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MODELLING OF A PERMANENT MAGNET ELECTRICAL MACHINE WITH STATOR CORE ANISOTROPY

MODELOWANIE MASZYN ELEKTRYCZNYCH WZBUDZANYCH MAGNESAMI TRWAŁYMI Z ANIZOTROPOWYM RDZENIEM STOJANA

A b s t r a c t

This article deals with the methodological aspects of the modelling and field analysis of machines with surface mounted permanent magnets on the rotor, taking into account the anisotropic properties of the stator core. These considerations are intended to show the effect of anisotropy on the structure of the mathematical model of the machine and answer the question of whether if based on classical modelling, it is possible to take into account the rotational magnetization phenomena. The results explain the question, how the magnetic anisotropy effect at rotational magnetization is important from an exploitation point of view.

Keywords: *magnetic anisotropy*, *PM synchronous machine*

S t r e s z c z e n i e

Temat podjęty w pracy dotyczy metodycznych aspektów modelowania oraz analiz wyników obliczeń polowych maszyn z powierzchniowo montowanymi magnesami trwałymi na wirniku z uwzględnieniem anizotropii magnetycznej rdzenia stojana. Rozważania te mają na celu pokazanie wpływu anizotropii na strukturę modelu matematycznego maszyny i udzielenie odpowiedzi na pytanie, czy przy oparciu się na klasycznych założeniach stosowanych w modelowaniu obwodowym możliwe jest jej uwzględnienie. Przedstawione wyniki obliczeń polowych wykonane dla maszyny modelowej pozwalają określić, na ile efekt anizotropii przy magnesowaniu obrotowym jest istotny z punktu widzenia eksploatacyjnego.

Słowa kluczowe: *anizotropia magnetyczna*, *maszyna synchroniczna z magnesami trwałymi*

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1. Introduction

The phenomenon of rotational magnetization is usually omitted in 'classical' models of electrical machines [1]. Studies considered in this paper refer to the modelling of machines which have permanent magnets (PM) mounted on the rotor surface. Anisotropic properties of the stator core can be taken into account with the assumption of linear magnetization characteristics along the rolling direction (RD) and the transverse direction (TD). On the one hand, these considerations are intended to show the influence of the anisotropy on the structure of a mathematical model of a given machine. On the other hand, we expect the answer as to whether it is possible to take into account the anisotropy based on classical assumptions of the circuit modelling.

The Lagrangian formalism is one of the most convenient methods for the circuit modelling of electrical machines [2, 3]. This formalism also refers to machines which have permanent magnets in their structure – under the assumption of certain general conditions. For today's rare earth magnets (NdFeB types) we can assume that the operating point of the permanent magnet moves along an unambiguous demagnetization curve caused by the impact of winding currents. Thanks to this assumption, we obtain possibilities of using the Lagrangian formalism for modelling machines with permanent magnets, even for cases where the magnetic circuit has anisotropic properties.

The total co-energy which occurs in the magnetic circuit of the permanent magnets machine (PMM) is a sum of the co-energy $E_{0\Theta}$ generated by winding currents and the co-energy $E_{0\mathrm{PM}}$ generated by permanent magnets in a currentless state. For typical constructions of PMMs, the equivalent air gap is relatively wide. Therefore, we can consider that corrections which allow modelling the anisotropic properties refer to the magnetic field excited by permanent magnets. As a result of the stator core anisotropy, differences between the magnetic permeability μ_{rFeRD} in the rolling direction RD and permeability μ_{rFeTD} in the transverse direction TD occur. The function of the field co-energy and the function of the equivalent air gap width should be appropriately modified with respect to the classical considerations [4, 5]. Determination of the mathematical model parameters of the machine with surface-mounted permanent magnets on the rotor, which describe the magnetic anisotropy of the stator core, requires finite element analysis FEA of the magnetic field.

Studies which present the results of research in this area concern mainly synchronous motors, and they focus on the impact of the stator core anisotropy on the cogging torque [6, 7]. Frequently, these studies relate to modelling methods of magnetic anisotropy in cores constructed of oriented or non-oriented steel sheets [8, 9]. Effects caused by magnetic core anisotropy were presented in [10]. Machines with permanent magnets are characterized by relatively high values of the air gap width. Thus, influence of the stator core anisotropy can be considered in the linear range with the use of formulations given in [2].

