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THE STRUCTURE OF NEUTRON STARS WITH LOCALIZED PROTONS

STRUKTURA GWIAZD NEUTRONOWYCH ZE ZLOKALIZOWANYMI PROTONAMI

A b s t r a c t

Strongly asymmetric nuclear matter becomes unstable with respect to proton localization above a specific critical nuclear density. For equation of state of Akmal, Pandharipande and Ravenhall the Tolman-Oppenheimer-Volkoff equations were solved and the radius of the spherical shell of a neutron star within which proton localization takes place was found.

Keywords: strongly asymmetric nuclear matter, proton localization, neutron stars

S t r e s z c z e n i e

Silnie asymetryczna materia jądrowa wykazuje niestabilność związaną z lokalizacją protonu powyżej krytycznej gęstości. Dla równania stanu Akmala, Pandharipande i Ravenhalla zostały rozwiązane równania Tolmana-Oppenheimera-Volkoffa i został wyznaczony promień powłoki gwiazdy neutronowej, wewnątrz której ma miejsce lokalizacja protonów.

Słowa kluczowe: silnie asymetryczna materia jądrowa, lokalizacja protonu, gwiazdy neutronowe

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1. Introduction

The structure of neutron stars is both interesting and complex and is a problem to be solved. A few years ago, the model with star matter having localized protons was proposed [1–5]. In high-density matter with a low proton fraction, the coupling of proton impurities with the density waves in neutron matter could lead to the localization of protons in the potential wells associated with the neutron density inhomogeneities. Such instability is a universal phenomenon in high density matter, although the proton localization threshold density depends on the equation of state [6].

This paper is organized as follows. In Section 2, there is a brief discussion of some of the features of the Akmal-Pandharipande-Ravenhall (APR) Hamiltonian [7] and the ingredients involved in their construction. In Section 3, the Tolman-Oppenheimer-Volkoff equations [8] are presented. In Section 4, these are solved with the APR equation of state and the radius of the shell below which protons in neutron stars are localized is calculated.

2. Akmal-Pandharipande-Ravenhall equation of state

The A18+δv+UIX* parametrization of the APR equation of state was chosen for nuclear interaction. In this approach, the Jastrow wave function is assumed and the expectation value of the Hamiltonian is cluster-expanded. Subsequently, parts of the higher-order cluster terms are resummed up by the Fermi Hypernetted Chain (FHCN) method [9]. Akmal, Pandharipande and Ravenhall performed the FHCN calculation [8] with Argonne v18 (Av18) two-body potential [10] and the Urbana IX* (UIX*) three-body potential [11] with boost correction. The obtained energy density reads:

$$\begin{aligned}\mu(n_N,n_P) &= \left(\frac{1}{2m_N}+\left((p_3+p_5)n_N+p_3n_P\right)e^{-p_4(n_N+n_P)}\right)\frac{3}{5}\left(3\pi^2\right)^{\frac{2}{3}}n_N^{\frac{5}{3}}+\\ &\left(\left(\frac{1}{2m_P}+\left((p_3+p_5)n_P+p_3n_N\right)e^{-p_4(n_N+n_P)}\right)\frac{3}{5}\left(3\pi_2\right)^{\frac{2}{3}}n_P^{\frac{5}{3}}+4n_Nn_P\left(p_1+p_2(n_N+n_P)+\right.\right.\\ &\left.p_6(n_N+n_P)^2+\left(p_{10}+p_{11}(n_N+n_P)\right)e^{-p_9^2(n_N+n_P)^2}\right)-\\ &(n_N-n_P)^2\left(\frac{p_{12}}{n_N+n_P}+p_7+p_8(n_N+n_P)+p_{13}e^{-p_9^2(n_N+n_P)^2}\right)-\\ &4n_Nn_P(n_N+n_P-p_{19})\left(p_{17}+p_{21}(n_N+n_P-p_{19})\right)e^{p_{18}(n_N+n_P-p_{19})}-\\ &(n_N-n_P)^2(n_N+n_P-p_{20})\left(p_{15}+p_{14}(n_N+n_P-p_{20})\right)e^{p_{16}(n_N+n_P-p_{20})},\end{aligned} \tag{1}$$

