	[2]
	xo=r1*cos(ζ)	yo=r1*sin(ζ)			x(p)=	y(p)	Xanm=	Yanm=
1	2,00000	0,00000		1	2,00000	0,00000	8,00000	0,00000
2	1,93185	0,51764		2	1,93185	0,51764	7,94922	1,14397
3	1,73205						255	2,24118
4	1,41421						639	3,24903
5	1,00000						513	4,13054
20	0,51764						345	-4,84984
21	1,00000						513	-4,13054
22	1,41421						639	-3,24903
23	1,73205	-1,00000		23	1,73205	-1,00000	7,81255	-2,24118
24	1,93185	-0,51764		24	1,93185	-0,51764	7,94922	-1,14397
25	2,00000	0,00000		25	2,00000	0,00000	8,00000	0,00000



Ill. 5. Exemplary screenshot captured from MSExcel: calculating the case of a circle (= designed geometry) and its transformation into an anamorphic image

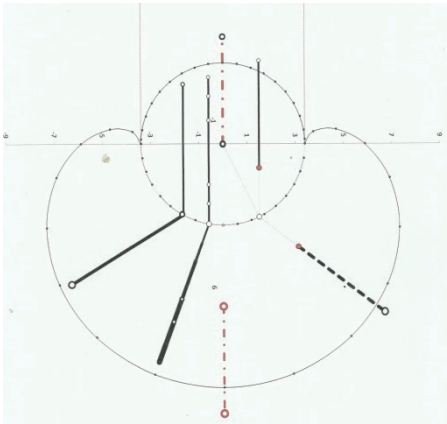
3. Example solutions

The images designed consist only of some parts of the curves (or straight lines) recorded by parametric equations. To the group of cases described here belong those which represent some special positions of geometric entities like the following in a reflected picture: vertical lines, intersecting lines, circles, ellipses etc. ...

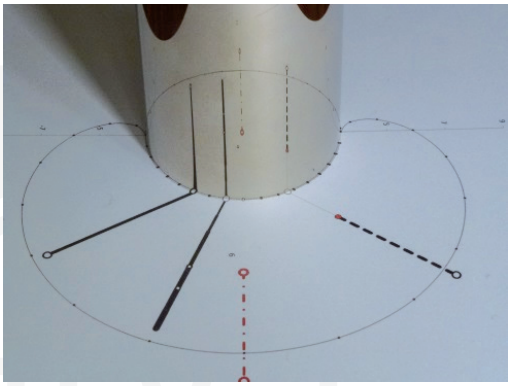
- Ill. 6 Vertical lines, i.e. the lines perpendicular to the anamorphic picture plane, as are the generators of the reflective cylinder's surface. Additionally, a vertical line parallel to the cylinder axis as positioned in the interior of the cylinder;
- Ill. 3 displays the image of the lines positioned in the design part of the cylinder together with their view contained in the active part of the anamorphic plane;
- Ill. 6 displays the view obtained after reflection in a cylindrical surface. It should be noted that anamorphic images of two lines are straight lines when they coincide with the surface generators. The other line – represented with a dashed line – will be reflected as a vertical

line that belongs to the interior of the cylinder. In addition, the line represented with a dash-dot reflects in the cylinder surface as its axis. Let us also notice that the width of the line belonging to the anamorphic picture plane changes along its length which results from the perspective image perception. This change in width will be required if we want to get a uniform linear image in the reflection;

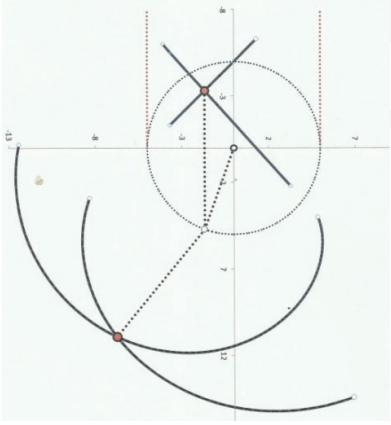
- Ill. 8 – the image of two intersecting lines;
- Ill. 10 – the image of two parallel lines;



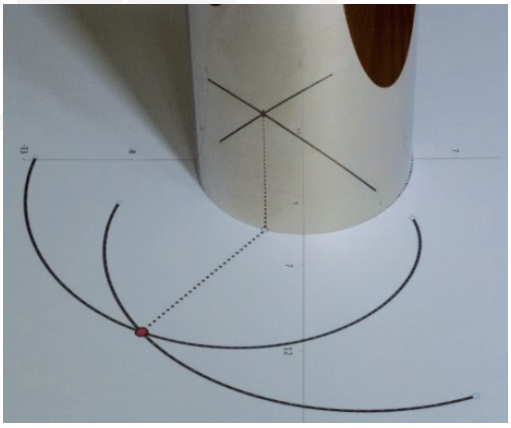
Ill. 6. Vertical lines – a design project in the passive part and the anamorphic image in the active part of the picture plane