2. Lagrange's equations of the machine excited by permanent magnets

As it was previously mentioned, the total magnetic field co-energy which occurs in the magnetic circuit of the PMM is a sum of the co-energy of introduced through winding currents and the co-energy generated by permanent magnets in the currentless state. A general form of this function for a three-phase machine can be written as follows:

$$E_0(\varphi, i_1, i_2, i_3) = E_{0\Theta}(\varphi, i_1, i_2, i_3) + E_{0\text{PM}}(\varphi) \tag{1}$$

where:

$E_{0\Theta}(\varphi, i_1, i_2, i_3)$ – co-energy introduced into the system through the winding currents in the presence of magnets,

$E_{0\text{PM}}(\varphi)$ – co-energy introduced into the system by PM in currentless state.

Lagrange's equations of the model of the PMM can be written in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial E_{0\Theta}(\varphi, i_1, i_2, i_3)}{\partial i_i} = u_i - R_\mathrm{s} \cdot i_i \quad \text{for} \quad i = 1, 2, 3$$

$$J\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = \frac{\partial E_{0\Theta}(\varphi, i_1, i_2, i_3)}{\partial \varphi} + \frac{\partial E_{0\text{PM}}(\varphi)}{\partial \varphi} + T_1 - D\frac{\mathrm{d}\varphi}{\mathrm{d}t} \tag{2}$$

Iron cores of the stator and rotor have a very high magnetic conductivity with respect to the air and magnet materials. Therefore, in classical considerations, it is usually assumed that the energy concentration of the magnetic field occurs mainly in the volume of the air gap and the permanent magnets. This means that the magnetic voltage drops in iron yokes of the given machine can be neglected. In order to take into account the effect of the stator core anisotropy, we should resign from classical assumptions and take into account magnetic voltage drops in iron parts of the stator and the rotor. It is worth underlining that differences between the iron magnetic permeability in the rolling direction (RD) and the permeability in the transverse direction (TD) occur during the rotational magnetization.

The easiest way to take into account the magnetic voltage drops in iron is to add these drops to the magnetic voltage drops occurring in the air gap, on this basis, the magnetic field distribution should be corrected. The consequence of this approach is the local increase of the air gap width for the transverse direction (TD). Thus, the co-energy function should be appropriately modified with respect to classical electrical machines.

3. Magnetic field distribution in PMM with taking into account the anisotropy

The equivalent air gap of the PMM is relatively large. Therefore, it can be assumed that the local increase of the air gap, which represents the anisotropy in the TD axis, occurs mainly for magnetic field lines associated with the flux of the permanent magnet. Inductances of the simplest models of classical machines are calculated on the basis of the distribution of the field radial component in the air gap, because it is possibly due to the specific structure of the magnetic circuit. The geometry of the magnetic circuit of the PMMs is varied and not always simple relations, which are sufficiently accurate for conventional machines, can be used for machines with permanent magnets. Regardless of winding inductances, for these machines the linkage fluxes and the co-energy should be determined as a dependence on the rotational angle in currentless state.

Firstly, this paper focuses on machines with a cylindrical rotor which has surface mounted magnets. For this case, in analytical models of the magnetic field distribution in the air gap, the geometry of the magnetic circuit is characterized by a function of the unit permeance [2–5]. For machines with a smooth surface air gap, these formulas are relatively simple. However, this problem gets complicated when the anisotropy of stator steel sheets need to be taken into account. In order to show the methodology of analysis of the field distribution in the air gap, we used a model of the PMM with a smooth stator (Fig. 1).

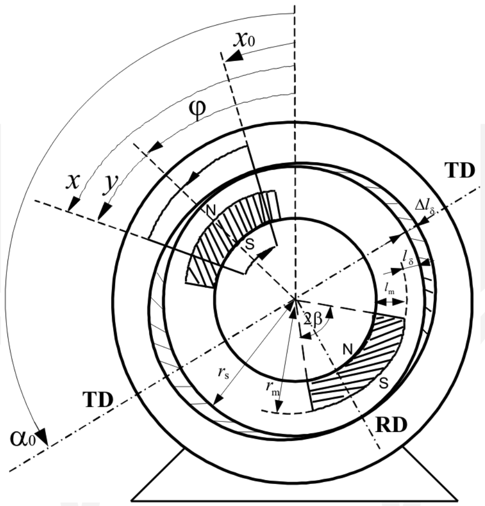

Fig. 1. Cross-section of the PMM with the stator anisotropy

We assumed that the demagnetization curve of the permanent magnet is linear $B_m = B_r + \mu_0 \cdot \mu_{rm} \cdot H_m$; this approximation is widely acceptable for today's rare earth permanent magnets. The form of Ampere's law for the selected contour marked in Fig. 1 allows us to formulate a relationship which describes a simplified one-dimensional field distribution in the air gap of the given PMM:

