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THE LOADING PATTERN'S INFLUENCE ON THE STRESS INCREMENT IN PRESTRESSING STEEL AND BENDING MOMENT RESISTANCE IN MULTI-SPAN MEMBERS POST-TENSIONED WITH UNBONDED TENDONS

WPŁYW SCHEMATU OBCIĄŻEŃ NA PRZYROST NAPRĘŻEŃ W STALI SPRĘŻAJĄCEJ ORAZ NOŚNOŚĆ NA ZGINANIE W ELEMENTACH WIELOPRZĘSŁOWYCH SPRĘŻONYCH CIĘGNAMI BEZ PRZYCZEPNOŚCI

A b s t r a c t

During the design process of members post-tensioned with unbonded tendons, deformations of the whole structure between anchorages, having impact on ultimate value of prestressing force, shall be considered. The conducted researches enabled the separation of several parameters influencing the stress increase in unbonded tendons – i.a. the loading pattern in multi-span members. This paper presents selected codes provisions and theoretical researches describing this factor. Values received from analytical calculations are shown and compared.

Keywords: *design process*, *stress increase in unbonded tendons*, *multi-span members*

S t r e s z c z e n i e

Podczas projektowania elementów sprężonych cięgnami bez przyczepności należy wziąć pod uwagę odkształcenia całej konstrukcji pomiędzy zakotwieniami mające wpływ na graniczną wartość siły sprężającej. W przeprowadzonych badaniach wyodrębniono kilka parametrów wpływających na przyrost naprężeń w cięgnach bez przyczepności – m.in. schemat obciążeń. Artykuł przedstawia wybrane przepisy normowe i podejścia teoretyczne opisujące ten wskaźnik. Zaprezentowano i porównano jego wartości otrzymane z obliczeń teoretycznych.

Słowa kluczowe: *projektowanie*, *przyrost naprężeń w cięgnach*, *elementy wieloprzęsłowe*

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1. Introduction

Knowledge of cross-section internal forces is required for the estimation of bending moment resistance; thus, compression forces in concrete and ordinary reinforcement, tension forces in ordinary and prestressing reinforcement. For this purpose, strain compatibility analysis, strains-stresses relationships for concrete and steel and equilibrium equations introduced both in Polish Code [3] and Eurocode 2 [2] can be used. However, problems connected with establishing the prestressing force value in unbonded tendons are encountered. Prestressing reinforcement stresses can be described using the equations:

– EC2 notation [2]

$$\sigma_{pmt} = \sigma_{pm\infty} + \Delta\sigma_{p,\mathrm{ULS}} \tag{1a}$$

– ACI 318M-14 notation [1]

$$f_{ps} = f_{se} + \Delta f_{ps} \tag{1b}$$

where:

σ_{pmt}, f_{ps} – stress in tendons at ultimate,
$\sigma_{pm\infty}, f_{se}$ – effective prestress in tendons,
$\Delta\sigma_{p,\mathrm{ULS}}, \Delta f_{ps}$ – stress increase in unbonded tendons at ultimate.

Although effective prestress determination can be performed with limited effort, stress increase in unbonded tendons due to external loading is not such an easy task. The encountered problem is caused by the fact that strains increase in unbonded tendons has rather global than local character in comparison with bonded tendons. Ideal bond assumption between concrete and prestressing reinforcement, which could be taken in case of bonded tendons, leads to the same strains changes in prestressing reinforcement and concrete at the tendon's level. On the other hand, neglecting friction between the prestressing reinforcement and the sheath in the case of unbonded tendons results in the same value of stresses along the tendon. Therefore, strain changes in prestressing reinforcement are equal to mean value of strain changes in concrete at tendon's level between anchorages.

Thanks to both theoretical and experimental research, several parameters influencing stress increase in unbonded tendons were distinguished. These are: concrete compressive strength, span-to-depth ratio, type of loading, ordinary and prestressing reinforcement ratios and finally, the loading pattern in the case of multi-span members. A large majority of them are presented in a previous article dealing with stress increase in unbonded tendons at ultimate [10].