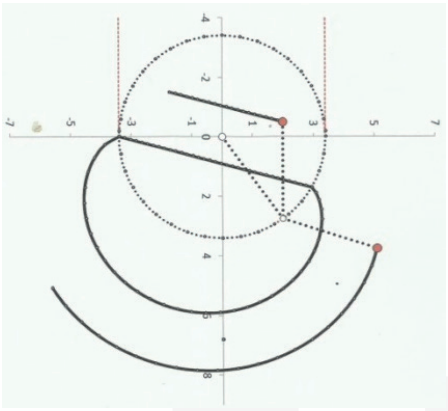
Ill. 7. Anamorphic image of vertical lines as reflected in the cylinder's surface



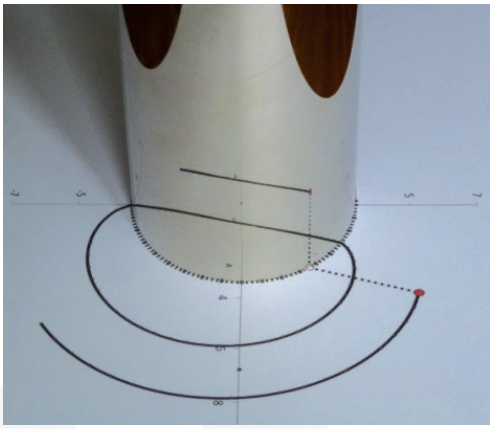
Ill. 8. Two intersecting segments – the design project in the passive part and the anamorphic image in the active part of the picture plane



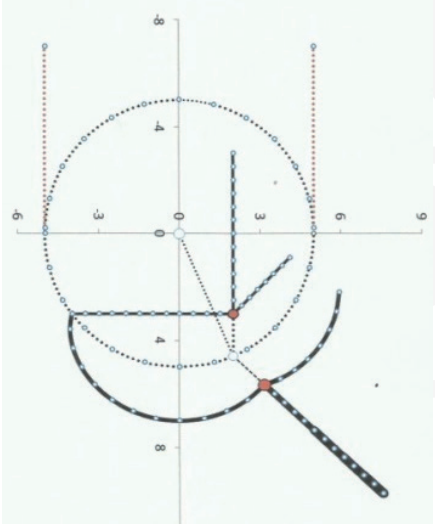
Ill. 9. Anamorphic image of two intersecting segments as reflected in the cylinder's surface



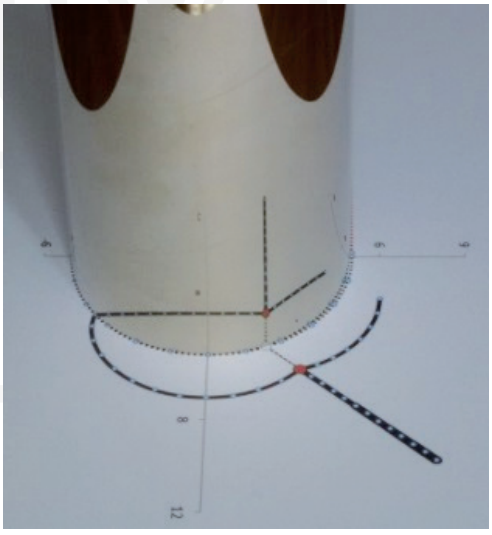
Ill. 10. Two parallel segments – a design project in the passive part and the anamorphic image in the active part of the picture plane



Ill. 11. Anamorphic image of two parallel segments as reflected in the cylinder's surface

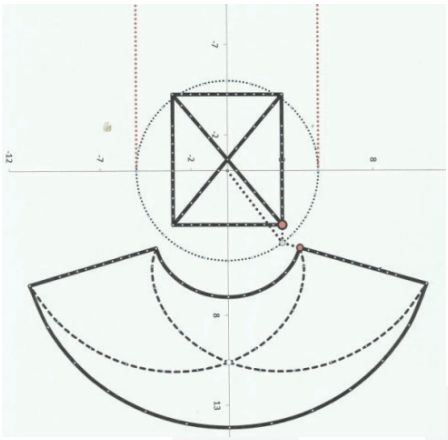


Ill. 12. Three mutually perpendicular segments – a design project in the passive part and the anamorphic image in the active part of the picture plane