$$B(x,\varphi,i_1,i_2,i_3) = B_\Theta(x,\varphi,i_1,i_2,i_3) + B_{\mathrm{PM}}(x,\varphi) \tag{3}$$

where:

$$B_\Theta(x,\varphi,i_1,i_2,i_3) = \Theta_{\mathrm{s}}(x,i_1,i_2,i_3)\cdot\lambda(x,\varphi) + C_\Theta(x,\varphi,i_1,i_2,i_3) \tag{4}$$

$$B_{\mathrm{PM}}(x,\varphi) = B_{\mathrm{m}}(x-\varphi)\frac{\lambda(x,\varphi)}{\lambda_{\delta\mathrm{m}}} + C_{\mathrm{PM}}(x,\varphi) \tag{5}$$

$B_\Theta(x,\varphi,i_1,i_2,i_3)$ – radial component of the magnetic field density distribution generated by winding currents along the circumference of the air gap,

$B_{\mathrm{PM}}(x, \varphi)$ – radial component of the magnetic field density distribution in the air gap generated by permanent magnets,

$\lambda_{\delta m} = \dfrac{\mu_0}{l_\delta + l_m / \mu_{rm}}$ – unit permeance for machines with stator smooth cylindrical surface,

$\Theta_s(x, i_1, i_2, i_3)$ – magnetomotive force (MMF) distribution of the stator windings,

$B_m(x - \varphi)$ – function describing one-dimensional distribution of the magnetic flux density along the circumference in the PMM in the currentless state.

The coefficients C_Θ and C_{PM} in formulas (4), (5), which result from Gauss's law for magnetism, are equal to zero in the considered cases. In general, a magnetic circuit can be characterized by the unit permeance function $\lambda(x, \varphi)$, the distribution of which can be written in the form of a double Fourier series [3]:

$$\lambda(x,\varphi) = \sum_{m \in M} \sum_{n \in N} \lambda_{m,n} \cdot e^{\mathrm{j}mx} \cdot e^{\mathrm{j}n\varphi} \tag{6}$$

The function of the unit permeance allows modelling of the real shape of the magnetic circuit, therefore in the general case, sets M and N in the formula (6) can include all integers. The function of the magnetic field distribution in the one-dimensional model in the currentless state (smooth stator and anisotropy is neglected) is the function of one variable.

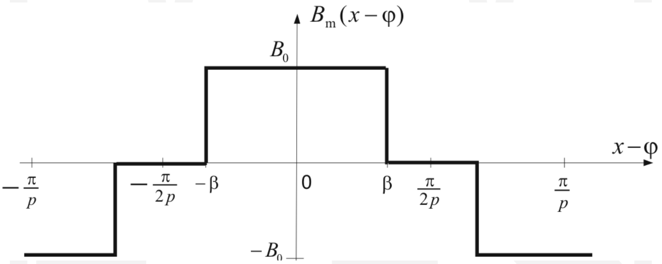

Fig. 2. Flux density distribution in the air gap in a slotless machine without taking into account the stator core anisotropy

The coefficients of the Fourier series of the field distribution function presented in Fig. 2 can be determined on the basis of the following formulas:

$$B_m(x-\varphi) = \sum_{\varsigma \in Q} B_{m\varsigma} \cdot e^{\mathrm{j}\varsigma(x-\varphi)} \tag{7}$$

where:

$$Q = \{\ldots -5p, -3p, -p, p, 3p, 5p \ldots\}; \;\; B_{m\varsigma} = \frac{2}{\pi}\frac{B_0}{\varsigma} p \cdot \sin(\varsigma \cdot \beta); \;\; B_0 = B_r \frac{l_m / \mu_{rm}}{l_m / \mu_{rm} + l_\delta}.$$

