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TEMPERATURE DEPENDENCE OF PROTON LOCALIZATION FOR SKYRME NUCLEAR INTERACTIONS

ZALEŻNOŚĆ TEMPERATUROWA LOKALIZACJI PROTONÓW DLA ODDZIAŁYWAŃ SKYRME'A

A b s t r a c t

In this paper, our earlier approach to proton localization in neutron star matter to finite temperatures is extended. The Skyrme forces were chosen to describe interactions in nuclear matter. The dependence of threshold density on temperatures for proton localization was obtained and these results were compared with those calculated earlier for the Friedman-Pandharipande-Ravenhall potential.

Keywords: temperature dependence of proton localization, the Skyrme forces

S t r e s z c z e n i e

W artykule rozszerzono wcześniejsze podejście do lokalizacji protonów w materii gwiazdy neutronowej do skończonych temperatur. Wybrano siły Skyrme'a do opisu oddziaływań w materii jądrowej. Otrzymano zależność gęstości progowej od temperatury dla lokalizacji protonów i porównano te wyniki z wcześniejszymi obliczeniami dla potencjału Friedmana-Pandharipande-Ravenhalla.

Słowa kluczowe: temperaturowa zależność lokalizacji protonu, siły Skyrme'a

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1. Introduction

We have shown in the earlier papers [1, 2] that in asymmetric nuclear matter, protons are localized in the core of neutron stars for realistic nuclear models. Protons that form the admixture tend to be localized above the threshold density n_{loc} depending on the model used. The localization effect occurs as a result of the interaction of protons with small density oscillations of the neutron background [2]. In paper [3], we have extended the effect of proton localization to finite temperatures for the Friedman-Pandharipande-Ravenhall potential. Here, the influence of temperature on the proton localization for another nuclear model, the Skyrme nuclear interaction, is investigated.

The plan of this paper is as follows. In Section 2, some of the features of the Skyrme forces are briefly discussed. In Section 3, model of proton admixture in neutron model is presented. In Section 4, the process of calculating entropy and free energy of nuclear matter is described. Numerical results are discussed in Section 5.

2. Skyrme forces parametrization

It was chosen to work with the Skyrme forces [4] to calculate the properties of the asymmetric nuclear matter. The Skyrme potential reads:

$$\varepsilon(n_N,n_P)=\left(\frac{1}{2m_N}+\frac{1}{4}\left(\left(1+\frac{1}{2}x_1\right)t_1+\left(1+\frac{1}{2}x_2\right)t_2\right)n_B+\frac{1}{8}\left(\left(1+2x_2\right)t_2-\left(1+2x_1\right)t_1\right)n_N\right)\tau_N+$$
$$+\left(\frac{1}{2m_{\mathrm{P}}}+\frac{1}{4}\left(\left(1+\frac{1}{2}x_1\right)t_1+\left(1+\frac{1}{2}x_2\right)t_2\right)n_B+\frac{1}{8}\left(\left(1+2x_2\right)t_2-\left(1+2x_1\right)t_1\right)n_P\right)\tau_P$$
$$+n_B^2\left(\frac{3}{8}t_0+\frac{1}{16}t_3n_B^{\gamma}\right)-\frac{1}{4}\left(n_N-n_P\right)^2\left(\left(\frac{1}{2}+x_0\right)t_0+\frac{1}{6}\left(\frac{1}{2}+x_3\right)t_3n_B^{\gamma}\right), \tag{1}$$

where $n_B = n_N + n_P$ (the density of baryon) and masses of nucleons are $m_P = 4.7549\ \mathrm{fm}^{-1}$ and $m_N = 4.7615\ \mathrm{fm}^{-1}$.

The local kinetic energy densities of neutrons and protons for plane waves become:

$$\tau_i=\frac{3}{5}\left(3\pi^2\right)^{2/3}n_i^{5/3},\quad i=N,P. \tag{2}$$

In our calculations, the following Skyrme force parameters were used: t_0 = –1057.3 MeVfm^3; t_1 = 235.9 MeVfm^5; t_2 = –100.0 MeVfm^5; t_3 = 14463.5 MeVfm$^{3+3\gamma}$; x_0 = 0.2885; $x_1 = x_2 = 0$; x_3 = 0.2257; $\gamma = 1$. These are the Vautherin and Brink [5] parameters modified as described in [6].