2. Codes provisions

Recommendations regarding stresses in unbonded tendons at ultimate could be encountered for the very first time in ACI 318 Code 1963 edition. Due to the small number of tests and corresponding lack of proper knowledge concerning these types of structures, the following very simple and conservative equation was proposed:

$$f_{ps} = f_{se} + 105 \text{ [MPa]} \tag{2}$$

The increasing amount of tests' data and theoretical researches conducted over dozens of years has led to the introduction of three parameters to the equations describing stress increment in unbonded tendons at ultimate. These are: concrete compressive strength f_c', prestressing reinforcement ratio ρ_p and span-to-depth ratio l_{eff}/d_p. Currently used equations are gathered in Table 20.3.2.4.1 of ACI 318-14 [1]. Equation (3) is used for calculating stress in unbonded tendons at ultimate for members with span-to-depth ratio not greater than 35 $(L/d_p \leq 35)$. Equation (4) is valid for more slender members.

$$f_{ps} = f_{se} + 70 + \frac{f_c'}{100 \cdot \rho_p} \text{ [MPa]} \tag{3}$$

with limitations $f_{ps} \leq f_{se} + 420$ MPa; $f_{ps} \leq f_{py}$ i $f_{se} \geq 0.5 f_{pu}$

$$f_{ps} = f_{se} + 70 + \frac{f_c'}{300 \cdot \rho_p} \text{ [MPa]} \tag{4}$$

with limitations $f_{ps} \leq f_{se} + 200$ MPa; $f_{ps} \leq f_{py}$ i $f_{se} \geq 0.5 f_{pu}$.

In chapter 5.10 of Eurocode 2 entitled 'Prestressed members and structures', recommendations regarding such types of structures are gathered. The most crucial facts concerning the above mentioned matter included in paragraph 5.10.8 are as follows:

– for prestressed members with permanently unbonded tendons, it is generally necessary to take the deformation of the whole member into account when calculating the increase of the stress in the prestressing steel,
– if no detailed calculation is made, it may be assumed that the increase of the stress from the effective prestress to the stress in the ultimate limit state is $\Delta\sigma_{p,\mathrm{ULS}}$ with the indication that the recommended value should equal 100 MPa,
– if the stress increase is calculated using the deformation state of the whole member, the mean values of the material properties should be used. The design value of the stress increase $\Delta\sigma_{pd} = \Delta\sigma_p \cdot \gamma_{\Delta P}$ should be determined by applying partial safety factors $\gamma_{\Delta P,\mathrm{sup}}$ and $\gamma_{\Delta P,\mathrm{inf}}$ respectively. The recommended values for $\gamma_{\Delta P,\mathrm{sup}}$ and $\gamma_{\Delta P,\mathrm{inf}}$ are 1.2 and 0.8 respectively. If linear analysis with uncracked sections is applied, a lower limit of deformations may be assumed and the recommended value for both $\gamma_{\Delta P,\mathrm{sup}}$ and $\gamma_{\Delta P,\mathrm{inf}}$ is 1.

Additionally, in chapter 7.2 Eurocode recommends that the mean value of the stress in prestressing tendons should not exceed $0.75 f_{pk}$.

Polish Code [3] (significantly based on Eurocode 2) does not contain different information regarding calculation of stress increase in the unbonded tendons. Recommendations concerning this type of prestressing are described in paragraph 7.1.10 entitled 'Structures post-tensioned without bond'. These recommendations are as follows:

– prestressing force value at Ultimate Limit State equals the design value of the force in the tendon enlarged by the mean increase of the concrete strain along the tendon's duct,

– it is assumed that the stress increase in the internal unbonded tendons equals 100 MPa for a single span length. In the case of a higher number of spans, this value should be reduced considering amount of spans,
– the number of tendons in continuous slabs should be chosen in such a way that releasing prestress in two adjacent tendons will not lead to the destruction of the construction,
– in the case of a single tendon failure, the redistribution of internal forces should be assured by ordinary reinforcement.

Moreover in chapter 7.1.2, Polish Code recommends that the mean value of the stress in prestressing tendons should not exceed 0.65 f_{pk}.

The above review of codes which are used in Poland during the design process indicates their conservativeness with regards to ULS of structures post-tensioned with unbonded tendons. The constant value equal to 100 MPa is given both in Polish Code [3] and Eurocode 2 [2], with no differentiation regarding the type of structures (beams, slabs, tanks etc.). This conforms to the state of art described by the ACI Code from 1963. Even though the Polish Code provides stress reduction necessity in the case of continuous members, it does not specify exact means to be taken in this regard. Although ACI Code [1] takes into account three parameters influencing stress increase in unbonded tendons, the loading pattern is not considered among them. Factors distinguished during regression analysis were calculated based on tests' researches both for single and multi-span members.