For special cases, when it is assumed a smooth cylindrical surface of the stator and a uniform air gap, the function of the unit permeance (6) of the anisotropic machine only becomes dependent on x because properties of permanent magnets are similar to air properties ($\mu_{rm} \cong 1$):

$$\lambda(x,\varphi) = \lambda(x) = \frac{\mu_0}{l_\delta + \Delta l_\delta(x) + l_m / \mu_{rm}} \tag{8}$$

where: $\Delta l_\delta(x)$ is a function which increases the equivalent width of the air gap due to the occurrence of magnetic voltage drops caused by the anisotropy of the stator core.
Thus, for anisotropic cores with smooth surfaces of the stator and the rotor, the set M contains harmonics {…, –4, –2, 0, 2, 4, …}, however, the set N only contains the zero harmonic {0}:

$$\lambda(x) = \sum_{m\in M} \lambda_{m,0} \cdot e^{jmx} \tag{9}$$

The consequence of this assumption is the fact that the inductances of the stator windings do not depend on the rotation angle, and the PM flux density distribution in the air gap takes a simplified form:

$$B_{\mathrm{PM}}(x,\varphi) = \sum_{m\in M}\sum_{\varsigma\in Q} B_{m\varsigma} \frac{\lambda_{m,0}}{\lambda_{\delta m}} e^{\mathrm{j}(m+\varsigma)x} \cdot e^{-\mathrm{j}\varsigma\varphi} \tag{10}$$

4. Form of the co-energy function of the PMM whilst taking into account the anisotropy

For methodological purposes, the form of the co-energy function is presented for a simplified case of the anisotropic machine with smooth cylindrical surfaces of the stator and the rotor. The functions of the winding characteristics have an additional element which represents the flux linkage produced by PM $\psi_{\mathrm{PM}a}(\varphi)$; this flux depends on the rotation angle φ. For example, the relationship between the flux and the current of the a winding can be written as follows:

$$\psi_a(\varphi, i_1, i_2, i_3) = L_{a1} \cdot i_1 + L_{a2} \cdot i_2 + L_{a3} \cdot i_3 + \psi_{\mathrm{PM}a}(\varphi) \tag{11}$$

As a consequence, the co-energy component dependent on winding currents takes the following form [2]:

$$E_{0\Theta}(\varphi, i_1, i_2, i_3) = \frac{1}{2}\sum_{a=1}^{3}\sum_{b=1}^{3} L_{ab} \cdot i_a \cdot i_b + \sum_{a=1}^{3} \psi_{\mathrm{PM}a}(\varphi) \cdot i_a \tag{12}$$

In order to determine the characteristics of the windings (flux dependences), we assumed that the Fourier spectrums contain the v harmonic which belong to the P set. Elements of this set for the windings with an integer number of slots per pole and phase are as follows: $P = \{\ldots, -5p, -3p, -p, p, 3p, 5p, \ldots\}$. Furthermore, we assumed that windings a and b, which have w_s number of turns, are characterized by winding factor $k_u^{|\nu|}$. Thus, inductances can be determined on the basis of the relation [2–4]:

$$L_{ab} = \sum_{\nu \in P} \sum_{m \in M} Q_1 \frac{2 r_s \cdot l_c}{\pi} \frac{(w_s)^2 \cdot k_u^{|\nu|} \cdot k_u^{|\nu+m|}}{|\nu| \cdot |\nu + m|} \lambda_{m,0} \cdot e^{\mathrm{j}(\nu+m)x_a} \cdot e^{-\mathrm{j}\nu x_b} \tag{13}$$

where:
l_c – equivalent axial length of the given machine,
Q_1 – parameter depends on elements of the P, M sets and it is defined as follows:

$$Q_1 = \begin{cases} 1 \Leftrightarrow \forall \nu, \forall m, \nu \in P \wedge m \in M \wedge (\nu + m) \in P \\ 0 \qquad \text{in opposite case} \end{cases} \tag{14}$$

For the three-phase stator, the angles between winding magnetic axes are equal to:

$$x_a = (a-1)\frac{2\pi}{3p}, \; x_b = (b-1)\frac{2\pi}{3p} \;\; \text{for } a, b = 1, 2, 3$$

The component of the magnetic flux which is linked with a winding in currentless state is written as [2]:

$$\Psi_{\mathrm{PM}a}(\varphi) = \sum_{\varsigma \in Q} \sum_{m \in M} D_1 \frac{2\, r_{\mathrm{s}}\, l_{\mathrm{c}}}{\lambda_{\delta m}} B_{m\varsigma} \frac{w_s\, k_{\mathrm{u}}^{|\varsigma+m|}}{|\varsigma + m|} \lambda_{m,0} \cdot \mathrm{e}^{\mathrm{j}(\varsigma+m)x_a} \cdot e^{-\mathrm{j}\varsigma\varphi} \tag{15}$$

