

# Simulation Model of the Bus Stop

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## Abstract

On the basis of the research conducted at the bus stops in Krakow and Warsaw, the movement processes, related with functioning of a bus stop have been identified. As a result, the simulation model imitating its functioning has been formulated and verified. The stop has been analyzed as a system, in which the input flow comprises of municipal bus transport, whereas the service time corresponds the dwell time. The formulas for time losses incurred due to queuing to reach the boarding and alighting zones have been derived, constituting a basis for operative capacity estimation of a bus stop and specifying the selection determinants for the number of boarding and alighting zones in regards to traffic intensity as well as boarding and alighting time.

## 1. Introduction

In the era of private transportation, ensuring high quality public transport services and boosting its competitiveness has become an increasingly important issue. Implementation of this strategy is only possible through shaping a proper transport infrastructure that would ensure the safety of passengers as well as enhance the service efficiency through simultaneous reduction of cost and negative aspects of its functioning.

One of the main elements characterizing the urban public transport is the stop. The complexity of bus stops is mostly related with the proper allocation and geometric designing. The first one is relatively well examined; however there is a lack of specific guidelines as to design of the bus stops, i.e. defining the adequate number of boarding and alighting zones and in consequence the length of the kerbside. The insufficient length of the bus stops may cause time losses on the expense of buses and passengers who are queuing to reach the loading area, what forces buses to

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stop and in consequence decrease the speed of transportation. Whereas, extending the stop line brings negative consequences of costs of building the bus bays and a decline in the service quality for awaiting passengers.

They comprise the following elements, such as moving the front side of the stop from the intersections and pedestrian crossings, obstructed passengers orientation or worsening traffic safety.

Current road design standards [6] recommend adjusting the stop line only according to the traffic intensity, what is definitely oversimplified. In order to properly define the adequate number of boarding and alighting zones with a given traffic intensity, we need to estimate the operative capacity of the bus stop, considering the standards of servicing. The existing models vary in describing the operative capacity of the bus stop. It might be defined as a part of the maximum capacity, specified in a deterministic way, or dependent on a ride time between the stops. The textbook [10] uses the failure rate of the stops, understood as the acceptable probability of a bus arrival at the stop at a time, when all <sup>1</sup>. Considering a diversification of boarding and alighting time, we get the quasi-deterministic model of the bus stop capacity. In previous analyses [5,8,9], the operative capacity, based on probabilistic model for bus stop operations, was dependant on the time loss, incurred due to queuing to servicing time ratio. The identification research, based on which that model was built, were conducted in 70, 80 and 90. In subsequent years, this topic had not been broached. At the same time, private transport increased in role due to legislative changes of the transportation procedures. All these factors have contributed to the change of a relevant part of the analysis results.

The creation purpose of the simulation model of the bus stop is the analysis of delays, incurred due to queuing to reach the boarding and alighting zones, that will enable to estimate the design capacity of the stop, and determine the adequate number of boarding and alighting zones.

## 2. The Bus Stop as a Mass Service System

Most of the processes that constitute the functioning image of a bus transportation have a random character, thus, it is important to use the probabilistic method of analysis. Considering its cumulating at stops, modelling of its operations is increasingly difficult. On the one hand, we have a random arrival timing of buses at the stops, and on the other hand, the flow of passengers – the number of boarding as well as alighting people, and finally its timing. The time lag between end of boarding and alighting as well as moving, is also random, resulting from the operational activity as well as interference of a different character. Moreover, at stops with more than one boarding and alighting zone, buses might block each other, what has a significant impact on capacity and the amount of time losses incurred

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<sup>1</sup> boarding and alighting zones are occupied

by buses. It also complicates the model of the bus stop itself. In [9], where the simulation model of the bus stop is described, the computer simulation is pointed as an universal method of analysis. With the insufficient number of research ranges and necessity to use laborious measures on the one hand, and limited application of the analytical methods in simple processes on the other hand, the simulation methods stand as the most effective in solving the bus stop capacity dilemmas.

The very realistic mathematical reflection of all undergoing processes at bus stops, requires modelling of the input flow of passengers, and its boarding and alighting processes. The existing regression models [1] condition the boarding and alighting time from filling capacity of buses reaching the stops. Such an approach would allow for analyzing the time losses incurred by passengers as the major beneficiaries of this system. The complications related with the particulation of the model, and difficulty with generalisation of the results for a large set of variables, incline towards choosing the simplified approach, in which the number of boarding and alighting passengers is only considered by boarding and alighting time.

