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DESIGNING OF STRUCTURAL SYSTEMS WITH APPLICATION OF PRINCIPLE OF SUPERPOSITION

PROJEKTOWANIE SYSTEMÓW KONSTRUKCYJNYCH Z ZASTOSOWANIEM ZASADY SUPERPOZYCJI

Abstract

The paper refers to a method of statical calculation and to the forms of structural systems developed as results of an appropriate application of the principle of superposition. The point of the two-stage method of calculation of the statically indeterminate trusses is to do a suitable reduction of scheme of a given truss in order to obtain the scheme of a statically determinate truss together with a double application of simple methods used for these purposes and then application of superposition of obtained values of forces in appropriate members of the truss. Discussed also, are the procedures of shaping of selected forms of structural systems where the principle of superposition has been applied. It will be shown on suitable examples of the forms of a lenticular girder and of a system of combined foundation and a high-rise building.

Keywords: method of statically calculation, structural system, designing

Streszczenie

Przedmiotem artykułu jest metoda obliczeń statycznych oraz formy systemów konstrukcyjnych opracowane w wyniku odpowiedniego zastosowania zasady superpozycji. Dwuetapowa metoda obliczeń kratownic statycznie niewyznaczalnych polega na odpowiednim zredukowaniu schematu danej kratownicy do układu kratownicy statycznie wyznaczalnej i dwukrotnym zastosowaniu prostych metod obliczeń używanych do tych celów, a następnie superpozycji otrzymanych wartości sił w stosownych prętach. Omówiono także procesy kształtowania wybranych postaci systemów konstrukcyjnych, gdzie zastosowano zasadę superpozycji. Jest to przedstawione na przykładach odpowiednich form dźwigara soczewkowego oraz zespolonego systemu fundamentu i budynku wysokiego.

Słowa kluczowe: metoda obliczeń statycznych, system konstrukcyjny, projektowanie

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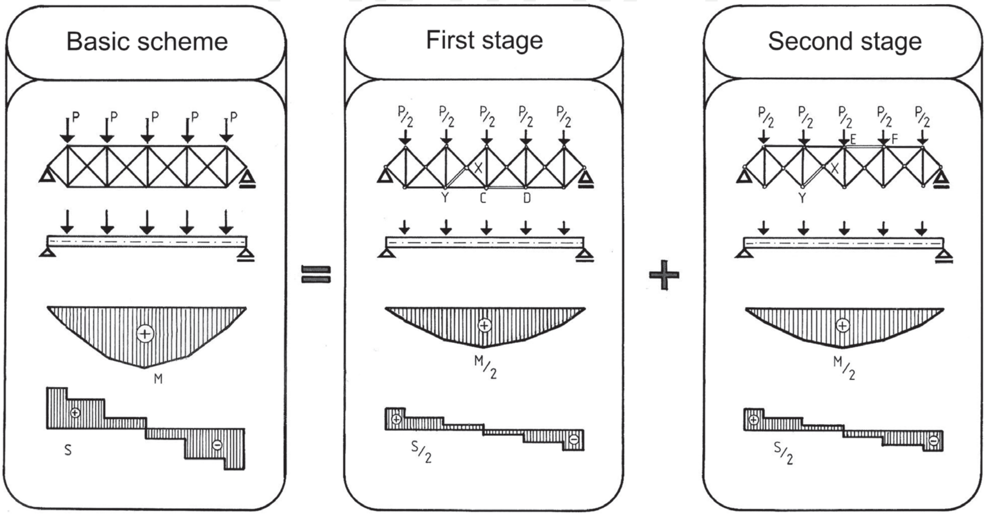
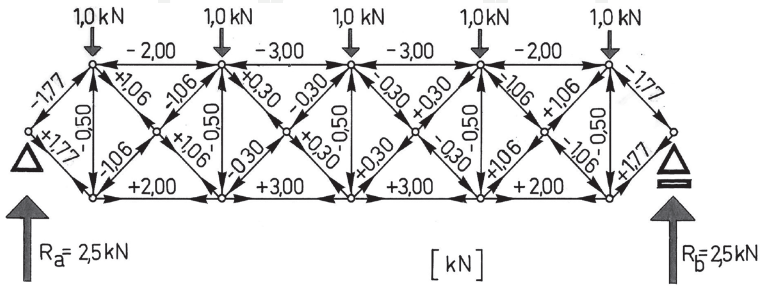
1. Two-stage method of calculation of statically indeterminate trusses