Parameter D_1 depends on the content of sets P, Q, M and it is defined as:

$$D_1 = \begin{cases} 1 \Leftrightarrow \forall \varsigma, \forall m, \varsigma \in Q \wedge m \in M \wedge (\varsigma + m) \in P \\ 0 \qquad \text{in opposite case} \end{cases} \tag{16}$$

The co-energy component which is independent of winding currents is associated with areas of energy occurrence (air gap, permanent magnets). Analytical calculation of the co--energy in the currentless state is not a simple task when real shapes of the magnetic circuit need to be taken into account. We propose a simplified approach which is based on 1-D field distribution. If the previously defined function of the unit permeance function (6) is taken into account, the co-energy component for the currentless state takes the form:

$$E_{0\mathrm{PM}}(\varphi) = \frac{r_m\, l_c}{2} \int_0^{2\pi} \frac{\left[B_{\mathrm{PM}}(x,\varphi)\right]^2}{\lambda(x,\varphi)} \, \mathrm{d}x \tag{17}$$

In the particular case in which the simplified forms of the permeance function (6) and the flux density distribution are taken into consideration, we obtain the relationship describing the co-energy in the currentless state in the following form:

$$E_{0\mathrm{PM}}(\varphi) = \frac{l_c \cdot r_m}{2(\lambda_{\delta m})^2} \int_0^{2\pi} [\lambda(x,\varphi) \cdot B_m(x-\varphi)^2]\, \mathrm{d}x \tag{18}$$

where:

$$B_m(x-\varphi)^2 = \sum_{k \in K} B_{mk}^2 \cdot \mathrm{e}^{\mathrm{j}k(x-\varphi)} \tag{19}$$

$$K = \{\ldots -4p, -2p, 0, 2p, 4p, \ldots\};\;\; B_{mk}^2 = \begin{cases} \dfrac{2}{\pi}(B_0)^2 \cdot p \cdot \beta & \text{for} \quad k = 0 \\ \dfrac{2}{\pi}\dfrac{(B_0)^2}{k}\, p \cdot \sin(k \cdot \beta) & \text{for} \quad k \neq 0 \end{cases} \tag{20}$$

After taking into account relationships (18), (19) and (20), and formal mathematical operations, we obtain the simplified formula describing the co-energy in the currentless state:

$$E_{0\mathrm{PM}}(\varphi) = \frac{\pi \cdot l_c \cdot r_m}{(\lambda_{\delta m})^2}\, \mathrm{Re}\, \{\sum_{k \in K} \lambda_{-k,0} \cdot B_{mk}^2 \cdot e^{-\mathrm{j}k\varphi}\} \tag{21}$$

5. Field calculations of the PMM whilst taking into account the anisotropy

Field analysis was carried out with the use of the MagNet package for the model of the exemplary machine which is shown in Fig. 1 under the assumption that the magnetization characteristics of iron and permanent magnets are linear. The anisotropy was taken into account by introducing appropriately different values of the magnetic permeability of the stator core for the rolling direction ($\mu_{r\mathrm{FeRD}}$ = 10 000) and for the transverse direction (μ_{rFeRD} = 5000). The assumption of the linear characteristics was related to methodological aspects of the analysis, these were intended to emphasize only the influence of the phenomenon of the anisotropy.

The basic construction parameters of the considered machine are as follows: external radius of the stator yoke – 120 mm; internal radius of the stator yoke r_s = 71 mm; packet length l_c = 100 mm; number of stator slots Z_s = 36; opening width of the stator slots 4 mm; number of pole pairs p = 1; double-layer winding composed of a 12 serial elementary coils; number of coils forming a one pair of poles – 6; total number of stator turns w_s = 120; stator winding pitch – 15; external radius of the rotor core – 64 mm; internal radius of the rotor core – 27 mm; length of the air gap over the magnet l_δ = 2 mm; thickness of the magnet l_m = 5 mm; PM angular pitch 2β = 120°; parameters of the PM: B_r = 1.1 T; H_c = 827.6 kA/m; μ_{rm} = 1.1.