From this perspective, within the queuing theory scope, the stop might be treated as a mass service system, described by the following elements:

- input process – as units coming into the system – flow of buses arriving at the stop,
- service process – time of standing on boarding and alighting zone, understood as a time interval between the consecutive service instants of another buses at the same zone,
- service channels – equivalent to a number of boarding and alighting zones at the stop.

In Kendall's notation [4] this system is described by the following parameters:

$$| 1 | 2 | 3 | 4 | \quad (1)$$

- 1 – input process,
- 2 – distribution of a random variable describing the input service,
- 3 – number of boarding and alighting zones,
- 4 – service sequencing.

In order to describe the above processes, we need to use the identification of the movement processes related with the bus service at the stop, presented in [3]. Within the measuring scope, the character of a theoretical random variable distributions of the processes related with the functioning of the stop has been described, as well as the variable values and constant parameters used in the model have been defined.

The input process, meaning the input flow of buses, is the Palma stochastic process with Gamma distribution of intervals between the consecutive rides of the buses through the given point [3, 5]. For the increased bus volume (above 20 – 30  $[A/h]$ ), the exponential distribution is more appropriate. If the stop is located behind the intersection with the traffic lights, its cyclical work interfere with the input flow. The simplified models that allow to transform the unobstructed input, determined

with the interval distribution of buses flow leaving the intersection with the fixed cycle signalisation, has been described in figure [3, 7].

The two extreme options of influence of the fixed-cycled traffic lights on the input of buses were considered. In the first case, the constant and even traffic intensity is assumed – the only random process in this model comprise the input instants of buses queuing at the traffic lights. For the purpose of the simulation model, the knowledge about the transformation of the input instants at the stop line  $t$  and at the junction outflow  $t_1$  shall be sufficient:

$$t_1 = T - G + \frac{G}{T}t \quad (2)$$

where  $T$  is the duration of the traffic lights cycle, and  $G$  the duration of the green interval [3].

In the second case, the traffic influence on the arrival instants through the stop line is omitted – it concerns the situation, when at the intake the bus uses a bus lane, or moves together with the relation of a low traffic intensity. Bus lane is not the critical lane - the duration of the green interval depends on parallel operated flows, and is much higher than it would be required to ensure the capacity at the inflow of the bus lane. The moment of crossing the stop line is equal to the input instant, while the bus reaches the intersection on a green light, or is equal to the beginning of the first possible green light, while stopping at the junction:

- for  $mT < t < (m + 1) \cdot T - G$ ,  $m = 0, 1, 2, \dots$ :

$$t_1 = T - G \quad (3)$$

- for  $(m + 1) T - G \leq t \leq (m + 1) T$ ,  $m = 0, 1, 2 \dots$ :

$$t_1 = t \quad (4)$$

The diagram 1 presents the method, in which both models transform the input instants into operation of a traffic lights.

The service process comprise:

- boarding and alighting time  $t_w$ ;
- time lag between end of boarding and alighting process, and moving  $t_l$ ;
- clearance time  $t_d$ .

The boarding and alighting time might be described by Gamma distribution, using the following parameters; the mean value of boarding and alighting time, and a distribution parameter  $k$ . The time lag is defined, as a difference between the moving from the first zone of the stop, and completing the boarding and alighting process. A given distribution might be described, using the combination of the normal and exponential distribution [3]. By applying the proper approximation, a dwell time being a sum of a time losses at the stop and the time needed for boarding and alighting process, might be also described using Gamma distribution.

