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REDISTRIBUTION OF BENDING MOMENTS IN MULTI – SPAN R/C BEAMS AND SLABS SUBJECTED TO FIRE

REDYSTRYBUCJA MOMENTÓW ZGINAJĄCYCH W WIELOPRZĘSŁOWYCH BELKACH I PŁYTACH ŻELBETOWYCH W WARUNKACH POŻAROWYCH

A b s t r a c t

This paper shows consideration of decrease in cross-section stiffness in commonly used in practice R/C beams and slabs in cases when only reinforcing bars or only concrete compressed zone is subjected to fire. Analyses were based on: a) standard fire curve, b) 500°C Isotherm Method assumptions, c) mechanical properties of reinforcing steel when heated to high temperatures. Afterwards, based on the estimated decrease of cross-sections stiffness, the redistribution of bending moments was calculated in some cases of two-span R/C beams and slabs subjected to fire from their bottom face. Due to the bending moment redistribution, one could expect a reduction of bending moments in span cross-sections and an increase of the support bending moment. As a result of this phenomenon, the ultimate limit state of the structural multi-span elements might occur after a shorter fire duration than could be expected when redistribution of bending moments is neglected.

Keywords: concrete, cross-section, fire design, stiffness

S t r e s z c z e n i e

W artykule rozważono zmniejszenie sztywności powszechnie stosowanych przekrojów belek i płyt żelbetowych w przypadkach, gdy tylko strefa prętów zbrojenia lub tylko strefa ściskana betonu wystawiona jest na działanie pożaru. Analizy oparto na: a) standardowej krzywej pożaru, b) założeniach metody izotermy 500°C [10], c) właściwościach mechanicznych stali zbrojeniowej w wysokiej temperaturze. Następnie, na podstawie oszacowania spadku sztywności przekrojów, obliczono redystrybucję momentów zginających w niektórych przypadkach dwuprzęsłowych belek i płyt żelbetowych, ogarniętych od spodu pożarem. Ze względu na redystrybucję momentów zginających można spodziewać się ich zmniejszenia w przekrojach przęsłowych oraz zwiększenia w przekrojach podporowych. W efekcie stan graniczny nośności wieloprzęsłowych elementów konstrukcyjnych może być osiągnięty po krótszym czasie trwania pożaru niż można byłoby się tego spodziewać w przypadku nieuwzględnienia redystrybucji momentów zginających.

Słowa kluczowe: beton, przekrój, projektowanie z uwagi na warunki pożarowe, sztywność

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Symbols

A_S – area of reinforcement [m]
B_I – stiffness is determined on the assumption that the cross-section is not cracked [kNm^2]
B_{II} – stiffness is determined on the assumption that the cross-section is fully cracked [kNm^2]
B_{fi} – stiffness of a section in fire condition [kNm^2]
$E_{c,eff}$ – modulus of elasticity of concrete [GPa]
E_s – modulus of elasticity of steel [GPa]
$E_{s,fi}$ – modulus of elasticity of steel in fire condition [GPa]
a – axis distance of reinforcement [cm]
b – breadth of beam [m]
f_{yk} – characteristic value of tension yield stress of steel [MPa]
$f_{sy,\theta}$ – characteristic value of tension yield stress of steel in fire condition [MPa]
g_k – characteristic permanent load [kN/m^2]
h – total height of a section [m]
l – span; length of an element [m]
q_k – characteristic variable load [kN/m^2]
t – time, duration [min]
α – angle [rad]
$\gamma_{G,sup}$ – partial safety factor for permanent [loads]
γ_Q – partial coefficient for the variable effects
$\varepsilon_{s,tot}$ – total elongation of reinforcement [‰]
ζ – factor related to the impact of tension stiffening
η – ratio of bending moment in fire condition against the bending moment in non-fire condition
θ_s – temperature of reinforcement [°C]
ξ – a reduction factor
ρ – ratio of tension reinforcement ($= A_s/bd$) [%]
σ_s – steel stress [MPa]
φ – concrete creep coefficient
ψ_0 – the ratio of the value of combining the variable effects
ψ_2 – the ratio of the quasi-permanent value of variable loads

1. Introduction

When reinforced concrete elements are exposed to fire, contraction of concrete and elongation of reinforcing bars can be much larger than found at room temperature [1–3]. This results in a significant reduction in cross-section stiffness, leading to high deformations (deflections) of the elements. Sometimes, the large deflection bending elements can be prevented by the formation of a static scheme of secondary structures (e.g. if the bending elements lean against partition walls) [4].

In predicting the fire resistance of roof elements, the worst fire scenario occurs when fire acts from the bottom of the elements. In this case, only reinforcement is heated in the span cross-sections, and in cross-sections of the support, only a compressed zone of concrete.

Relative changes in the stiffness of the support and span cross-sections may result in the redistribution of bending moments under fire conditions.

The study analyzed computationally changes in stiffness encountered in the practice of slabs and reinforced concrete beams cross-sections exposed to fire only on the tensile reinforcement, or only from the compression zone of concrete. Then, using the two span elements, what may be the impact of relative variation of stiffness of the sections to redistribution of bending moments and the projected capacity fire of elements was determined. The impact of fire was considered above the slab/beam, as less profitable situation.

It should also be noted that in case of fires in complex public or industrial buildings, in practice very rarely is the structure exposed to intense heat from all sides. Most parts (elements) are heated only from the zone of compression, or only from the expanded zone. Competent forecasting of changes in the stiffness of individual elements (cross-sections) and the prediction of the appropriate redistribution of internal forces may be crucial during a global analysis of complex reinforced concrete structures under fire conditions.

2. Stiffness of cross-section R/C slabs under fire conditions

2.1. Assumptions

Four cases of two-span slabs were examined with the span, thickness and reinforcement chosen so that the conditions of the limit state bearing capacity and usability were fulfilled [5–6]. Adopted variable load q_k = 5.00 kN/m^2, which corresponds to the category of use such as C or D [7–8] and a permanent load (g_k), which is the sum of the weight of its own slabs and the value of 1.25 kN/m^2 were assumed. A more unfavorable combination of loads specified by the formulas [7] has been adopted for the calculation of the ultimate limit state (ULS):

$$p = \gamma_{G,sup} \cdot g_k + \gamma_Q \cdot \psi_0 \cdot q_k \tag{1a}$$

$$p = \xi \cdot \gamma_{G,sup} \cdot g_k + \gamma_Q \cdot q_k \tag{1b}$$

For the calculation of serviceability limit state (deflection), a quasi-permanent combination of loads was assumed according to the formula:

$$p_{qp} = g_k + \psi_2 \cdot q_k \tag{2}$$

In these formulas:

$\gamma_{G,sup}$ – partial safety factor for permanent loads; $\gamma_{G,\mathrm{sup}}$ = 1.35,
γ_Q – the partial coefficient for the effects variable; γ_Q = 1.50,
ψ_0 – the ratio of the value of combining the variable effects; ψ_0 = 0.7,
ξ – a reduction factor; ξ = 0.85,
ψ_2 – the ratio of the quasi-permanent value of variable loads; ψ_2 = 0.6.

It is assumed that all slabs are made of C30/37 concrete and reinforced with steel of characteristic yield strength f_{yk} = 500 MPa. The distance from the axis of cross-section bars is 30 mm. Table. 1 shows the most important information about the slabs.

It should be noted that the combination of loads determined by the formula (2) is also suitable for the analysis of fire conditions [7, 9–10]. Given in Table 1, the coefficient η is the ratio of bending moment in fire condition against the bending moment adopted to check the ULS.

Stiffness of slab cross-sections in the fire conditions has been determined basing on assumptions of the 500°C-Isotherm Method, recommended in [10] to calculate the bearing capacity of reinforced concrete elements exposed to standard fire [9, 11]. This method assumes that the concrete in the outer part of cross-section, where the temperature exceeds 500°C, is completely destroyed. In the rest, the inside of the section it is assumed that the strength of concrete is the same as at room temperature. The mechanical properties of reinforcement have been accepted depending on its temperature, regardless of whether the bars are located inside or outside the area limited by the position of the 500°C isotherm.