Test calculations were performed on the basis of the field model of the two-poles machine – this model was formulated in the MagNet 2D Static and is shown in Fig. 3. In the first step, magnetic field distributions in the air gap of machines with the isotropic and anisotropic stator were compared.

a) b)

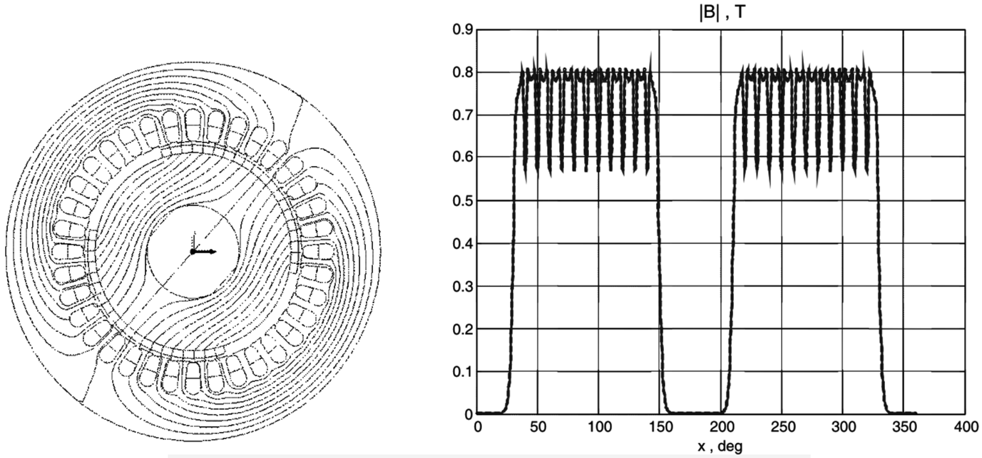

Fig. 3. Diagram of the magnetic field lines for the anisotropic machine core a) and the distribution of the radial component module of the flux density in the air gap ($\varphi = \pi/2$, $\alpha_0 = \pi/2$) b) isotropic case – dashed line, anisotropic case – continuous line

The results shown in Fig. 3b) indicated that the influence of the anisotropy on the field distribution in the gap is practically unnoticeable. For this reason, further studies focused on integral values of the magnetic field i.e. the co-energy and linkage fluxes of the windings in the currentless state.

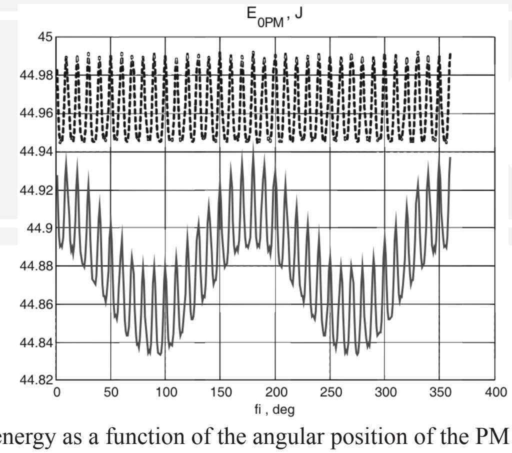

Fig. 4. Changes of co-energy as a function of the angular position of the PM rotor with respect to the stator in the currentless state; isotropic case – dashed line, anisotropic case – continuous line

Figure 4 shows a comparison of changes of the co-energy function according to rotor positions in the currentless state. When the stator is anisotropic, the co-energy function also contains the second harmonic in addition to the constant component and slot harmonic. The Fourier spectrum of this function is presented in Figs. 5a) and b); values of amplitudes are given in (dB) with respect to the accepted reference level of 0.1 mJ.

a) b)

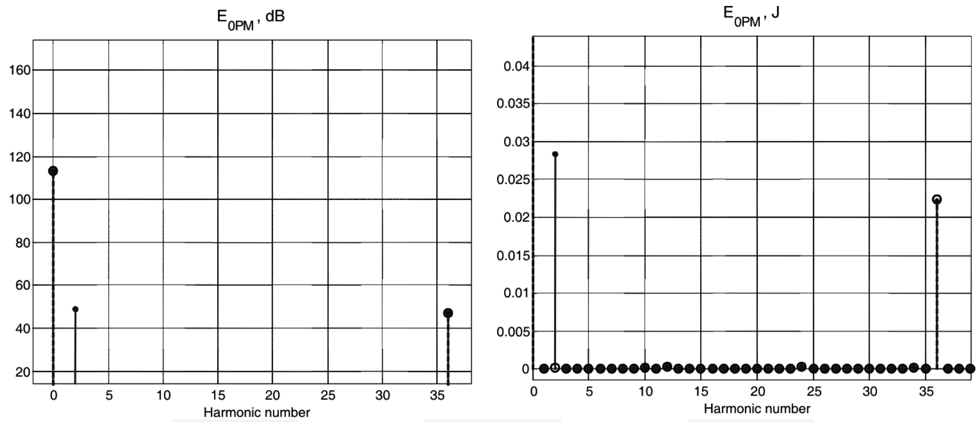

Fig. 5. Amplitude spectrum of the co-energy function in (dB) and variable components of the co-energy in energy unit (J) isotropic case – dashed line O, anisotropic case – continuous line •

The amplitude of the second harmonic of the co-energy is close to the level of the 36th slot harmonic, which has practically the same value in both cases, as is shown in Fig. 5.