3. Theoretical researches

Description trials of the above mentioned phenomenon were conducted by various authors. Two trends could be emphasised among the proposed theories. The first trend introduces the prestressing reinforcement strain reduction factor which allows conducting calculations in a similar way as for members post-tensioned with bonded tendons [8, 11]. The second one refers to plastic hinge length which occurs at ultimate [4–7]. Only theories considering the loading pattern as a parameter used for estimating stress increase in unbonded tendons are described below.

3.1. Naaman et al.

Strain reduction factor defined as mean strain increase in unbonded tendons to strain increase in equivalent bonded tendons in critical cross-section was introduced by Naaman. This is expressed by the equation below:

$$\Omega_u = \frac{(\Delta\varepsilon_{psu})_{av}}{(\Delta\varepsilon_{psb})_m} = \frac{(\Delta\varepsilon_{psu})_{av}}{(\Delta\varepsilon_{cps})_m} \tag{5}$$

where:

$(\Delta\varepsilon_{psu})_{av}$ – average strain increase in unbonded tendons beyond ε_{pe},
$(\Delta\varepsilon_{psb})_m$ – maximum strain increase in bonded tendons beyond ε_{pe},
$(\Delta\varepsilon_{cps})_m$ – maximum strain increase in concrete at the level of the tendon.

During regression analysis, two parameters were taken into consideration: type of loading (one-point loading, third-point loading and uniformly distributed loading) and span-to-depth ratio varying within the limits of 7.8 to 45 – this covers the majority of commonly used beams and slabs. The best convergence between test results and analytical calculations was obtained for the following values of Ω_u:

$$\Omega_u = \frac{2.6}{\left(\frac{L}{d_{ps}}\right)} \quad \text{for one-point loading} \tag{6}$$

$$\Omega_u = \frac{5.4}{\left(\frac{L}{d_{ps}}\right)} \quad \text{for third-point or uniformly distributed loading} \tag{7}$$

Equation describing the value of stress in unbonded tendons at ultimate:

$$f_{ps} = f_{pe} + \Omega_u \cdot E_{ps} \cdot \varepsilon_{cu} \cdot \left(\frac{d_{ps}}{c} - 1\right) \cdot \frac{L_1}{L_2} \tag{8}$$

where:

E_{ps} – prestressing steel modulus of elasticity,
ε_{cu} – ultimate compressive strain in the concrete,
d_{ps} – effective depth of a cross-section,
c – concrete compressive stress block depth,
L_1 – sum of spans lengths under loading,
L_2 – tendon length between anchorages.

3.2. Harajli et al.

In one of the first papers, Harajli [4] connected stress increase in unbonded tendons with the number and length of plastic hinges which occur at ultimate. After few operations, plastic hinge length L_0 could be expressed as a function of loading type and span-to-depth ratio described by the equation below:

$$L_0 = d_p \left[\frac{L}{d_p} \cdot \left(\frac{0.95}{f} + 0.05\right) + 1\right] \tag{9}$$

where:

$f = \infty$ – for one-point loading,
$f = 6$ – for uniformly distributed loading,
$f = 3$ – for third-point loading.

In equation (11), which describes value of stress increment in unbonded tendons, plastic hinge length was expressed by usage of the γ parameter (10). Additionally, by means of regression analysis, two factors α and β (depending on type of loading) were introduced.

$$\gamma = \left[1 + \frac{1}{\frac{L}{d_p} \cdot \left(\frac{0.95}{f} + 0.05 \right)} \right] \cdot \frac{n_0}{n} \tag{10}$$

$$f_{ps} = f_{pe} + \gamma \cdot f_{pu} \cdot \left(\alpha - \beta \cdot \frac{c}{d_p} \right) \tag{11}$$

where:

$\alpha = 0.10$; $\beta = 0.18$ – for one-point loading ($f = \infty$),
$\alpha = 0.25$; $\beta = 0.44$ – for uniformly distributed loading ($f = 6$),
$\alpha = 0.40$; $\beta = 0.70$ – for third-point loading ($f = 3$),
n_0 – number of spans under loading,
n – total number of spans.