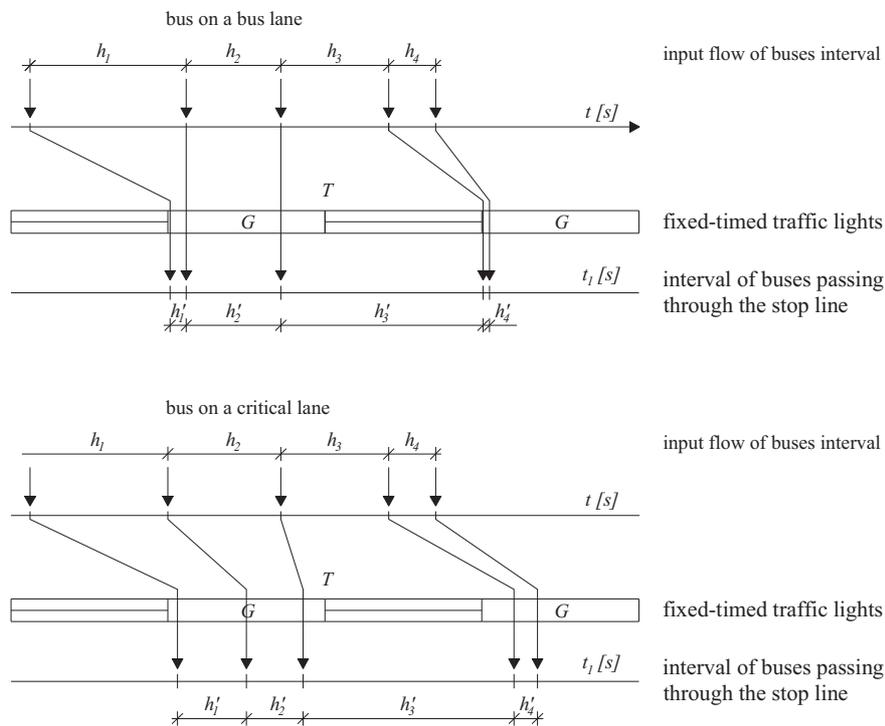


Fig. 1. Ideogram illustrate the impact of traffic lights on intervals of buses reaching the stop according to analysed models

The clearance time  $t_d$  is equal to the time differences between the moving of the last bus occupying the boarding and alighting zone, and starting the process of boarding and alighting by the first bus reaching the zone after queuing. Thus, it depends on the length and queuing position of the bus. If a bus is serviced without incurring the time losses, than  $t_d$  is equal 0.

The number of service channels is equal to the number of boarding and alighting zones – that is, the number of vehicles that can run the boarding and alighting process simultaneously. In practice, there are stops with one, two or three such zones. Limiting the stop analysis to three zones seems to be reasonable due to the fact, that platform extension obstructs the passengers orientation and prolong the time needed to get to the bus, arriving at relatively distant zone. In regard to the blocking of buses at stops, it leads to the increase in dwell time and reduction of a positive effect of extending the stop. According to [10] the estimated growth of operative capacity for bus stop with four, in relation to the stop with three boarding and alighting zones, comprise only 8%.

In practice, despite of defining the bus stops regulations by transport and communication boards, the number of boarding and alighting zones is directly related with the length of the platform. If the stop is used by the buses of a different length,

the number of zones constitute the length function of the stop and the type of buses that occupy the stop. It is unfavourable for the awaiting passengers, because it obstructs the accuracy in predicting a standing position of the bus.

In the case, where more than one zone is occupied, the vehicles can not bypass each other – the buses standing in the rear can only move, when the previous bus moved. It is difficult case to analyse, nevertheless is based on proven measures. The buses usually stop close to each other, blocking at the same time the possibility to bypass the bus in the front side. What is more, such manoeuvres are impossible for the bay stops. Through an analogy, the occupation of free zones by the oncoming (or queuing) buses is excluded, if farther zones (2 or 3) are occupied. Only if the last boarding and alighting zone is free, the queuing bus occupies the first one. As a result of these assumptions, the following graph of possible occupational steps of this system have been shown, figure 2. The numbers describe the occupation of the first, second and third (only in b) boarding and alighting zone. A zero means that the zone is free, and one – occupied by the bus.

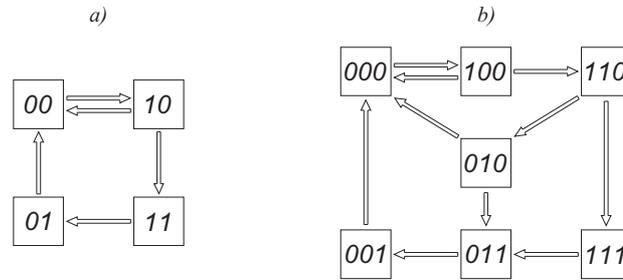


Fig. 2. Graph presenting the occupational steps of the boarding and alighting zones for the maximum number of zones a) two, b) three

As a result, buses are serviced according to the sequence of arrival – this is a standard case of FIFO (first in first out) sequencing without priorities. A size of the system is not limited – each bus arriving at the stop must be serviced, no matter how long the queue is and how big is the incurred time loss due to the servicing. For the above assumption, the system will be described as:

$$|G| E_k |L| F | \quad (5)$$

where:  $G$  – any distribution defined by the user,

$E_k$  – Erlang distribution with parameter  $k$ ,

$L$  – number of boarding and alighting zones equal to 1,2, or 3,

$F$  – service in regard to input sequencing.