Table 1

Key parameters of considered slabs

Length	Height	Load [kN/m²]		Span reinforcement			Support reinforcement		
[m]	[cm]	ULS	SLS	$H = p_{qp}/p$	A_s	ρ [%]	η	A_s	ρ [%]
7.20	25	16.13	10.50	0.61	ø10/100	0.36	0.65	ø12/140	0.54
6.00	20	14.69	9.25	0.59	ø10/120	0.38	0.63	ø16/210	0.56
4.80	16	13.54	8.25	0.57	ø10/150	0.40	0.61	ø12/140	0.58
3.60	12	12.39	7.25	0.55	ø8/130	0.42	0.59	ø8/90	0.62

In the case of heating the tension zone (span cross-section) a cross-section of unchanged dimensions was considered, taking into account only the increase in the elongation of the reinforcement. In the case of the compression zone heating, the cross-section of reduced dimensions and unchanged mechanical characteristics of reinforcement has been examined. The following presents the calculation procedure that was used:

2.2. The procedure for calculating cross-sectional span stiffness (heated reinforcement)

The first calculation was performed for the beginning of the fire t = 0 min. The data used for the calculation are suitable for the calculated accidental situation of fire:
– bending moment calculated for the load combinations according to the formula (2); Table 1,
– mechanical characteristics of concrete and reinforcing steel,
– concrete creep coefficient of φ = 2.09÷2.30; $E_{c,eff}$ = 10.63÷9.94 GPa (not including the impact of high temperatures).

Stiffness of the cross-section was calculated by the formula [5]:

$$B_{fi} = \frac{1}{\frac{1-\zeta}{B_{\mathrm{I}}} + \frac{\zeta}{B_{\mathrm{II}}}} \tag{3}$$

where:

- B_{I} – stiffness is determined on the assumption that the cross-section is not cracked,
- B_{II} – stiffness is determined on the assumption that the cross-section is fully cracked,
- ξ – is a factor related to the impact of tension stiffening.

In the next step – the calculations for the duration of the fire, $t = 30$ min.

Reinforcement temperature (θ_s) is estimated on the basis of Figure 1a [12]. They are the guidelines for a simplified prediction of reinforcement temperature and isotherms of 500°C position in cross-sections of R/C slabs. For the duration of the fire, $t = 30$ min obtained $\theta_s = 220$°C.

Then the total elongation of reinforcement ($\varepsilon_{s\,tot}$) has been estimated. In this regard, graphs given in Figure 2 [13] have been used. They represent the stress-strain relationship developed on the basis of [12] of the free thermal elongation of steel.

The vertical axis shows stress in the reinforcement calculated for the previously considered duration of the fire. The horizontal axis represents the full extension of the reinforcement, depending on the temperature. In the case of slabs with spans of 7.2 m, tension in the reinforcement at the beginning of the fire was $\sigma_s = 280$ MPa, temperature of the reinforcement $\theta_s = 220$°C, and elongation of reinforcement read $\varepsilon_{s\,tot} = 4.18$‰ (in fact, instead of charting, analytical files have been used for their preparation).

Position of 500°C-isotherm:
Duration of of standard fire:
$t = 30$ min. – $a_{500} = 1.0$ cm
$t = 60$ min. – $a_{500} = 2.0$ cm
$t = 90$ min. – $a_{500} = 3.0$ cm
$t = 120$ min. – $a_{500} = 3.5$ cm
$t = 180$ min. – $a_{500} = 5.0$ cm
$t = 240$ min. – $a_{500} = 6.0$ cm

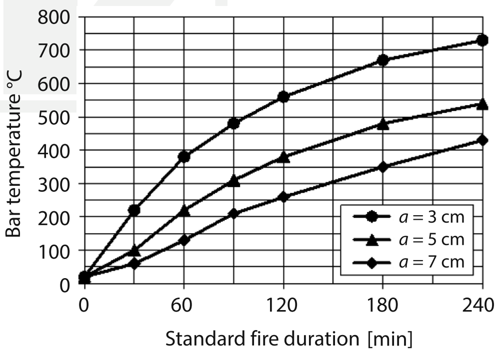


Fig. 1. Recommendation for a simplified evaluation of the temperature in the bars when using the "500°C – Isotherm Method": a) position of the 500°C – isotherm and, b) the temperature of the reinforcing bars (a – the axis distance of reinforcement) [13]

Resultant modulus of elasticity of steel was calculated by the formula:

$$\tan\alpha = E_{s,fi} = \frac{\sigma_s\left(t=0\right)}{\varepsilon_{s,\mathrm{tot}}} \tag{4}$$

Taking into account the impact of the elastic modulus defined by the formula (4), cross-sectional stiffness (B_{fi}) has been calculated, according to formula (3), and the adjusted value of the stresses in the reinforcement (σ_s) has been used. In the next steps calculation procedure described above was repeated for successive durations of fire. Computations have been done applying the calculated load bearing capacity. The cross-section load bearing capacity was calculated according to the 500°C-Isotherm Method, depending on the calculated tensile strength of steel ($f_{sy,\theta} = k_{s,\theta}\, f_{yk}$) defined by Fig. 3. Tables 2a–d present the main results of the calculations.

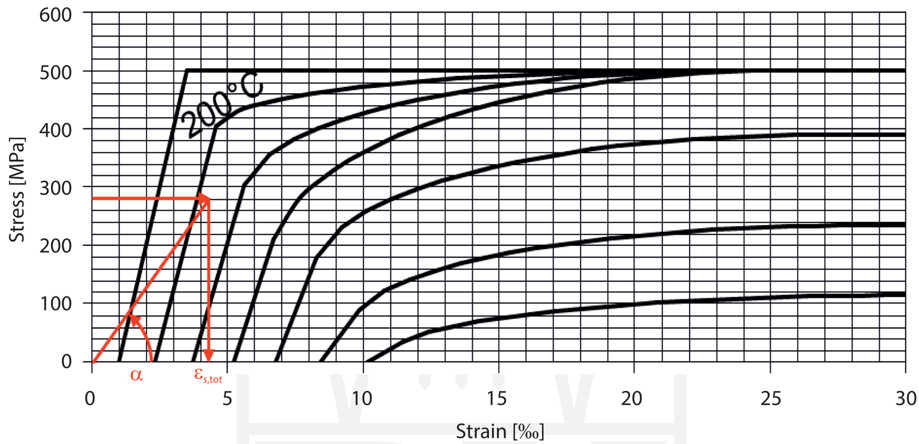


Fig. 2. The stress-strain relationship (included steel free thermal elongation) for hot-rolled reinforcing steel (f_{yk} = 500 MPa) at high temperatures [13]. Looking from the left side of the figure, the successive lines refer to temperatures 100, 200, 300, 400, 500, 600, and 700°C respectively

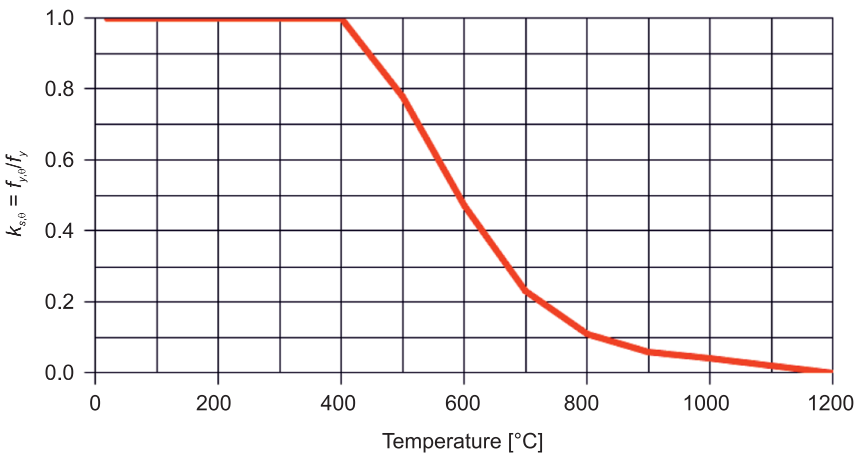


Fig. 3. The reducing factor of the yield strength of reinforcing steel [10]

Table 2a

Key results of the calculations – a slab span of 7.2 m (*h* = 25 cm)

t [min]	θ_s [°C]	σ_s [MPa]	$f_{sy,\Theta}$ [MPa]	$\varepsilon_{s,tot}$ [‰]	$E_{s,fi}$ [GPa]	B_I [kNm^2]	B_{II} [kNm^2]	B_{fi} [kNm^2]	B_{fi}/B_{fi} (t = 0)
0	20	280	500	1.40	200.0	15 175	4 742	5 691	1.00
30	220	268	500	4.18	67.0	12 905	1 925	2 343	0.41
60	380	264	500	6.91	38.8	11 106	1 192	1 410	0.25
90	480	262	412	9.52	27.7	10 075	881	1 024	0.18
120	560	259	297	17.05	15.4	8 953	511	584	0.10
133	600	257	260	26.59	9,7	7 515	333	370	0.07