3. Proton admixture in neutron matter

Protons in strong asymmetric nuclear matter tend to localize [1, 2] in potential wells which correspond to neutron matter inhomogeneities created by the protons in neutron medium. The energetically favorable ground state of asymmetric nuclear matter was found by comparing the energy of two phases: a normal phase (in Wigner-Seitz approximation) and a phase with localized protons. The cells are assumed to be spherical and the volume is $V = 1/n_P$, where the proton density $n_P = xn_N$ for a small proton fraction x. In normal phase, protons are not localized and their wave functions are plane waves. The energy of the cell reads:

$$E_0 = V\varepsilon\left(n_N, n_P\right), \tag{3}$$

where $\varepsilon(n_N, n_P)$ is the energy density. In the phase with localized protons, the energy of the Wigner-Seitz cell E_{loc} is [7]:

$$E_{\text{loc}} = \int d^3r \psi_P^*(r)\left(-\frac{\nabla^2}{2m} + \mu_P\left(n(r)\right)\right)\psi_P(r) + \int d^3r\varepsilon\left(n(r)\right) + B_N\int d^3r\left(\nabla n(r)\right)^2. \tag{4}$$

The first term is the energy of the proton confined to an effective potential well $v_{\text{eff}} = \mu_P(n(r))$. The two other terms in eq. (4) describe the contributions to the energy arising from local change of the neutron Fermi momentum and the gradient of the neutron distribution, respectively, in the Thomas-Fermi approximation. Here, $\varepsilon(n(r))$ is the local neutron matter energy per unit volume. The parameter B_N is the curvature coefficient for pure neutron matter.

We assume a simple trial form of the proton wave function:

$$\Psi_{\text{P}}(r) = \left(\frac{2\pi}{3}R_P^2\right)^{-\frac{3}{4}} \exp\left(-\frac{3r^2}{4R_P^2}\right), \tag{5}$$

where R_P is the root mean square (r.m.s.) radius of the localized proton probability distribution. We treat this quantity as a variational parameter and minimize $\Delta E = E_{\text{loc}} - E_0$. The results are presented in Section 5.

4. Entropy and free energy of nuclear matter

The internal energy density of uniform nuclear matter is given by eq. (1) and inhomogeneities one by eq. (4). The entropy densities S_i reads:

$$S_i = -k_B \sum_{\alpha}\left(n_{\alpha,i} \log n_{\alpha,i} + (1 - n_{\alpha,i})\log\left(1 - n_{\alpha,i}\right)\right). \tag{6}$$

Here, $n_{\alpha,i}$ are the occupation numbers of the simple-particle orbitals $\Phi_{\alpha,i}(x)$ and $i = N$ or P. Upon integration by parts, the entropy per baryon has the particularly simple form

$$S_i = \frac{5}{3}\frac{1}{n_i}\frac{1}{4\pi^2}\left(2m_i^*T\right)^{\frac{3}{2}} J_{\frac{3}{2}}\left(\eta_i\right) - \frac{1}{2}\eta_i, \tag{7}$$

where m_i^* denotes the effective nucleon masses $i = N, P$. The quantity η_i is calculated from the relation:

$$n_i = \frac{2}{(2\pi)^2}\left(2m_i^*T\right)^{\frac{3}{2}} J_{\frac{1}{2}}\left(\eta_i\right), \tag{8}$$

where Fermi integrals are defined as follows:

$$J_\nu\left(\eta\right) = \int_0^\infty dx \frac{x^\nu}{1+e^{x-\eta}} \quad . \tag{9}$$

Knowing the internal energy ε and the entropy per baryon S_i the free energy per baryon is:

$$F = (\varepsilon\left(n_N, n_P, T\right) - T\left(n_N S_N + n_P S_P\right)) / n_B . \tag{10}$$

At finite temperatures, the ground state is found by minimizing the free energy.

5. Proton localization in finite temperatures

The internal energy difference between the localized state of protons and the state with uniform matter $\Delta E = E_{\mathrm{loc}} - E_0$ for the Skyrme forces was calculated. The difference ΔE as the function of variational parameter R_P for various neutron density at $T = 0$ is shown in Fig. 1.