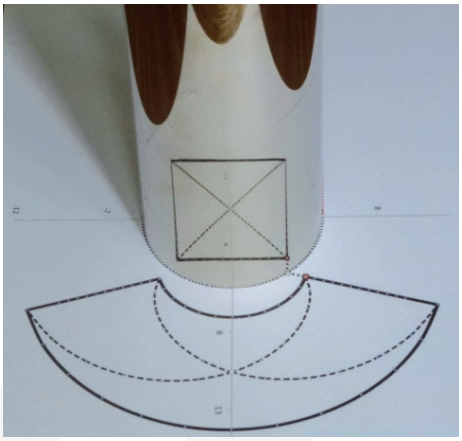


Ill. 13. Anamorphic image of three mutually perpendicular segments as reflected in the cylinder's surface

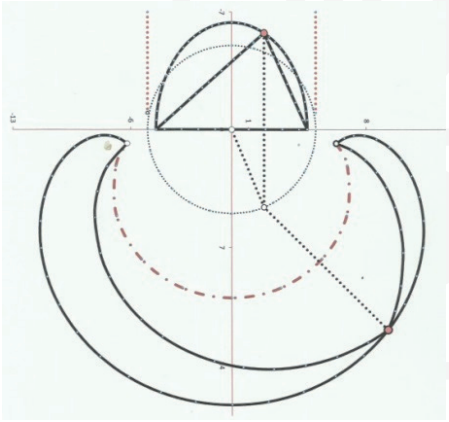
- Ill. 12 – two mutually perpendicular lines intersecting at a single point;
- Ill. 14, 16, 18 – the images of some basic geometric figures: a square, a rectangle, a circle, an ellipse;



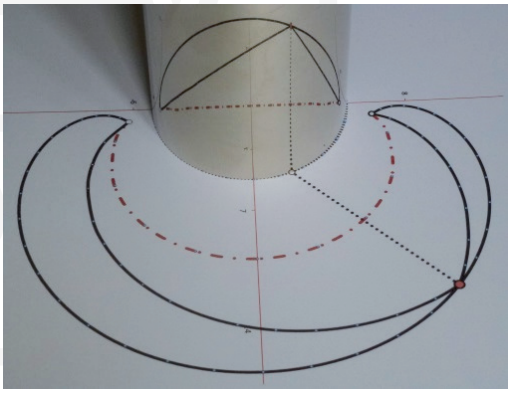
Ill. 14. A square perpendicular to the anamorphic picture plane – a design project in the passive part and the anamorphic image in the active part of the picture plane



Ill. 15. Anamorphic image of a square perpendicular to the anamorphic picture plane as reflected in the cylinder's surface

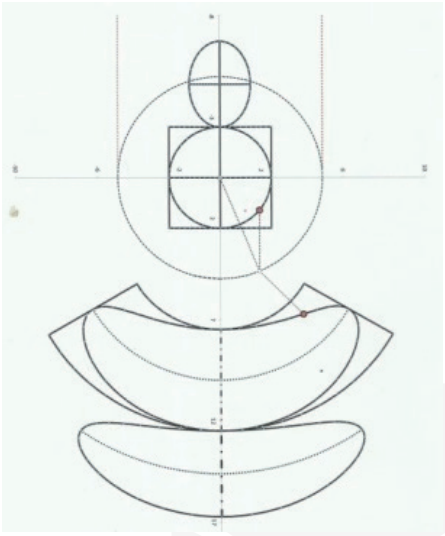


Ill. 16. Right angle inscribed into a semicircle – a design project in the passive part and the anamorphic image in the active part of the picture plane

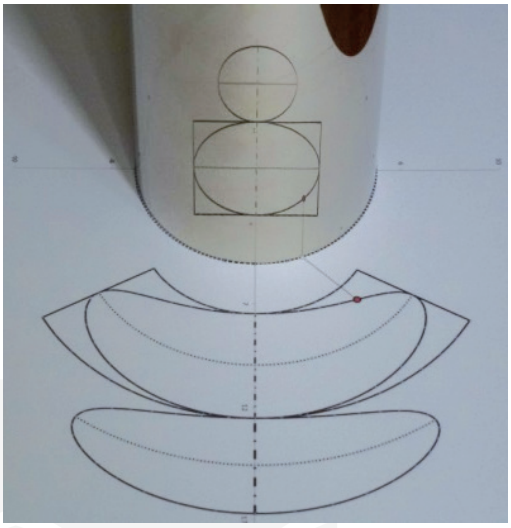


Ill. 17. Anamorphic image of a right angle inscribed into a semicircle as reflected in the cylinder's surface