In the next step, the asymmetry degree of the induced voltages in the stator windings was examined. The basis for the analysis were FEA results recalculated for the rotor speed which corresponds to a frequency of 50 Hz. Waveforms of induced voltages, which are shown in Fig. 6a), have small but noticeable differences in the amplitudes for the basic harmonic.

a) b)

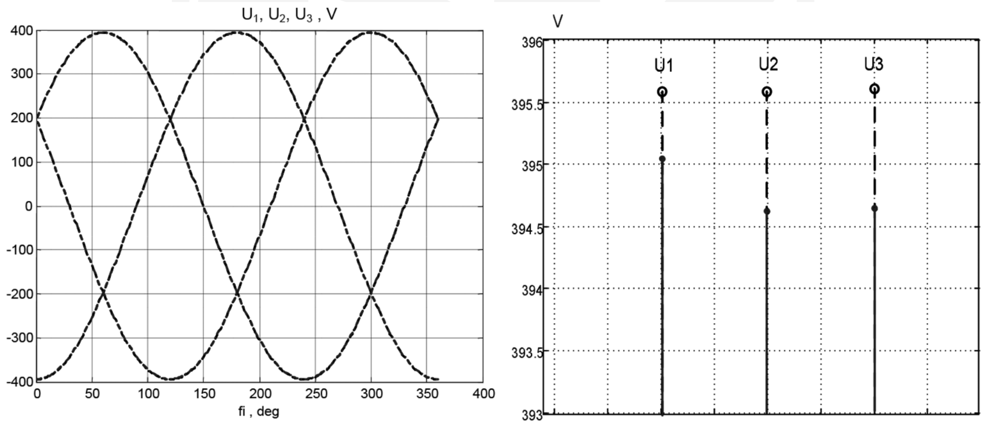

Fig. 6. Time waveforms of voltages induced in stator windings a) and amplitudes for base harmonic of particular phases b) isotropic case – dashed line O, anisotropic case – continuous line •

From the comparison of amplitude values of the basic harmonic of the phase voltages (Fig. 6) it follows that the anisotropy of the stator core voltage introduces an asymmetry in phase 1 at 0.5 V with respect to phase 2 and 3. A very important aspect in PMMs is the occurrence of the cogging torques. The calculation results of these torques are shown in Figs. 7a) and b) – numerical analysis was carried out with the use of the virtual work method.

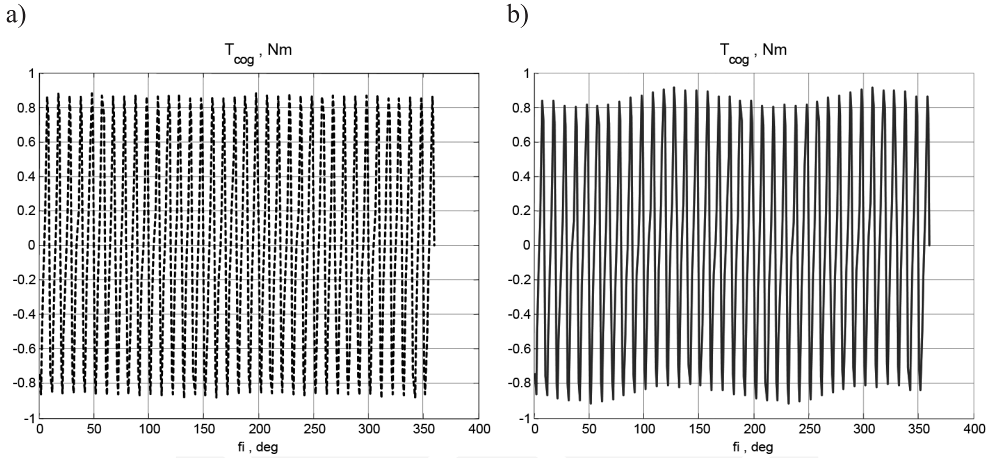

Fig. 7. Cogging torque produced for changes of rotor position: a) isotropic case, b) anisotropic case

Harmonic analysis of these torques (Fig. 8) shows that the second harmonic caused by the anisotropy is about ten times smaller than the harmonics caused by the stator slotting. However, the variable component of the co-energy caused by the anisotropy is comparable with the component associated with the stator slotting.