Table 2b

Key results of the calculations – a slab span of 6.0 m (*h* = 20 cm)

t [min]	θ_s [°C]	σ_s [MPa]	$f_{sy,\Theta}$ [MPa]	$\varepsilon_{s,tot}$ [‰]	$E_{s,fi}$ [GPa]	B_I [kNm^2]	B_{II} [kNm^2]	B_{fi} [kNm^2]	B_{fi}/B_{fi} (t = 0)
0	20	270	500	1,35	200.0	7 689	2 319	2 829	1.00
30	220	258	500	4,13	65.0	6 403	931	1 140	0.40
60	380	255	500	6,78	38.1	5 322	580	683	0.24
90	480	253	412	9,22	27.6	4 710	434	499	0.18
120	560	250	297	15,91	15.9	4 057	261	294	0.10
136	590	248	250	28,18	8.9	3 811	151	169	0.06

Table 2c

Key results of the calculations – a slab span of 4.8 m (*h* = 16 cm)

t [min]	θ_s [°C]	σ_s [MPa]	$f_{sy,\Theta}$ [MPa]	$\varepsilon_{s,tot}$ [‰]	$E_{s,fi}$ [GPa]	B_I [kNm^2]	B_{II} [kNm^2]	B_{fi} [kNm^2]	B_{fi}/B_{fi} (t = 0)
0	20	256	500	1.28	200.0	3 874	1 073	1 390	1.00
30	220	245	500	4.05	63.0	3 145	420	537	0.39

t	θ_s	σ_s	$f_{sy,\Theta}$	$\varepsilon_{s,\text{tot}}$	$E_{s,fi}$	B_{I}	B_{II}	B_{fi}	$B_{fi}/B_{fi}\ (t = 0)$
60	380	241	500	6.62	37.0	2 499	262	315	0.23
90	480	239	412	8.84	27.2	2 140	200	231	0.17
120	560	237	297	14.47	16.5	1 769	126	142	0.10
142	598	235	237	27.85	8.5	1 590	68	75	0.05

Table 2d

Key results of the calculations – a slab span of 3.6 m (*h* = 12 cm)

t [min	θ_s [°C]	σ_s [MPa]	$f_{sy,\Theta}$ [MPa]	$\varepsilon_{s,\text{tot}}$ [‰]	$E_{s,fi}$ [GPa]	B_{I} [kNm^2]	B_{II} [kNm^2]	B_{fi} [kNm^2]	$B_{fi}/B_{fi}\ (t = 0)$
0	20	257	500	1.29	200.0	1 493	364	518	1.00
30	220	245	500	4.05	63.4	1 159	145	193	0.37
60	380	241	500	6.63	37.0	849	90	109	0.21
90	480	240	412	8.85	27.3	684	69	79	0.15
120	560	237	297	14.49	16.5	522	44	48	0.09
140	596	235	240	24.86	9.5	332	26	28	0.04

2.3. The procedure for calculating the stiffness of the support section (heated compressed zone of concrete)

The first calculation was performed for the beginning of the fire (t = 0 min.), assuming the data presented in Section 2.2. Then, based on formula (3) cross-sectional stiffness (B_{fi}) was calculated for successive durations of fire. According to the assumptions of the 500°C – Isotherm Method [10, 11] it dealt with a reduced cross-sectional height (Fig. 4), and unchanged mechanical characteristics of reinforcement (E_s = 200 GPa, f_{yk} = 500 MPa) and concrete ($E_{c,eff}$ = 10.63, 10.39, 10.21, 9.91, GPa for h = 25, 20, 16, 12, cm respectively). The location of the 500°C isotherms (a_{500}) in cross-section was estimated on the basis of Figure 1a [12].

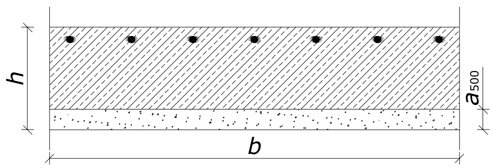


Fig. 4. Reduced cross-section of a slab considered in calculation

For the slabs with a thickness of 25 and 20 cm (range 7.2 and 6.0 m), it was calculated that load bearing capacity of cross-section is not exhausted even after 240 minutes of fire duration. The slabs of smaller thickness calculations were carried out until the calculated load bearing capacity. Tables 3a–d present the main results of the calculations.

Table 3a

Key results of the calculations – a slab span of 7.2 m (*h* = 25 cm)

t [min]	a_{500} [cm]	M_{Rd} [kNm]	M_{qp} [kNm]	B_{I} [kNm^2]	B_{II} [kNm^2]	B_{fi} [kNm^2]	$B_{fi}/B_{fi}(t = 0)$
0	0.0	123.1	68.1	15 797	6 449	6 847	1.00
30	0.9	117.8	68.1	14 540	5 865	6 177	0.90
60	2.0	111.3	68.1	13 152	5 190	5 421	0.79
90	2.7	107.1	68.1	12 352	4 784	4 974	0.73
120	3.5	102.4	68.1	11 514	4 342	4 492	0.66
180	4.7	95.3	68.1	10 402	3 722	3 827	0.56
240	5.7	89.4	68.1	9 601	3 244	3 321	0.48

Table 3b

Key results of the calculations – a slab span of 6.0 m (*h* = 20 cm)

t [min]	a_{500} [cm]	M_{Rd} [kNm]	M_{qp} [kNm]	B_{I} [kNm^2]	B_{II} [kNm^2]	B_{fi} [kNm^2]	$B_{fi}/B_{fi}(t = 0)$
0	0.0	76.8	41.6	7 945	3 078	3 295	1.00
30	0.9	72.5	41.6	7 142	2 718	2 878	0.87
60	2.0	67.2	41.6	6 279	2 312	2 420	0.73
90	2.7	63.9	41.6	5 794	2 071	2 155	0.65
120	3.5	60.0	41.6	5 300	1 814	1 876	0.57
180	4.7	54.3	41.6	4 669	1 463	1 501	0.46
240	5.7	49.5	41.6	4 240	1 201	1 225	0.37

Table 3c

Key results of the calculations – a slab span of 4.8 m (*h* = 16 cm)

t [min]	a_{500} [cm]	M_{Rd} [kNm]	M_{qp} [kNm]	B_{I} [kNm^2]	B_{II} [kNm^2]	B_{fi} [kNm^2]	B_{fi}/B_{fi} (t = 0)
0	0.0	46.2	23.8	3 974	1 406	1 540	1.00
30	0.9	42.8	23.8	3 458	1 193	1 283	0.83
60	2.0	38.6	23.8	2 923	958	1 012	0.66
90	2.7	36.0	23.8	2 633	824	862	0.56
120	3.5	33.0	23.8	2 347	683	708	0.46
180	4.7	28.5	23.8	2 002	498	511	0.33
228	5.5	25.4	23.8	1 823	393	401	0.26

Table 3d

Key results of the calculations – a slab span of 3.6 m (*h* = 12 cm)

t [min]	a_{500} [cm]	M_{Rd} [kNm]	M_{qp} [kNm]	B_{I} [kN/m^2]	B_{II} [kN/m^2]	B_{fi} [kN/m^2]	B_{fi}/B_{fi} (t = 0)
0	0.0	23.6	11.8	1 623	490	558	1.00
30	0.9	21.1	11.8	1 333	384	423	0.76
60	2.0	18.0	11.8	1 047	274	292	0.52
90	2.7	16.0	11.8	902	214	225	0.40
120	3.5	13.8	11.8	768	156	162	0.29
156	4.0	12.4	11.8	700	124	128	0.23

2.3. Analysis of stiffness reduction in cross-section of slabs

Figure 5 shows the ratio of the stiffness of the cross section, calculated for successive durations of the fire against the initial stiffness (B_{fi}/B_{fi} (t = 0)), for span and support cross-sections (Table 2a–d, 3a–d). Figure 6 shows the ratio of stiffness of the cross-sectional span against support cross-section stiffness, depending on the duration of the fire.

A relative decrease in span cross-section stiffness (with heated reinforcement) occurs much faster than support cross-sections (with heated concrete compression zone). In all examined span cross-sections, regardless of their height in the initial phase of the fire (t = 30 min.) more than 60% reduction in stiffness has was been reached. The support cross-sections of the stiffness was reduced, the faster, the lower section height. The ratio of stiffness of the span cross-section against support cross-section stiffness fell twice after thirty minutes of fire for all the thicknesses. A significant change in the proportion of the stiffness of the span and support cross-sections should lead to a substantial redistribution of bending moments, which is an increase support moments and reducing the span moments.