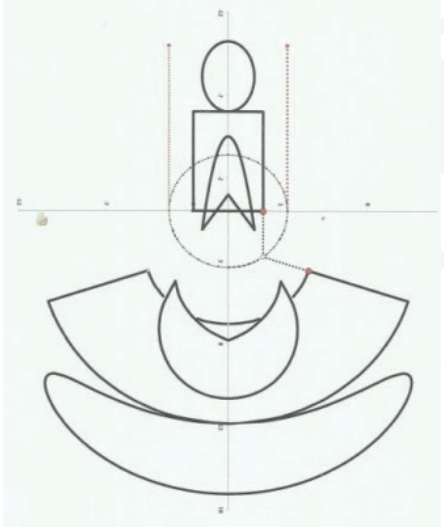
- Ill. 20, 22, 24, 26 – images of some spatial compositions which have been compiled from selected cases:
  - Ill. 20 – a composition of, a rectangle, a circle with a segment of a parabola cut out with a right notch;
  - Ill. 22 – anamorphic image of a right circular cylinder standing on the horizontal picture plane;



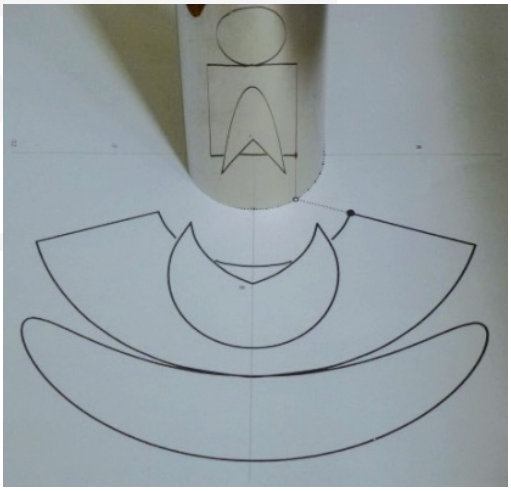
Ill. 18. A circle and an ellipse inscribed into a rectangle – a design project in the passive part and the anamorphic image in the active part of the picture plane



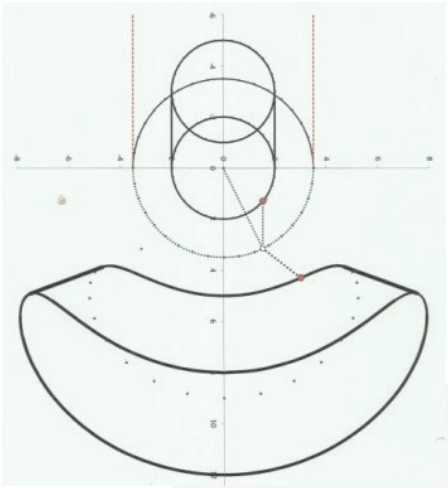
Ill. 19. Anamorphic image of a circle and an ellipse inscribed into a rectangle as reflected in the cylinder's surface



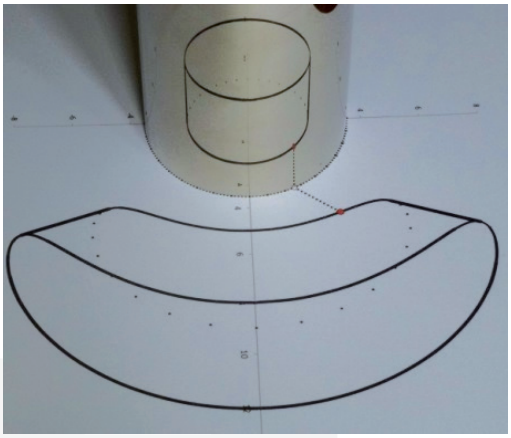
Ill. 20. Two-dimensional composition of planar figures – a design project in the passive part and the anamorphic image in the active part of the picture plane



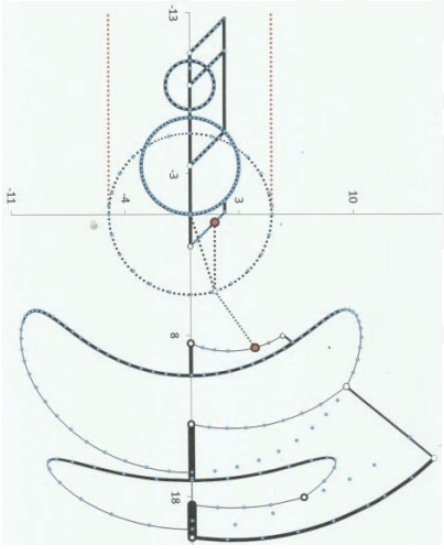
Ill. 21. Anamorphic image of a two-dimensional composition as reflected in the cylinder's surface