where the neutron and the proton densities fulfil the following conditions $n_N + n_P > p_{19}$ or: $n_N + n_P > p_{20}$. The parameters have the following values: p_1 = 337.2 MeVfm^3; p_2 = –382.0 MeVfm^6; p_3 = 89.8 MeVfm^5; p_4 = 0.457 fm^3; p_5 = –59.0 MeVfm^5; p_6 =19.1 MeVfm^9; p_7 = 214.6 MeVfm^3; p_8 = –384.0 MeVfm^6; p_9 = 6.4 fm^3;

p_{10} = 69.0 MeVfm^3; p_{11} = –33.0 MeVfm^6; p_{12} = 0.35 MeV; $p_{13} = p_{14} = p_{21} = 0$;
p_{15} = 287.0 MeVfm^6; p_{16} = –1.54 fm^3; p_{17} = 157.0 MeVfm^6; p_{18} = –1.45 fm^3;
p_{19} = 0.32 fm^{-3}; p_{20} = 0.195fm^{-3}.

3. Tolman-Oppenheimer-Volkoff equations

The structure of spherically symmetrical non-rotating neutron stars is described by the celebrated the Tolman-Oppenheimer-Volkoff (TOV) equations [12], which form a coupled set of first-order differential equations of the following form:

$$\frac{dP}{dr} = -\frac{G\rho m}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}, \tag{2}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho, \tag{3}$$

where:

$P(r)$ – denotes the pressure (at radius r),
$\varrho(r)$ – mass density,
$m(r)$ – the mass enclosed within the radius r,
G – the gravitational constant,
c – the speed of light.

For a given fluid element in the star, hydrostatic equilibrium is attained by adjusting the pressure gradient to exactly balance the gravitational pull.

The second equation defines the total mass contained in the sphere of radius r. Thus at $r = 0$, m must be zero and at $r = R$, m is the total mass M of the star. The unknowns in these two equations are ϱ, P and m – hence, a third equation is needed to close the system. This third equation is the equation of state (EOS) $P = P(\varrho)$. Thus, the input to the calculations is the EOS and the output yields the masses of neutron stars as a function of their radius for a given central density. Given the stellar radius R, which is defined by zero pressure at the stellar surface, the gravitational mass is as follows:

$$M(R) = 4\pi\int_0^R \rho(r) r^2 dr. \tag{4}$$

In hydrostatic equilibrium the neutron star is perfectly balanced by the action of two forces – gravity and pressure. The pressure gradient is negative so the pressure decreases monotonically with distance until it vanishes at the edge of the star. The pressure at the center must be enormous in order to be able to support the full weight of the star. This implies that models of the EOS will have to encompass high and low density ranges. This is an example of how the microscopic physics (EOS) can potentially be ‘observed’ from astrophysical data, namely from the mass and radius of the star.

4. The results

The aim of this work was to compare the energies of two phases. The energetically favorable ground state of matter is found by comparing the energy of a normal phase E_0 with uniform density and a phase with localized protons E_L. Based on our papers [1–5] we calculate $\Delta E = E_L - E_0$ versus neutron density n_N (Fig. 1) and we establish that above n_L = 0.819 fm^{-3}, protons in neutron matter for the A18+δv+UIX* potential are localized.

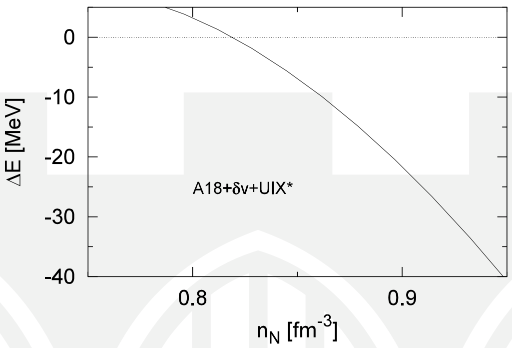

Fig. 1. Difference $\Delta E = E_L - E_0$ versus neutron density for the A18+δv+UIX* parametrization