The curves are labelled with the neutron matter density n_N. One can notice that for the Skyrme (*Sk*) interaction, a local minimum above a certain density for the proton r.m.s. radius R_P appears to strongly decrease with neutron density (Fig. 2). With increasing neutron matter density n_N, the depth of the minimum ΔE increases. Above the threshold density, the energy difference becomes negative. The negative value $\Delta E < 0$ means that the energy of

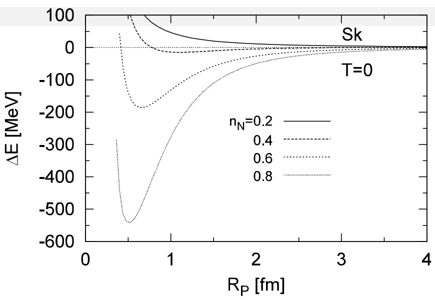

Fig. 1. The energy difference $\Delta E = E_{\mathrm{loc}} - E_0$ as a function of the proton r.m.s. radius R_P for various neutron densities at zero temperature for the Skyrme forces

the localized proton is lower than the energy of a non-localized proton. This means that the localized proton state is preferred energetically for $n > n_{loc}$.

To investigate the influence of temperatures on proton localization, we calculate the free energy difference $\Delta F = F_{loc} - F_0$ in the same manner as in Section 2 using eq. (10). The relation ΔF versus proton radius presents Fig. 3.

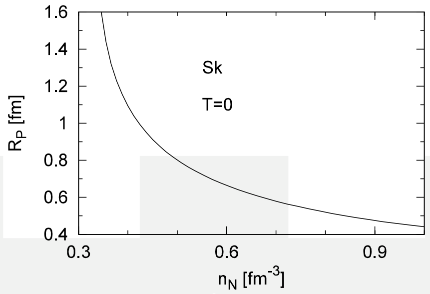

Fig. 2. The r.m.s. radius of proton wave function at zero temperature as the function of density

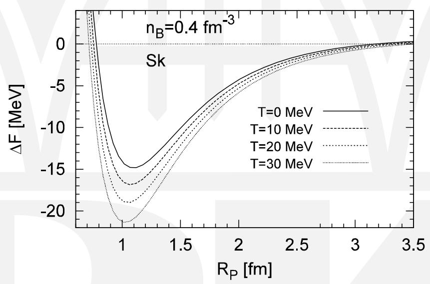

Fig. 3. The difference of free energy between the localized and delocalized states as a function of proton radius at fixed baryon density and for various temperatures

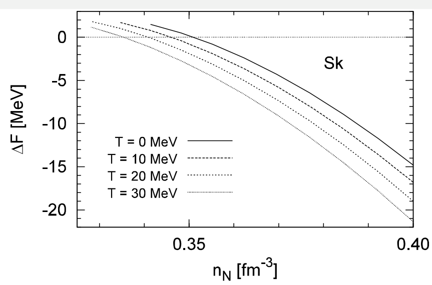

Fig. 4. The free energy difference at the minimum ΔE as a function of density at different temperatures for the Skyrme nuclear forces

When the temperature increases, the minimum value of ΔF as a function of the average neutron number density is shown in Fig. 4. A value of zero corresponds to the threshold density n_{loc} above which the state with localized proton occurs.

A strong influence of temperatures on proton localization was observed (Fig. 5). Temperature inclusion lowers the localization threshold density and diminishes the size of the proton wave function (Fig. 2). Thus, the localization is present even in the case of very high temperature. This means that temperature and baryon density cooperate to achieve proton localized state in dense asymmetric nuclear matter.

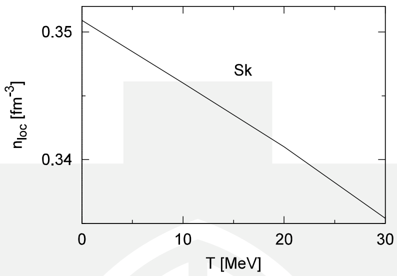

Fig. 5. The threshold density n_{loc} above which the localization is occurred as a function of the temperature of neutron matter

5. Conclusions

Finally, the results show that for low values of x, the state with localized single protons has a lower energy than a uniform configuration for $n > n_{loc}$, even for high temperatures. We have also found that the threshold localization temperature relationship for the Friedman-Pandharipande-Ravenhall parametrization [3] turns out to be surprisingly close to that which was obtained in our calculation for the Skyrme forces. This fact indicates universal character of temperature influence on the proton localization in dense nuclear matter.

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