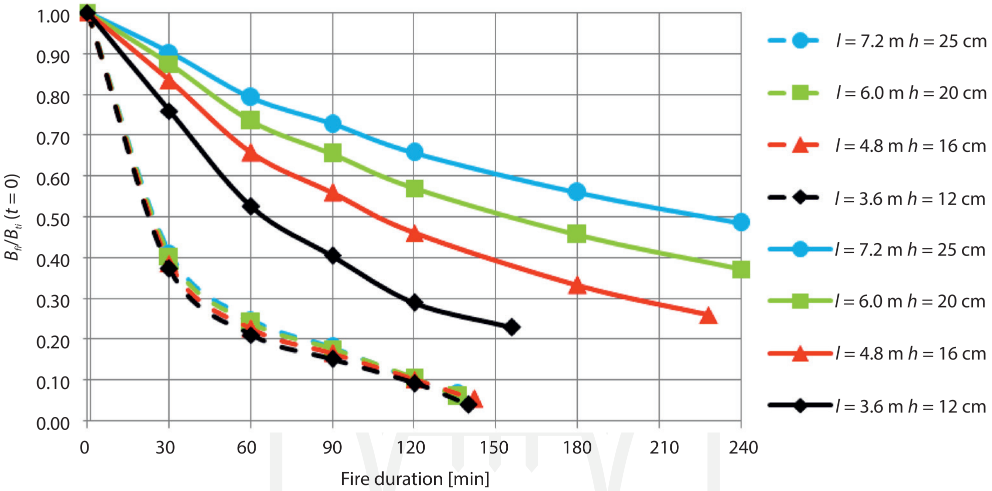


Fig. 5. Stiffness ratio of slabs cross-section, calculated for successive durations of fire, against the initial stiffness, continuous lines – support cross-sections, dashed lines – span cross-sections

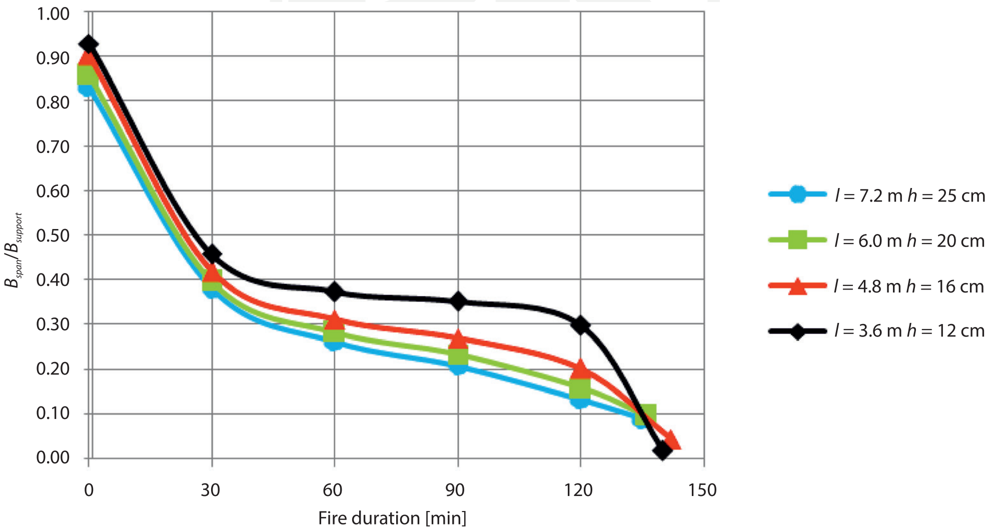


Fig. 6. The ratio of span cross-section stiffness against the support cross-section stiffness, depending on the duration of fire

3. Stiffness of cross-section R/C beams under fire conditions

3.1. Assumptions

Two beams (two – span) were considered with a span 6.0 m and 7.5 m. It was assumed that the beam with a span l = 6.0 m takes the load from the slab with a width of 4.20 m (h = 25 cm), and the beam with a span l = 7.5 m takes the load from the slabs width 7.5 m (h = 25 cm). Slabs variable loads and combinations of load were assumed in the same way as described in Section 2.1. C 30/37 concrete and main reinforcement steel with characteristic yield strength f_{yk} = 500 MPa were assumed, distance from the axis of the main bars from the edge of the cross-section was 50 mm. Cross-sections of beams and reinforcement bars were chosen such that they satisfy the terms of the boundary condition, load bearing capacity and serviceability for the degree of reinforcement span cross-section close to 1%. In Table 4 are the most important items of information about the beams.

Table 4

Key parameters considered beams

Lenght	Cross-section	Moment [kNm]		Span reinforcement			Support reinforcement		
l [m]	$b \times h$ [m]	span	support	η	A_s	ρ [%]	η	A_s	ρ [%]
6.00	0.25 × 0.50	130.8	178.1	0.62	4ø20	1.10	0.66	6ø20	1.68
7.50	0.35 × 0.70	369.5	583.4	0.62	7ø20	0.97	0.66	12ø20	1.66

Stiffness of the beam cross-sections were determined based on assumptions of the 500°C-Isotherm Method, as described in Chapter 2. The following section presents important information on procedures for the performed calculations.

3.2. The procedure for calculating cross-sectional stiffness (heated reinforcement)

First, similar to Section 2.2, the calculations were performed for the outbreak of fire, t = 0 min. For beams l = 6.0 m and l = 7.5 m there were adopted accordingly: φ = 2.37, $E_{c,eff}$ = 9.74 GPa; φ = 2.26, $E_{c,eff}$ = 10.07 GPa. In order to calculate the stiffness of the cross-section on fire conditions, it is necessary to estimate the temperature of reinforcement (θ_s) and the corresponding resultant modulus of steel ($E_{s,fi}$). Resultant modulus of elasticity of steel is calculated analogously to the slab on the basis of Fig. 2 and formula (4).

Reinforcement temperature (θ_s) is estimated on the basis of Fig. 7 [12]. They are the guidelines for a simplified prediction of the temperature of the reinforcement in reinforced concrete beams cross-sections exposed to standard fire conditions.

In considering the beams, it should be taken into account that the temperature of the bars located in the corners of the cross-section is higher than that of bars located in the central part. This is reflected in the guidance given in Fig. 7. In the calculations, the corner and the middle bar temperatures has been set, then the resultant temperature was estimated as a weighted average depending on the number of corner bars and middle bars. Table 5 shows the estimated bars temperature values.

On the basis of the average temperature value of resultant (secant) modulus elasticity of steel was specified, based on the graphs shown in Fig. 2.

Stiffness of the cross-section was calculated by the formula (3). Calculations have been done for the time at which ULS has occurred. Tables 6a and 6b present the main results of the calculations.

Table 5

Estimated temperature of bars

Fire	Estimated temperature of bars [°C]					
t	L = 6.0 m (2ø cor + 2ø midd)			L = 7.5 m (2ø cor + 5ø midd)		
min	corner	middle	average	corner	middle	average
0	20	20	20	20	20	20
30	180	155	168	180	130	144
60	340	290	315	340	240	269
90	500	425	463	500	350	393
120	570	495	533	570	420	463
180	710	635	673	710	560	603
240	850	775	813	850	700	743

Table 6a

Key results of the calculations – a beam span of 6.0 m (25 × 50 cm)

t [min]	θ_s [°C]	σ_s [MPa]	$f_{sy,\Theta}$ [MPa]	$\varepsilon_{s,\mathrm{tot}}$ [‰]	$E_{s,fi}$ [GPa]	B_{I} [kNm^2]	B_{II} [kNm^2]	B_{fi} [kNm^2]	B_{fi}/B_{fi} (t = 0)
0	20	249	500	1.24	200.0	58 662	37 409	38 513	1.00
30	168	242	500	3.22	77.3	62 297	16 215	16 754	0.44
60	315	240	500	5.48	44.2	52 189	9 721	9 979	0.26
90	463	238	430	8.37	28.7	42 808	6 477	6 607	0.17
120	533	237	340	20.82	20.8	37 001	4 791	4 870	0.13
148	597	235	238	26.40	9.0	28 876	2 138	2 163	0.06