In consecutive paper [5] plastic hinge length was expressed as a function of compression zone depth c and type of acting loading (12). The stress increment in continuous members could be calculated as a variable of plastic hinges number n_p and length L_p (13) – its form is described by equation (14) in which compression zone depth c_y is counted by assuming yield strength both in ordinary and prestressing reinforcement (15).

$$L_p = \left(\frac{20.7}{f} + 10.5 \right) \cdot c \tag{12}$$

$$\Delta f_{ps} = E_{ps} \varepsilon_{cu} \left(\frac{d_p - c}{c} \right) \cdot \frac{n_p \cdot L_p}{L} \tag{13}$$

$$f_{ps} = f_{pe} + \phi_{ps} E_{ps} \varepsilon_{cu} \left(\frac{d_p - c_y}{L / n_p} \right) \cdot \left(\frac{20.7}{f} + 10.5 \right) \leq f_{py} \tag{14}$$

$$c_y = \frac{A_{ps} f_{py} + A_s f_y}{0.85 \beta_1 f_c' b} \tag{15}$$

where:

ϕ_{ps} – partial safety factor,
n_p – number of formed plastic hinges at ultimate,
A_{ps} – prestressing reinforcement area,
f_{py} – yield strength of prestressing reinforcement,
A_s – ordinary reinforcement area,
f_y – yield strength of ordinary reinforcement,
β_1 – factor in the Whitney stress block,
b – width of concrete compressive stress block.

Figure 1 shows differences in calculations of the loading pattern factor for two- and three-span members according to equations (8), (10) and (14). It should be pointed out that in all cases, expressions L_1/L_2, n_0/n and n_p/n are equal to 1 for simply supported beams. It can be observed that failure mechanism analysis by considering number of plastic hinges at ultimate enables to obtain greater values of loading pattern factor. Another benefit is that it takes into account and differentiates which span is loaded – the external or the internal. It should be emphasised that applying some loading patterns could lead to a value greater than 1 which is assumed for simply supported members.

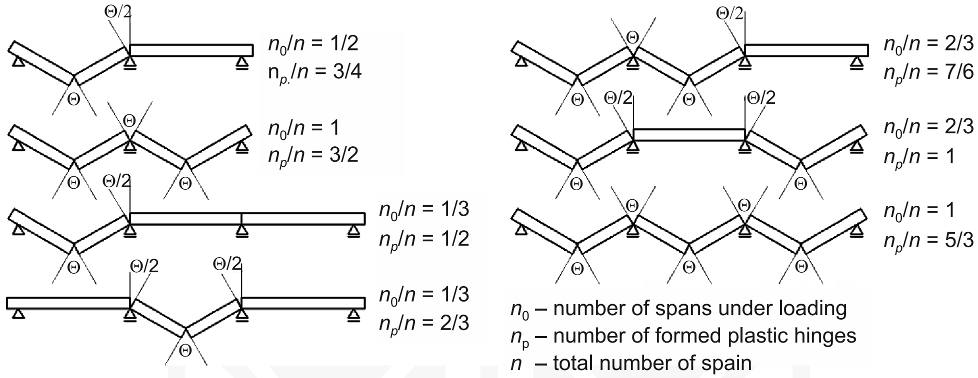

Fig. 1. Load pattern factor values – two- and three-span members

Some discrepancies can be found in over-mentioned theory. In (14) factor f which depends on loading type expresses only one plastic hinge length. It should be added that plastic hinge length might differ in span where different types of loading could be acting (f = 3, 6 or ∞) and at support where reaction should be rather associated with one-point loading ($f = \infty$).

The next paper [6] deals with these doubts by introducing distinction for plastic hinges formed in spans n_p^+ and at supports n_p^-. Both of these are connected and expressed by N_p factor (16). Equation (17) for calculating the stress increase in unbonded tendons at ultimate is a modification of the former equation (14).

$$N_p = \left(\frac{20.7}{f} + 10.5\right) \cdot n_p^+ + 10.5 \cdot n_p^- \tag{16}$$

$$f_{ps} = f_{pe} + \frac{\phi_{ps} \cdot N_p \cdot E_{ps} \cdot \varepsilon_{cu}}{L / d_p} \cdot \left[1 - \frac{c_y}{d_p}\right] \le 0.95 f_{py} \tag{17}$$

The method of calculating the N_p factor in accordance with equation (16) is presented in Fig. 2. The numbers of plastic hinges in spans n_p^+ and at supports n_p^- are presented. Moreover, two values of this factor which depend on the type of loading are presented for one-point loading (1P) and uniformly distributed loading (q) respectively. It is worth emphasising that this value for simply supported beams is 10.5 and 14 accordingly.