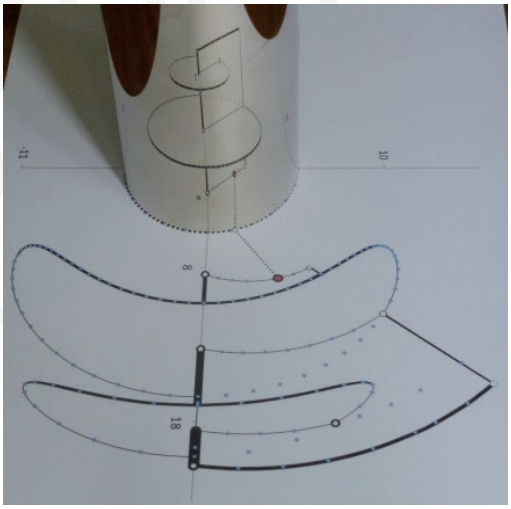
Ill. 22. Cylinder of revolution – a design project in the passive part and the anamorphic image in the active part of the picture plane



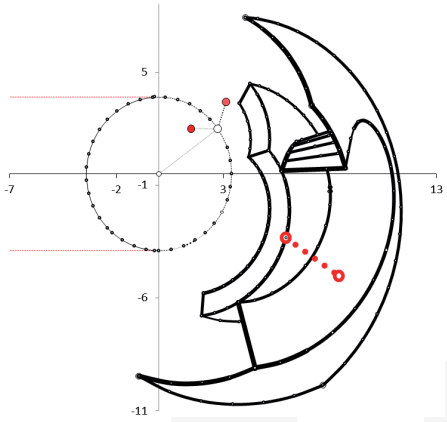
Ill. 23. Anamorphic image of a cylinder of revolution as reflected in the cylinder's surface



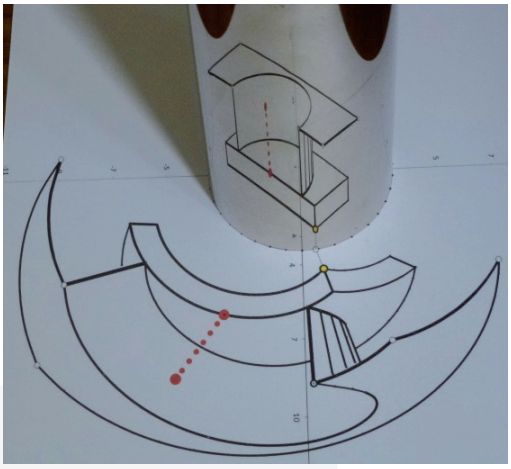
Ill. 24. Spatial object composition – a design project in the passive part and the anamorphic image in the active part of the picture plane



Ill. 25. Three-dimensional anamorphic composition as reflected in the cylinder's surface



Ill. 26. Spatial object – a design project in the passive part and the anamorphic image in the active part of the picture plane



Ill. 27. Three-dimensional anamorphic composition as reflected in the cylinder's surface

All photographs by Marcin Jonak, Andrzej Zdziarski

- Ill. 24 – a composition of two horizontal circles cut half-through with a vertical rectangle;
- Ill. 26 – reflection of the anamorphic image positioned in the active part of the anamorphic picture plane. As the design part of the anamorphic picture the plane has not been explicitly presented here; the result viewed in the reflective cylindrical surface is even more surprising.

The pictures described above have been created as compositions of primitive elements. All the pictures have their design image that have been fixed in the "passive part" and the anamorphic images have been placed in the active part of the anamorphic picture plane (Ill. 6, 8, 10, 12, 14, 16, 18, 20, 22, 24 and 26). The views reflected in the mirroring surface of the cylinder of revolution present spatial compositions as being observed from a specified view-point (Ill. 7, 9, 11, 13, 15, 17, 19, 21, 23, 25 and 27).

In practice, the images generated as described above, or rather the contours of the objects that are contained in the images, may play a special role in design. Further artistic elaboration of the images will usually undergo artistic finishing by means of adding textures, the colours and lineweights. One of the advantages of the method presented here is that the viewed anamorphic images have no deformations and give a good spatial impression when they are observed from a carefully specified viewing point. Besides this, the clarity of the image which can be seen in the reflection is high. Sometimes it happens that the observer will be surprised by the result obtained after seeing the reflected image.

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