The principle of superposition is constantly applied in numerous types of mathematic operations. Its suitable application constitutes the base of the two-stage method of the statically indeterminate trusses [1]. Procedure of an exact determination of the force values in this type of statical systems requires application of the appropriate calculation methods, which are taking into account differences of stiffness of members creating these structures. The aim of the proposed method is to make possible the initial and easy calculation of such type of trusses by means of the simplest possible way of the approximate determination of forces acting on their members. The method was worked out during the initial calculations of a statically indeterminate truss, which was a simplified scheme of a vertical cross-section of a certain group of the statically indeterminate spatial tension-strut structures [2] composed of cross-braces made as struts and tension members creating their external layers and vertical members. Structures built in this way have to be suitably pre-stressed, but if they will be overloaded by appropriately applied forces then the pre-stress effect disappears, results, that usually members of the upper layer are excluded from process of the force transmission. In this case it becomes a statically determinate truss and for want of the calculation of forces, one can apply the Cremona's, Ritter's or other suitable method [3–6]. A scheme of a simple procedure of the proposed method is presented in Ill. 1.



Ill. 1. Schemes of exemplary shapes of trusses calculated in successive stages of the proposed method

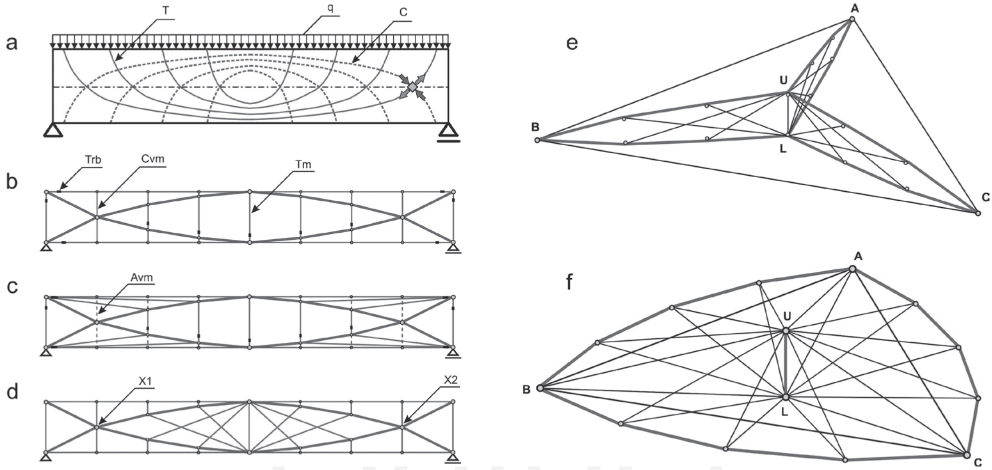
An exemplary shape of the basic flat statically indeterminate truss is loaded by forces P applied to nodes on the upper chord. Singular members of the external chords and the vertical posts are 1.00 meter in length. In the first stage of this method one should remove a number of e.g., the upper members, which equals the degree of statically indetermination of the basic truss and to apply to the same nodes the load of half the value of forces (P/2) than applied previously. In the second stage, one should reduce exactly the same number of members, but this time e.g. from the lower chord, due to which like in the first stage, it becomes the statically determinate truss and has to be loaded by forces of value P/2 applied to the above mentioned nodes. In each stage the truss has the same clear span like the basic truss and the necessity of application of the twice smaller values of forces resulting follows from the general conditions of static equilibrium of the coplanar force system. Force values calculated in this way have to be summed up by means of appropriate application of the basic rules of superposition. The total values of forces acting in cross-braces are obtained by the sum of the values calculated in each stage for each particular member, see Ill. 2. The force values acting in members of the lower chord are determined in the first stage, while the sizes of forces acting on members of the upper chord are calculated in the second stage of the proposed method. Because the methods assigned to the calculation of the statically determinate systems are used in both stages, that is why the two-stage method gives as a result, the approximate values of forces. The method does not take into consideration the various stiffness of the members joined to the nodes in the way of force distribution. In spite of this circumstance, the differences between values of forces obtained in this method and force values calculated by application of suitable computer software are relatively little for the trusses built by means of members having not a greatly differentiated range of lengths. Due to this feature, the proposed two-stage method can be applied to the preliminary processes of the design of this type of structure. After taking into account the suitable numerical procedures together with the application of appropriate co-factors, this method can be applied to obtain very exact results from the statical calculations [2, 7].