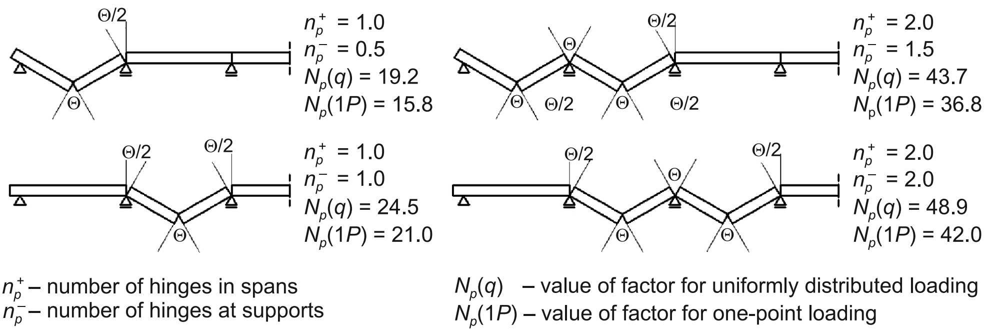

Fig. 2. Load pattern factor calculation with span and support hinge distinction taken into account

The following assumptions and limitations regarding the formation of plastic hinges are introduced:

- all plastic hinges behave similarly i.e. concrete compressive block depth c, depth and area of prestressing reinforcement d_{ps} and A_{ps} and ordinary reinforcement d_s and A_s are the same or very similar in all spans and support cross-sections,
- the section is rectangular or has rectangular section behaviour,
- stress increase at ultimate above the effective prestress Δf_{ps} is assumed to be not greater than $(0.95f_{py} - f_{se})$ – this ensures that the stress in tendons will not reach yield strength of prestressing reinforcement.

4. Conclusions

The above presented code recommendations treat unbonded tendons stress increase in continuous members in a superficial manner. ACI Code design equations for calculating stress increase do not make distinctions between simply supported and continuous members. The opportunity to achieve lower values of stress increase in multi-span members compared to simply supported elements in the case of loading which does not act at all spans simultaneously is disregarded by EC 2. Even though such a possibility is mentioned in Polish Code, no detailed provisions are given.

Due to this fact, directions for solving the problem of stress increment in multi-span unbonded members are searched and can be successfully found in theories proposed by various authors. All of the above presented equations in continuous members are expansions of equations derived firstly for simply supported members. Hence, the loading pattern factor for simply supported beams is equal to 1.

Table 1 contains a comparison of parameters needed to calculate the loading pattern factor utilising equations (10, 14 and 16) for members for which the span number is not greater than 3. Span names are signed with first alphabet letters as shown in Fig. 3. Due to symmetry and assumption that all spans lengths are equal not all combinations are being considered (e.g. in three-span member separate loading of external spans A and C, external and internal spans A+B and B+C will produce the same value of loading pattern factor).

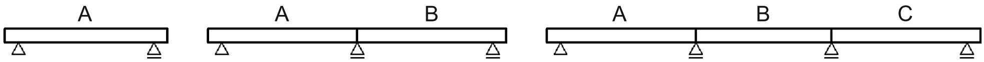

Fig. 3. Span nomenclature – simply supported, two-span and three-span members

Table 1

Loading pattern factor value for simply supported, two-span and three-span members

Type of member	**One-span**	**Two-spans**		**Three-spans**				
Loaded spans	**A**	**A**	**A+B**	**A**	**B**	**A+B**	**A+C**	**A+B+C**
Total number of spans n	1	2		3				
Number of loaded spans n_0	1	1	2	1	1	2	2	3
Value of factor n_0/n – eq. (10)	**1**	**1/2**	**1**	**1/3**	**1/3**	**2/3**	**2/3**	**1**
Number of plastic hinges n_p	1	1 1/2	3	1 1/2	2	3 1/2	3	5
Value of factor n_p/n – eq. (14)	**1**	**3/4**	**1 1/2**	**1/2**	**2/3**	**1 1/6**	**1**	**1 2/3**
Number of plastic hinges n_p^+	1	1	2	1	1	2	2	3
Number of plastic hinges n_p^-	0	1/2	1	1/2	1	1 1/2	1	2
$N_p(1P)$ value – eq. (16) $f = \infty$	10.5	15.8	31.5	15.8	21.0	36.8	31.5	52.5
$N_p(q)$ value – eq. (16) $f = 6$	14.0	19.2	38.4	19.2	24.5	43.7	38.4	62.9
Factor $N_p(1P)/(n{\cdot}N_p\,(1\ \text{span}))$	**1**	**3/4**	**1 1/2**	**1/2**	**2/3**	**1 1/6**	**1**	**1 2/3**
Factor $N_p(1q)/(n{\cdot}N_p\,(1\ \text{span}))$	**1**	**2/3**	**1 3/8**	**1/2**	**3/5**	**1**	**1**	**1 1/2**