Table 6b

Key results of the calculations – a beam span of 7.5 m (35 × 70 cm)

t [min]	θ_s [°C]	σ_s [MPa]	$f_{sy,\Theta}$ [MPa]	$\varepsilon_{s,tot}$ [‰]	$E_{s,fi}$ [GPa]	B_I [kNm^2]	B_{II} [kNm^2]	B_{fi} [kNm^2]	B_{fi}/B_{fi} (t = 0)
0	20	276	500	1.38	200.0	202 491	140 242	142 500	1.00
30	144	271	500	3.01	91.7	240 802	703 511	72 294	0.51
60	269	268	500	4.90	55.3	206 929	44 210	45 237	0.32
90	393	266	500	7.24	37.0	181 891	30 437	31 031	0.22
120	463	266	430	29.43	29.4	166 877	24 513	24 938	0.18
148	578	263	267	25.29	10.5	136 263	9 138	9 259	0.06

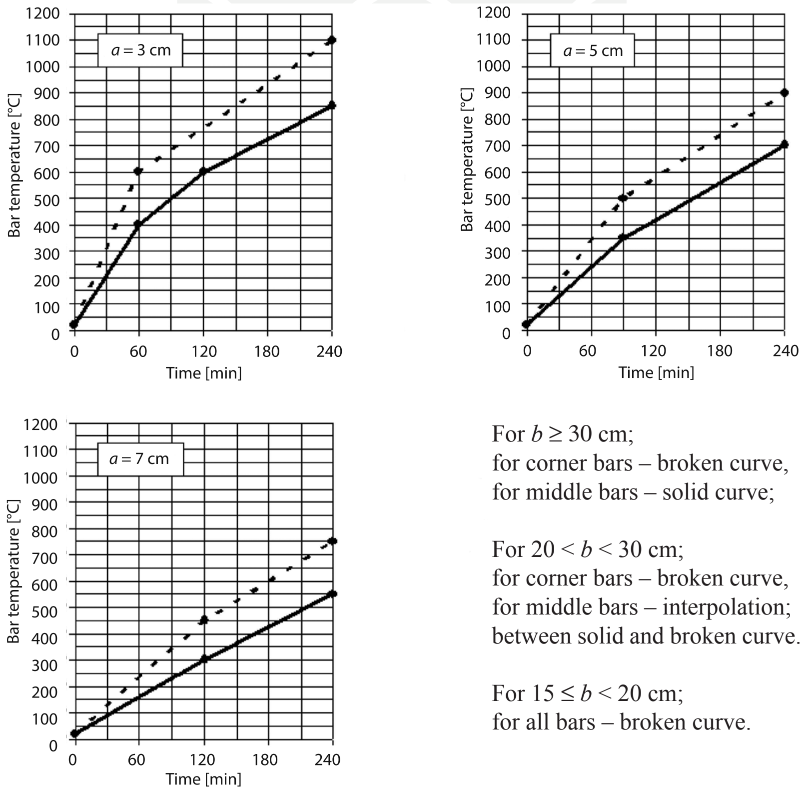


Fig. 7. Temperature of the reinforcing bars in R/C beams subjected to standard fire [12] (*a* – distance between the bar axis and the surface of the concrete; *b* – cross-section width)

3.3. The procedure for calculating the stiffness of the support cross-section (heated compressed zone of concrete)

First, calculations for the beginning of the fire were performed. Then, based on formula (3), calculated cross-sectional stiffness (B_{fi}) for successive durations of fire was established. According to the assumptions of the 500°C-Isotherm Method [10, 11] it dealt with a reduced cross-sectional size (Fig. 8), and unchanged mechanical characteristics of reinforcement (E_s = 200 GPa, f_{yk} = 500 MPa) and concrete ($E_{c,eff}$ = 10.63 and 9.94 GPa for h = 25 and 12 cm respectively).

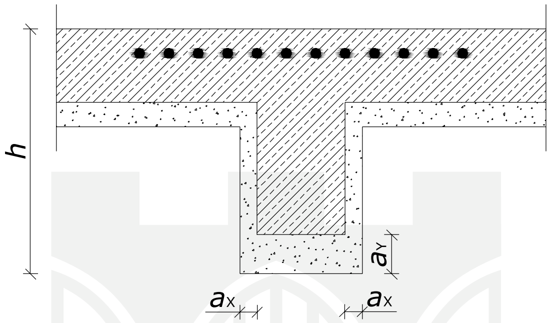


Fig. 8. Reduced cross-section of a beam considered in calculation

Location of 500°C isotherms in the cross-section was estimated on the basis of Fig. 9 [12]. They are the guidelines for a simplified forecasting 500°C isotherm distance from the side edge (a_X) and lower edge (a_Y) of the cross-section reinforced concrete beams exposed to standard fire. Tables 7a and 7b show the main results of the calculations.

Table 7a

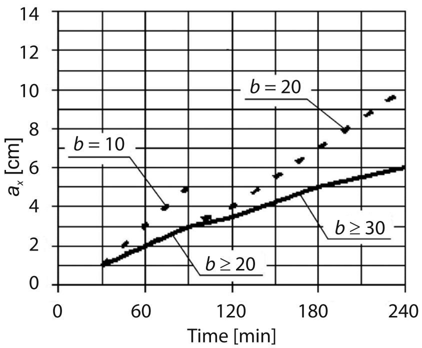
Key results of the calculations – a beam span of 6.0 m (25 × 50 cm)

t [min]	a_X [cm]	a_Y [cm]	M_{Rd} [kNm]	M_{qp} [kNm]	B_I [kNm^2]	B_{II} [kNm^2]	B_{fi} [kNm^2]	B_{fi}/B_{fi} (t = 0)
0	0.0	0.0	366.1	206.2	37 296	28 463	28 790	1.00
30	1.0	2.0	339.6	206.2	66 708	24 456	24 880	0.86
60	2.0	3.0	321.6	206.2	56 062	22 050	22 318	0.78
90	3.0	4.8	294.1	206.2	43 962	18 754	18 896	0.66
120	3.5	6.5	271.7	206.2	36 469	16 323	16 412	0.57
165	5.8	9.3	206.9	206.2	20 531	11 315	11 333	0.39

Table 7b

Key results of the calculations – a beam span of 7.5 m (35 × 70 cm)

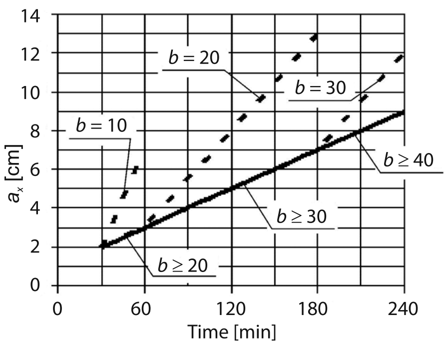
t [min]	a_X [cm]	a_Y [cm]	M_{Rd} [kNm]	M_{qp} [kNm]	B_1 [kNm^2]	B_{II} [kNm^2]	B_{fi} [kNm^2]	B_{fi}/B_{fi} (t = 0)
0	0.0	0.0	1041.2	583.4	153 669	119 706	121 026	1.00
30	1.0	2.0	989.7	583.4	283 328	107 916	109 831	0.91
60	2.0	3.0	955.9	583.4	251 194	100 718	102 112	0.84
90	3.0	4.8	904.6	583.4	212 888	90 611	91 520	0.76
120	3.5	6.5	862.7	583.4	187 701	82 876	83 537	0.69
180	6.0	10.0	737.4	583.4	123 388	63 875	64 087	0.53
240	8.0	12.0	630.7	583.4	87 325	52 027	52 104	0.43



Time t < 90 min:
broken line for b = 10 cm,
solid line for $b \geq 20$ cm.

Time ≤ 90 min:
broken line for b = 20 cm,
solid line for $b \geq 30$ cm.

Interpolate in intermediate cases



Time t < 60 min:
broken line for b = 10 cm,
solid line for $b \geq 20$ cm.

Time 60 ≤ t < 180 min:
broken line for b = 20 cm,
solid line for $b \geq 30$ cm.

Time $t \geq 180$ min:
broken line for b = 30 cm,
solid line for $b \geq 40$ cm.

Interpolate in intermediate cases

Fig. 9. Position of the 500°C – isotherm in R/C beams subjected to standard fire [12] (a_X – measured from the lateral side and a_Y – measured from the bottom side)

3.4. Analysis of stiffness reduction in cross-section beams

Figure 10 shows the ratio of the stiffness of the cross-section, calculated for successive durations of the fire against the initial stiffness ($B_{fi}/B_{fi}(t = 0)$ for span and support cross-sections (Tables 6a–b and 7a–b). Figure 11 shows the ratio of span cross-section stiffness against the stiffness of the support cross-section depending on the duration of the fire.