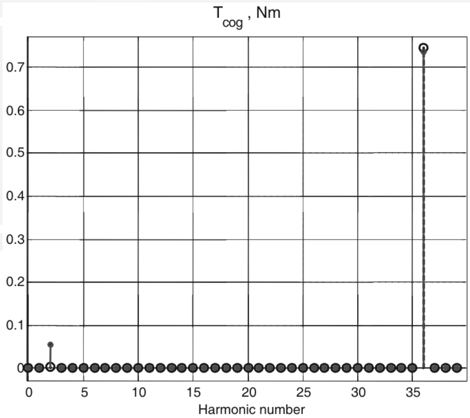

Fig. 8. Amplitude spectra of the cogging torque, isotropic case – dashed line O, anisotropic case – continuous line •

6. Components of the permeance function caused by the anisotropy

Due to the specificity of the magnetic circuit of the PMMs with the anisotropic stator, analytical relationships [2] describing the unit permeance function, which are sufficiently accurate for isotropic machines, need to be complemented with components allowing us to model the anisotropy. Based on the results of the distribution of the co-energy function obtained by FEA, additional components of the unit permeance function caused by the anisotropy can be determined qualitatively. This mainly concerns the second harmonic, which is confirmed by Figs. 5a) and b). For the model two-poles machine with the anisotropic stator and the smooth cylindrical rotor, the form of the simplified one-dimensional distribution of the unit permeance function can be written as follows:

$$\lambda(x) \approx \lambda_{0,0} + 2\lambda_{2,0} \cdot \cos(2x) + \cdots \tag{22}$$

For such a defined function of the unit permeance, we can formulate a Fourier series which approximates the co-energy function in the currentless state whilst taking the anisotropy into account:

$$E_{0\,\mathrm{PM}}(\varphi) \approx E_0 + E_2 \cdot \cos(2\varphi) + \cdots \tag{23}$$

The coefficients of the first terms of this series are defined as:

$$E_0 = \frac{2l_c \cdot r_m \cdot (B_0)^2}{(\lambda_{\delta m})^2} \lambda_{0,0} \cdot \beta \quad E_2 = \frac{4l_c \cdot r_m \cdot (B_0)^2}{(\lambda_{\delta m})^2} \frac{\lambda_{2,0}}{2} \cdot \sin(2\beta) \tag{24}$$

Taking into account the co-energy components in the currentless state determined on the basis of FEA, we obtain basic coefficients of the distribution of the unit permeance function which correspond to anisotropy:

$$\lambda_{0,0} = 1.96 \cdot 10^{-4} \frac{H}{m^2}; \ \frac{\lambda_{0,0}}{\lambda_{\delta m}} = 1.02; \ \lambda_{2,0} = 1.47 \cdot 10^{-7} \frac{H}{m^2}; \ \frac{\lambda_{2,0}}{\lambda_{\delta m}} = 0.77 \cdot 10^{-3}$$

7. Conclusions

Methodological aspects of the modelling presented in this paper show possibilities of how to take into account the phenomenon of the rotational magnetization in stator core of PMM. In order to include the anisotropy into the machine model, we must determine the corrections of the unit permeance function which are important for the second harmonic of its distribution. Determination of these corrections is only possible through modelling of the field distribution using numerical methods FEA. As a result, we can show the influence of the anisotropy on the form of the co-energy function in the currentless state and stator winding characteristics.

The proposed mathematical model of the anisotropic machines, despite significant simplifications, gives us the possibility to perform calculations the results of which may be useful from an operational point of view. Results of field calculations show that for the machine with assumed structure of the magnetic circuit characteristic effects are poorly unnoticeable. The asymmetry of induced phase voltages during a generator operation is at a level of 0.2%, and the cogging torque, important for motor operation, is smaller of one order than the torque caused by stator slotting. Despite such weak effects, we can obtain their qualitative compatibility with the effects which are predicted on the basis of the analytical description in field calculations. As a consequence, it proved possible to determine additional coefficients of the distribution of the unit permeance function caused by the anisotropy – these coefficients complete the basic classical parameters of the mathematical model. The presented research has been carried out for a two-pole machine because the anisotropic effects are then the strongest.

It is worth underlining that the presented conclusions can be dissimilar with respect to switching machines with different relationships between numbers of stator teeth and rotor poles and different dimensions of slots and teeth.

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