Ill. 2. Values of forces acting on members of an exemplary truss calculated by means of the two-stage method

2. Shaping of new types of lenticular girder

A group of new forms of the lenticular girder, as the plane and also spatial structural systems, has been shaped as result of the aim to design an assortment of simple, economic and easy to assemble structural systems of roof covers [8]. They should be built of compression members having a relatively small buckling length and of relatively long tension members. Moreover the new shape of such a girder, should be composed of typical members connected together in nodes having the simplest possible shape and to create structures, which can take the load forces applied to them at optional directions on their main planes. The inspiration in the process of their design were trajectories of the main stresses in a free-end beam, see Ill. 3a. Schemes of examples of flat forms of such girders are presented in Ills 3b-d. Red lines indicate positions of the compression members while the arrangement of tension members is indicated by blue lines. Girders built in this way have to be pre-stressed, which can be done by means of e.g., suitable shortening of selected tension members [9, 10].

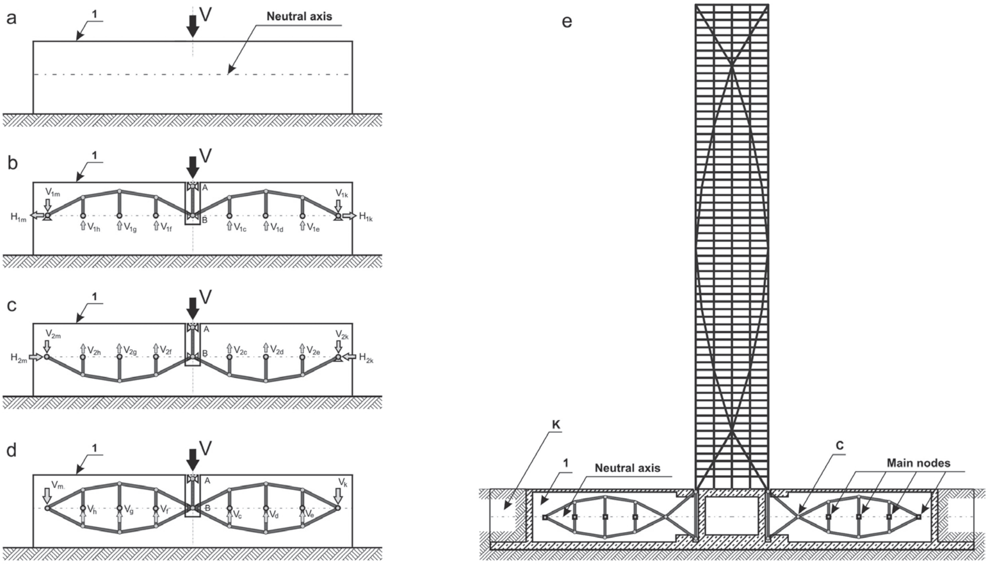


Ill. 3. Scheme of main vertical cross-section of a building designed by means of combined form of structural system

The structural concept described above has been adapted in the process of designing the spatial forms of the lenticular girders the selected shapes of which are shown in Ill. 3e and in Ill. 3f. These modules can be suitably connected together and can create double- or multi-layer bearing structures of the roof covers, particularly of large spans whose covers can be spaced over almost any optional shape of the base projection and can also take optional geometrical forms.

3. Shaping of the combined structural system of a foundation and of a building

The structure of a foundation, having transmitting loads from the building to the subsoil, has to be characterized by appropriate reliability and by a suitable set of technical conditions ensuring the safe foundation of the whole building on the ground having previously defined the parameters determining its load carrying ability. The proposed system of the combined foundation has been worked out mainly for objects located on the subsoil of small load capacity and in earthquake areas [10–14]. During the process of its design the principle of superposition was also applied [15] demonstrating an arrangement of component parts in the space of this system, as presented in a visual way in Ills 4a-d. In the narrow space between concrete elements (1) an upper set of the intermediate members, Ill. 4b, is supplemented



Ill. 4. Simplified schemes of structural system of the combined foundation and of the tall building

by means of the lower set of the intermediate members, Ill. 4c, which in both sets are symmetrically located towards the neutral axis of the concrete elements (1) creating in this way an integral system, Ill. 4d. The main nodes are joined to the matter of the concrete elements (1) in a theoretically articulated way, except for the nodes of type C, see Ill. 4e. The foundation endings (K), shaped in an appropriate way, are used as additional stabilization of these zones of the whole foundation structure. When in the aboveground structure of the storeys a bracing system shaped on the pattern of an appropriate form of the lenticular girder is applied, see Ill. 4e and moreover be supported on the proposed type of foundation, then the whole structure is referred to as the combined structural system of a tall building [16, 17].

4. Closing remarks and conclusions

Suitable application of the superposition's principle makes possible an invention of however approximate, but simple and very efficient method of calculating statically indeterminate systems. Moreover by skilful application of this principle it is possible to design very economical and safe types of structural systems assigned for objects of large formal variation and of different functional purposes.

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