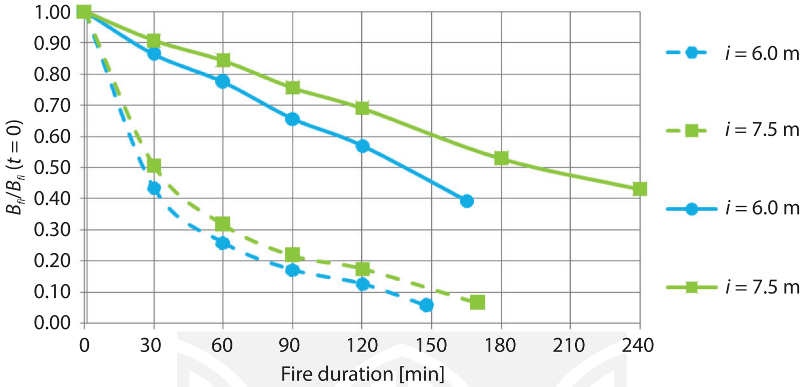


Fig. 10. Stiffness ratio of the beam cross-section calculated for successive durations of fire to the initial stiffness; solid lines – support cross-sections, broken lines – span cross-sections

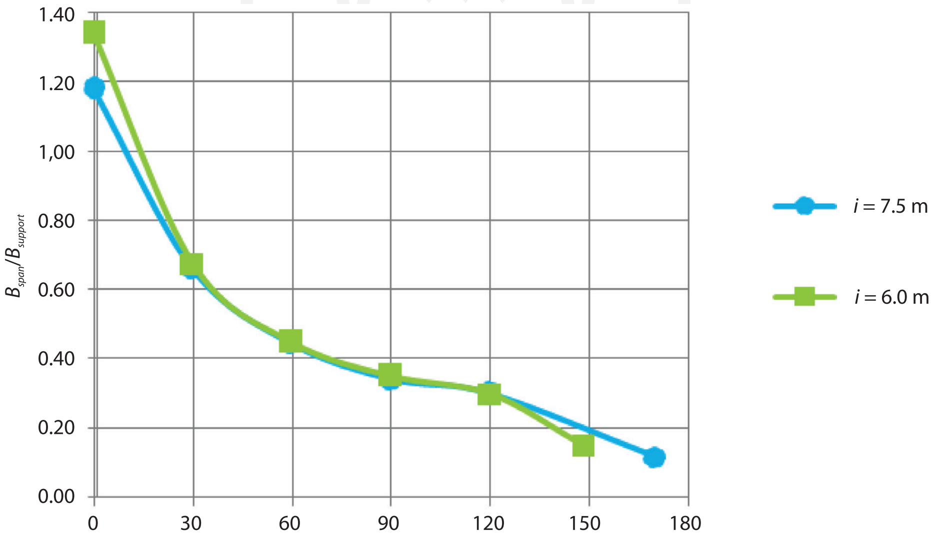


Fig. 11. The ratio of span cross-section stiffness beams against the stiffness of the support cross-section, depending on the duration of the fire

As in the case of the examined slabs, the relative reduction of span cross-sections stiffness (with heated reinforcement) occurs much faster than cross-sections stiffness of the support (within the heated concrete compression zone). In span cross-sections, already in the initial phase of the fire (t = 30 min.) more than 50% reduction in stiffness was reached. Stiffness in the cross-sections of the support is reduced more quickly the smaller the cross-sectional dimensions are. The ratio of span stiffness to the support stiffness decreased approximately twice after thirty minutes of fire duration. A significant change in the proportion of the stiffness of the span and support cross-sections should lead to a substantial redistribution of bending moments, i.e. increasing support moments and reducing the span moments.

4. Redistribution of bending moments

4.1. The assumptions and calculation procedure

Effect of changes in the stiffness of cross-sections in fire conditions on the redistribution of bending moments is defined in the examples: (1) two span beams with span length of 7.5 m, cross-section $b \times h = 30 \times 70$ cm, (2) two span slabs with span length 7.2 m, cross-section height $h = 25$ cm.

Using a computer program (Finite Element Method), calculations of bending moments in the designed permanent situation and accidental situation for the successive durations of fire in slabs and beams have been made. In the locations of the (positive) sagging bending moments for the calculation of the heated sections of the reinforcement (span), the stiffness defined in Section 2.2 or 3.2 has been taken. In the locations of the (negative) hogging bending moments assumed rigidity of the heated sections of concrete compression zone (the support), has been taken as defined in Section 2.3 or 3.3.

The paper presents the calculation results obtained in two the less favorable cases of the variable load location and operation of the fire (Fig. 12):

a) variable load is located on one span only, which is subjected to the action of fire from the bottom (Fig. 12a),
b) variable load is placed on both spans, which are subjected to the action of fire from the bottom (Fig. 12b).

a) b)

Fig. 12. Cases of variable loads and location of the impact of fire

4.2. Beam with a 7.5 m span length, cross-section $b \times h = 30 \times 70$ cm

Figure 13 shows diagrams of bending moments in the beam exposed to variable load and fire in one span only (according to Fig. 12a) in a persistent design situation, and in subsequent fire durations. Red horizontal lines correspond to the values of calculated load bearing capacity.

Figure 14 shows the comparison between the calculated bending moment and calculated load bearing capacity, in span and support cross-sections of the beam shown in Fig. 13.

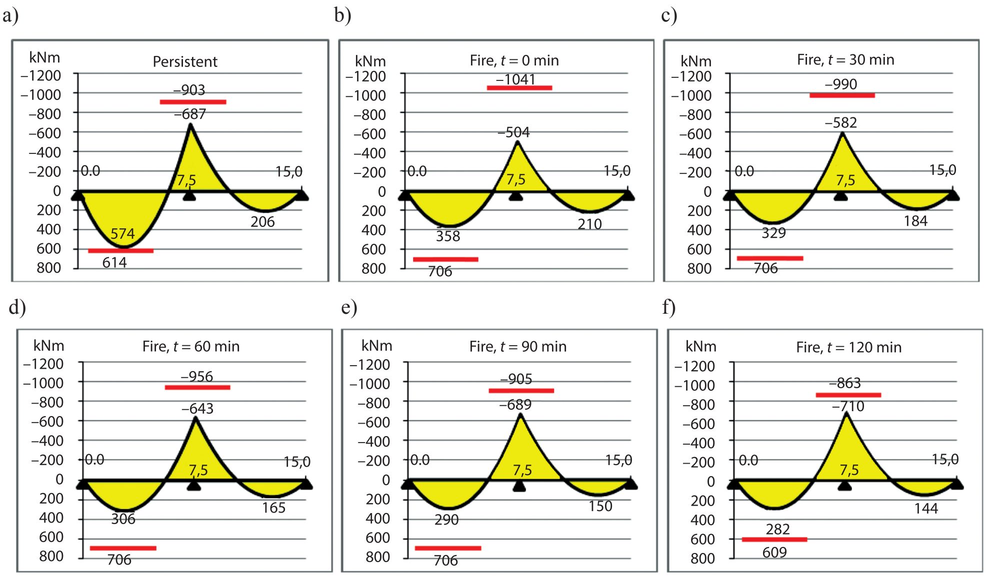


Fig. 13a–f. Bending moments for 7.5 m span length beam, 35 × 70 cm cross-section, variable load and fire in one span only

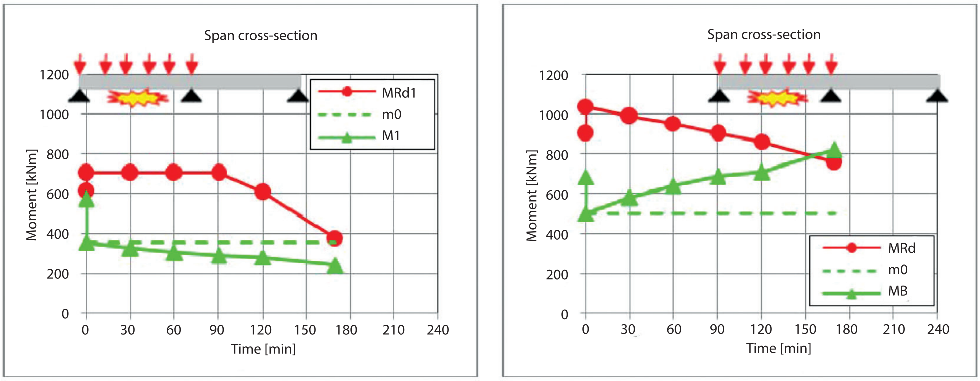


Fig. 14. Comparison of calculated bending moments and calculated load bearing capacity in span and support cross-sections: 7.5 m span length beam, 35 × 70 cm cross-section, variable load and fire in one span only. Red curve – calculated load bearing capacity, green solid line – calculated bending moment, redistribution considered; green broken line – calculated bending moment, redistribution neglected

Figures 15–16, in the same manner, present graphs of bending moments in the same beam, where a variable load and fire operate on both spans (according to Fig. 12b).