Observation of loading pattern factor values gathered in Table 1 leads to the following conclusions regarding two- or three-span members:

– in the case of only one span loaded stress increase in unbonded tendons will be lower than that calculated for simply supported member. This effect is greater for three-span than for two-span members; therefore, for members containing additional spans, this phenomenon will intensify,

– in the case of all spans loaded in two-span or at least two spans loaded in three-span, member stress increase in unbonded tendons will be greater or at the very least, the same as for simply supported members. Once again, this effect is greater for three-span than for two-span members; therefore, for members containing additional spans this phenomenon will intensify,

– loads acting in internal spans give greater values of load pattern factor than in the case of external span loading – this is caused by a higher number of formed plastic hinges.

Usually, when designing ordinary reinforced structures or structures post-tensioned with bonded tendons, we get into the habit of checking resistance in crucial cross-sections using loading patterns which give maximum bending moments. In the case of three-span members, the maximum moment in span A occurs when spans A and C are loaded simultaneously. The maximum moment at internal support appears when spans A and B (or B and C) are loaded simultaneously.

The design of multi-span structures post-tensioned with unbonded tendons could be different to the one described in the above scheme. In some cases, Ultimate Limit State could be reached by different loading patterns; for example, both span and support bending moments caused by loading only one exterior span (A) would be lower than loading both exterior spans (A+C) and exterior and interior span (A+B) respectively. It should be emphasised that stress increase in unbonded tendons which has an influence on the bending moment resistance would also be lower. In some cases, loading of only one span could result in the bending resistance reduction being greater than the decrease of bending moment produced by external loading. It could be anticipated that this effect would be greater for members with a higher number of spans. Ultimate Limit State would not be reached by a sophisticated loading pattern but for simple scheme where one, external span is loaded.

References

[1] Building Code Requirements for Structural Concrete (ACI 318M–14).

[2] PN-EN 1992-1-1:2004 Eurokod 2: Projektowanie konstrukcji z betonu – Część 1: Reguły ogólne i reguły dla budynków (Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings).

[3] PN-B-03264:2002 Konstrukcje betonowe, żelbetowe i sprężone. Obliczenia statyczne i projektowanie (Plain, reinforced and prestressed concrete structures. Analysis and structural design).

[4] Harajli M., Hijazi S., *Evaluation of the Ultimate Steel Stress in Partially Prestressed Concrete Members*, PCI Journal, Vol. 36, No. 1 1991, 62-82.

[5] Harajli M., *On the Stress in Unbonded Tendons at Ultimate: Critical Assessment and Proposed Changes*, ACI Structural Journal, Vol. 103, No. 6, 2006, 803-812.

[6] Harajli M., *Tendon Stress at Ultimate in Continuous Unbonded Post-Tensioned Members: Proposed Modification of ACI 318, Eq. (18-4) and (18-5)*, ACI Structural Journal, Vol. 109, No. 2, 2012, 183-192.

[7] Lee L.H., Moon J.H., Lim J.H., *Proposed Methodology for Computing of Unbonded Tendon Stress at Flexural Failure*, ACI Structural Journal, Vol. 96, No. 6, 1999, 1040-1048.

[8] Naaman A., Alkhairi F., *Stress at Ultimate in Unbonded Post-Tensioning Tendons: Part II – Proposed Methodology*, ACI Structural Journal, Vol. 88, No. 6, 1991, 683-692.

[9] Politalski W., *Przyrost naprężeń w wewnętrznych cięgnach bez przyczepności w betonowych, sprężonych elementach zginanych*, Rozprawa doktorska (*Stress increase in internal unbonded tendons in prestressed concrete flexural members. Doctoral thesis*), July 2015.

[10] Seruga A., Politalski W., *Graniczne naprężenia stali sprężającej w elementach zginanych sprężonych cięgnami bezprzyczepnościowymi*, Czasopismo Techniczne, 1-B/2008, 109-133.

[11] Yang K. H., Kang H.-K., *Equivalent Strain Distribution Factor for Unbonded Tendon Stress at Ultimate*, ACI Structural Journal, Vol. 108, No. 2, 2011, 217-226.