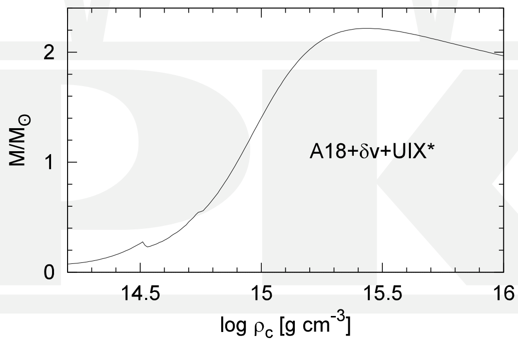

Fig. 2. Mass-central density relation for the A18+δv+UIX* equation of state

The TOV equations are solved with the A18+δv+UIX* parametrization introduced in Section 2. Fig. 2 presents the dependence of neutron star masses on the logarithm of central density. For central density above 10^{15} g cm^{-3}, we have neutron stars with masses higher than the solar mass ($M_\odot$). It transpires that the mass of the neutron star has a maximum value (Fig. 2) as a function of central density, above which the star becomes unstable and collapses to a black hole. The value of the maximum mass depends on the nuclear EOS. The considered solutions of the TOV equations with the A18+δv+UIX* equation of state is compatible with the largest mass observed up until now, which is measured to be $2.01 \pm 0.04\ M_\odot$ [13].

We have also plotted (Fig. 3) the density versus the distance from the centre of the neutron star for a given central density (log ϱ_c = 15.4) and found that the radius of the neutron star equals approximately 10 km. The obtained value is compatible with the observed radii of neutron stars [12].

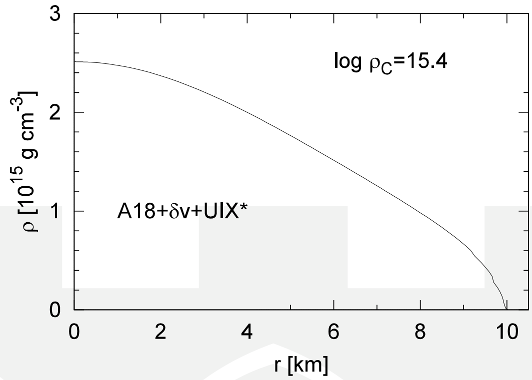

Fig. 3. Density of neutron star versus distance from centre for log ϱ_c = 15.5

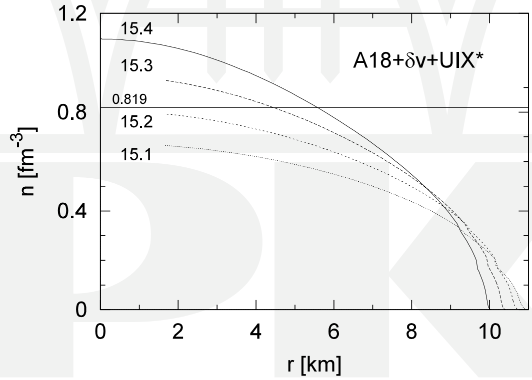

Fig. 4. Baryon number density versus distance from the centre of the neutron star. The curves are labelled by the logarithm of the central density of the star. The straight line indicates the threshold density of localization

Next, the density profiles of neutron stars were calculated. For the various central densities, changes of neutron star matter density versus distance from its centre for the A18+δv+UIX* equation of state were calculated (Fig. 4). Comparing the profile shape with the value of proton localization threshold (straight line on Fig. 1) n_L = 0.819 gives a radius of spherical shell r_L within which proton localization takes place. The curves are labelled by the logarithm of the central densities. In our case, for log ϱ_c = 15.3 we have r_L = 4.4 km (for neutron star radius R equals 10.4 km) and for log ϱ_c = 15.4 we have r_L = 5.6 km (R = 10.9 km).

5. Concluding remarks

The solution of the Tolman-Oppenheimer-Volkoff equations with the A18+δv+UIX* equation of state indicates that the structure of neutron stars is inhomogeneous – in the central area, up to r_L protons are localized and above r_L to R (radius of neutron star) protons and neutrons are delocalized.

The phase with localized protons inside neutron star cores has profound astrophysical consequences. As has been shown [14], the cooling of neutron stars proceeds in quite different ways for localized and delocalized phases. The presence of such a localized proton phase results in more satisfactory fits of the observed temperatures of neutron stars.
Further studies applying other equations of state are in progress.

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