With the “transition” from a persistent design situation to accidental situation of fire (Fig. 13a, b, Fig. 15a, b) the calculated load bearing capacity greatly increases. Then, as the fire takes effect, span moments decrease and support moments increase.

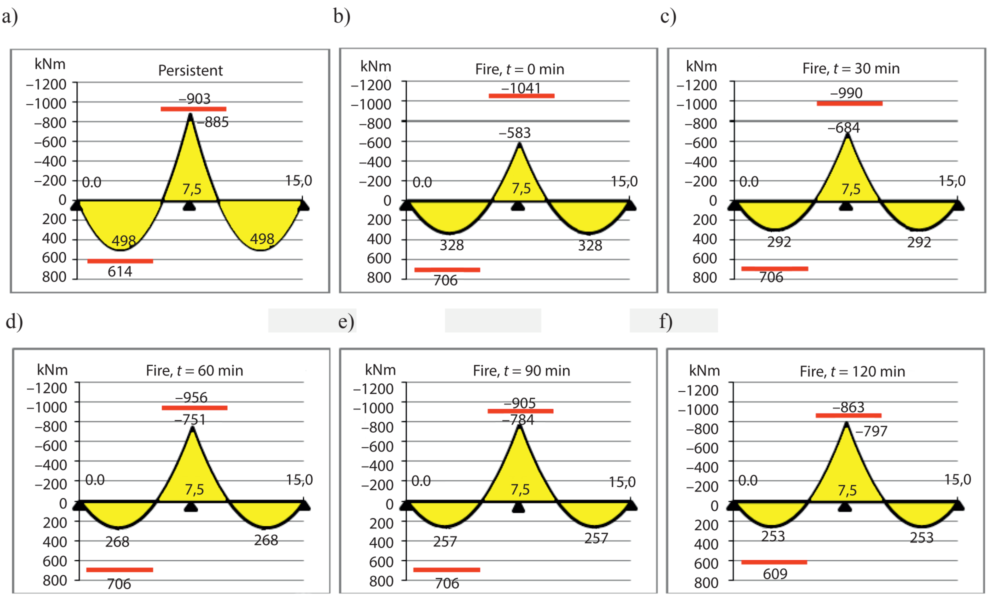


Fig. 15a÷f. Bending moments for 7.5 m span length beam, 35 × 70 cm cross-section, variable load and fire in both spans

In span cross-sections (Fig. 14, 16), slightly decreasing the calculated moment “moves away” from the rapidly decreasing calculated load bearing capacity. This causes a slight delay of a computational load limit state in cross-sections of the span.

In the cross-sections of the support (Fig. 14, 16) rapidly growing calculated moment soon “approaches” the decreasing calculated load bearing capacity. This results in a significant acceleration of the ultimate limit state in cross-sections of the support.

Consequently, as a result of redistribution of bending moments, the calculated ultimate limit state occurs firstly in the support cross-section, and then in cross-section of the span. If omitted, the redistribution of bending moments would cause the reverse situation. ULS occurs first in the span cross-section, and then in the support cross-section.

Please note that the occurrence of ULS in a support cross-section will result in a “descending” graph of bending moments, which was not considered in this paper.

In conclusion, it can be estimated that in the case of load and fire on the two spans (Fig. 16), as a result of redistribution of bending moments, calculated destruction of the beam occurs about 40 minutes earlier than it would appear according to the calculations with neglected redistribution of bending moments. In the case of load and fire in one span only, inclusion or omission of redistribution of bending moments is not essential for a fixed computational time of the beam destruction.

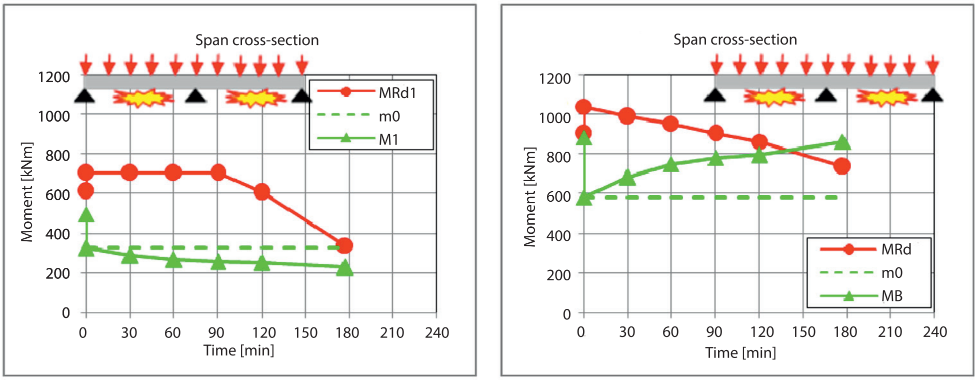


Fig. 16. Comparison of calculated bending moments and calculated load bearing capacity in span and support cross-sections: 7.5 m span length beam, 35 × 70 cm cross-section, variable load and fire in both spans. Red curve – calculated load bearing capacity, green solid line – calculated bending moment, redistribution considered; green broken line – calculated bending moment, redistribution neglected

4.3. Slab with a span of 7.2 m, height cross-section h = 25 cm

Figures 17 to 20 present graphs of bending moments in two span slabs of 7.2 m span length, cross-section height h = 25 cm, in the same manner as in the previous chapter.

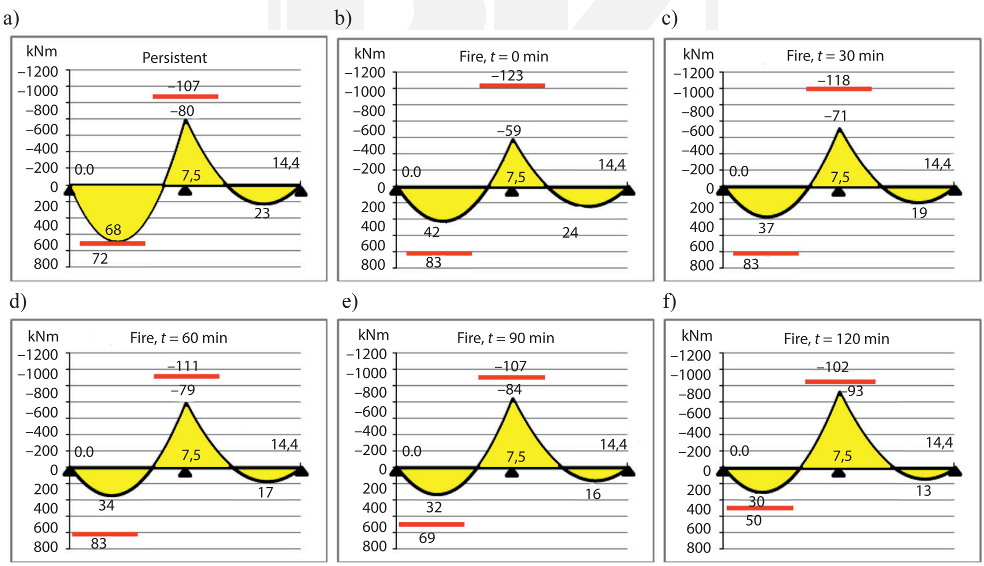


Fig. 17a÷f. Bending moments for 7.2 m span length slab, depth 25 cm, variable load and fire in one span only

In the examined slab, a similar effect of redistribution of bending moments has been observed, as in the beam considered in the chapter 4.2. As a result of the redistribution of bending moments, the ultimate limit state occurs firstly in the cross-section of a support, and then in cross-section of span. In the case of the slab, redistribution of bending moments, however, does not cause acceleration of ULS.

The Fig. 18 and 20 shows the two aforementioned fire situations to cross-sections of the span and the support of the two span slabs of 7.2 m span. Red curve – calculated load bearing capacity, green solid line – calculated bending moment; redistribution considered, green broken line – calculated bending moment; redistribution neglected.

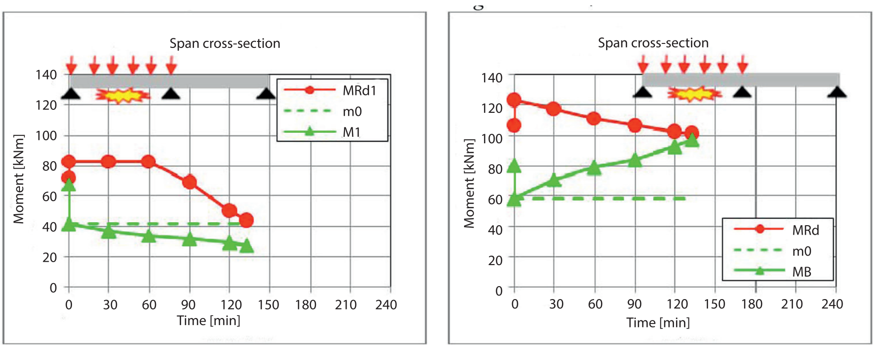


Fig. 18. Comparison of the calculated bending moments and the calculated load bearing capacity in span and support cross-sections: 7.2 m span length slab, 25 cm cross-section depth, variable load and fire in one span only. Red curve – calculated load bearing capacity, green solid line – calculated bending moment; redistribution considered, green broken line – calculated bending moment; redistribution neglected

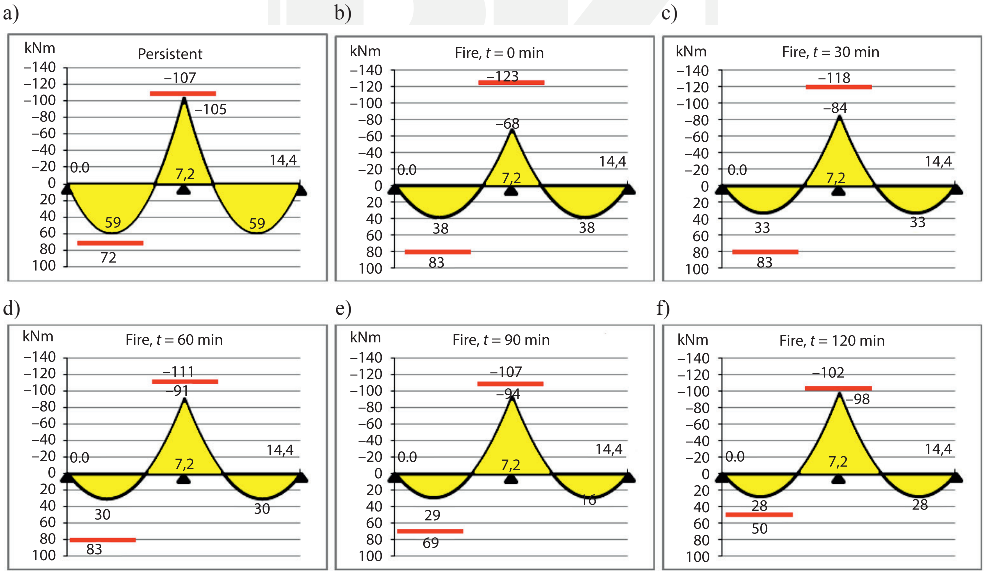


Fig. 19a÷f. Bending moments for 7.2 m span length slab, cross-section depth 25 cm, variable load and fire in both spans

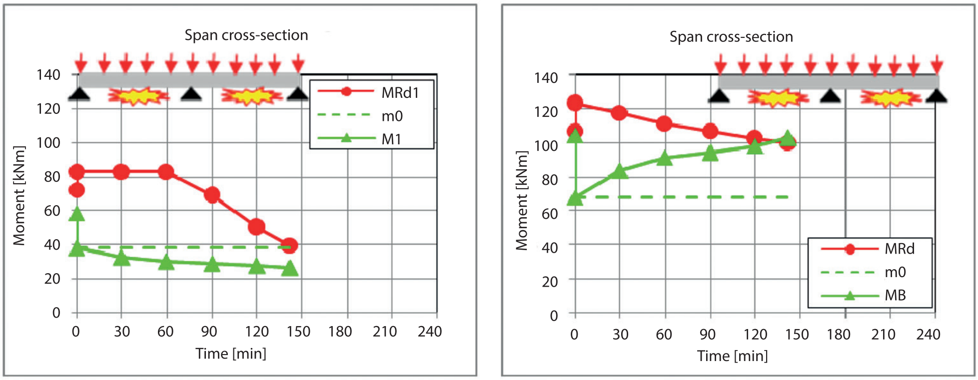


Fig. 20. Comparison of the calculated bending moments and the calculated load bearing capacity in span and support cross-sections: 7.2 m span length slab, 25 cm cross-section depth, variable load and fire in both spans. Red curve – calculated load bearing capacity, green solid line – calculated bending moment, redistribution considered; green broken line – calculated bending moment, redistribution neglected

5. Conclusions

In the first part of the paper, there has been analyzed computationally, how the stiffness of the encountered in practice cross-sections of slabs and cross-sections of reinforced concrete beams exposed to fire only on the tensile reinforcement, or only from the compression zone of concrete changes. The calculations were based on assumptions of the 500°C-Isotherms Method recommended in [10] to calculate the load bearing capacity of reinforced concrete elements exposed to standard fire.

Reduction of span cross-sections stiffness (with heated reinforcement) occurs much faster than the reduction of support cross-sections stiffness (with heated concrete compression zone). Already in the initial phase of the fire ($t = 30$ min) a ratio of span cross-section stiffness to the stiffness of the support cross-section decreased approximately twice. This significant change in the proportion of the stiffness of the span and support cross-section can cause the redistribution of bending moments.

In the second part of the paper, for example of two span elements, it has been estimated the impact of changes in stiffness of the cross-sections on redistribution of bending moments.

As a result of the redistribution of bending moments, a slight decrease in the span moments and relatively significant increase in the support moments should be expected. Consequently, the calculated ultimate limit state occurs firstly in the cross-section of a support, and then in the span cross-section. If omitted, the redistribution of bending moments would cause the reverse situation. ULS would occur firstly in the span cross-section, and then in the support cross-section.

The ultimate limit state in elements with a relatively large cross section (as a result of redistribution of bending moments due to changes in stiffness of the cross-section) may occur slightly earlier than could be expected when the impact of redistribution is neglected. In elements with a relatively small cross-section, the redistribution of bending moments should not have a significant impact on the time in which the ultimate limit state occurs.

References

[1] Buchanan A.H., *Structural Design for Fire Safety*, John Wiley and Sons Ltd., 2002.

[2] fib Bulletin 46/2008, *Fire design of concrete structures – structural behaviour and assessment, State-of-art report*, International Federation for Structural Concrete (fib), July 2008.

[3] fib Bulletin 38/2007, *Fire design for concrete structures – materials, structures and modelling, State-of-art report*, International Federation for Structural Concrete (fib), April 2007.

[4] Kowalski R., Król P., *Experimental Examination of Residual Load Bearing Capacity of RC Beams Heated up to High Temperature*, Sixth International Conference "Structures in Fire", Michigan State University, East Lansing, Michigan, USA 2010. Proceedings edited by V.K.R. Kodur and J.M. Fransen, DEStech Publications Inc.

[5] Eurocode 2, Fire Design of concrete structures – Part 1–1: General rules and rules for buildings, EN 1992-1-1: 2004.

[6] Kordina K., Design of concrete buildings for fire resistance. Chapter 6 in: Structural concrete, Textbook on behaviour, design and performance. Second edition. Vol. 4, fib bulletin 54, 2010.

[7] Eurocode 0, Basis of structural design, EN 1990: 2002.

[8] Eurocode 1, Actions on structures – Part 1–1: General actions – Densities, self-weight, imposed loads for building, EN 1991-1-1: 2002.

[9] Eurocode 1, Actions on structures – Part 1–2: General actions – Actions on structures exposed to fire, EN 1991-1-2: 2002.

[10] Eurocode 2, Design of concrete structures – Part 1–2: General rules – Structural fire design, EN 1992-1-2: 2004.

[11] Taerwe L.R., *Fire Design of Concrete Structures According to the Eurocodes: A Review*, SP-255-4, 75-96 in: Design of Concrete Structures for Fire Safety, ACISP-255, ACI, Detroit 2008.

[12] Kowalski R., *On the Identification of the Reference Isotherm in the Simplified Analysis of R/C Members in Fire*, Studies and Researches, ed. by Politecnico di Milano and Italcementi, pub. by STARRYLINK (Brescia, Italy), 2010.

[13] Abramowicz M., Kowalski R., *Stress-strain relationship of reinforcing steel subjected to tension and high temperature*, International Conference: Applications of Structural Fire Engineering, Prague 2009, Conf. proc.