Limiting the value of a parameter  $k$  of the Gamma distribution for integers exclusively, a special case of the Erlang distribution has been obtained, whose density function is easier to implement. Despite this for such a complicated system – taking into consideration the impact of traffic lights on input flow as well as blocking of

buses at stops – finding the analytical solution seems to be rather unreasonable. However, it is possible for simple instance of this system, as a special case of the examined:

$$|M| E_k | 1 | \quad (6)$$

being a Markov system, having the exponential input flow (lack of signalization), Erlang service time distribution and one boarding and alighting zone. Time losses incurred by buses  $d_a$  describes Pollaczek–Khintchine formula:

$$d_a = \frac{(k + 1)\rho \cdot \bar{t}_z}{2k(1 - \rho)} \quad (7)$$

in which:  $k$  – is Erlang distribution parameter,

$\rho$  – is the coefficient of saturation, defined as a ratio of arrival bus intensity  $q[A/s]$  and service time at the stop  $t_z$ :

$$\rho = q \cdot \bar{t}_z \quad (8)$$

Probability of possible queuing  $P_{w1}$  describe the formula:

$$P_{w1} = 1 - (1 - \rho) \left(1 + \frac{\rho}{k}\right)^k \quad (9)$$

### 3. Simulation Model of the Bus Stop

Based on presented in point 2. identification of the movement processes, the mathematical model has been developed, constituting an algorithm for the stop operations, independent from the number of boarding and alighting zones. Its implementation with using C programming language stands as a basis for a computer program counting the selected bus service indicators at stops for the given input data.

The basic input parameters comprise:

- traffic intensity of buses (10 – 90 A/h),
- average time needed for boarding and alighting of passengers (5 – 50 s),
- number of boarding and alighting zones (1 – 3),
- length of the kerbside (20 – 60 m),
- share of particular transportation buses in the input (0 – 100%),
- parameters of a traffic light program (the duration of a cycle and green light).

The selection of the remaining parameters, like theoretical distribution parameters is possible, however most of them are considered as default values.

The three types of a bus fleet have been determined, that are characterized by a different length. The standard division into classes has been employed, with the following establishing; mini (8 m), medium (12 m) and articulated (18 m). The

length of the bus fleet directly influence the number of available zones as well as the clearance time  $t_d$ . Modelling of a bus flow, described by different service parameters, has not been considered.

The relevant parameter, characterizing the service time, is related with the randomness of its duration. It has been observed, that together with growth of the average boarding and alighting time and dwell time, its standard deviation also increases. It might be caused by the increase in the number of variables obstructing the boarding and alighting process through its longer duration. The figure 3, characterizes a simple regression dependence between the standard deviation of a boarding and alighting time (response variable) and its average value (explanatory variable) in form of:

$$s = 0.42 \cdot \bar{t}_w \quad (10)$$

where:  $s$  – standard deviation of a boarding and alighting time at the stop,  
 $\bar{t}_w$  – mean boarding and alighting time at the stop.

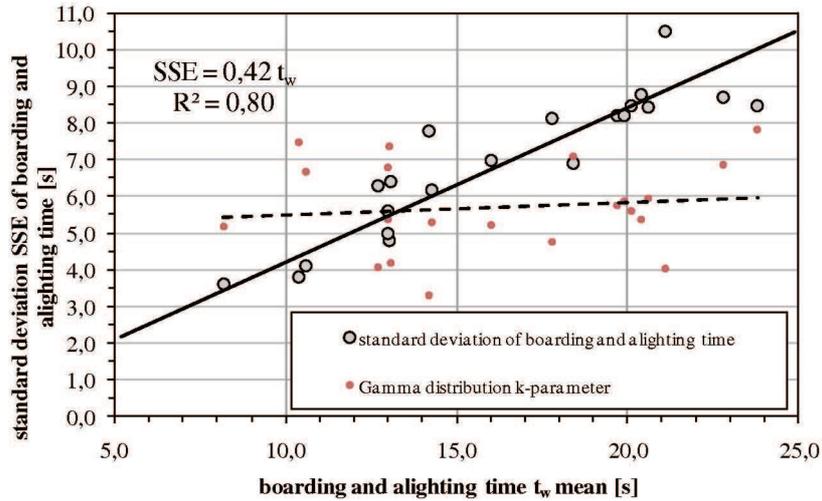


Fig. 3. Dependence of a standard deviation and coefficient  $k$  of the theoretical Gamma distribution from the average boarding and alighting time, based on the measurements conducted at the stops in Krakow and Warsaw

While integrating the above equation into the formula for the Gamma distribution  $k$ -parameter, we get its constant value:

$$k = \frac{\bar{t}_w^2}{s^2} = \frac{\bar{t}_w^2}{(0.42 \bar{t}_w)^2} = \frac{1}{(0.42)^2} \approx 6.0 \quad (11)$$

Finally, the Gamma distribution of a boarding and alighting time, describe the average boarding and alighting time and parameter  $k = 6.0$ .

The average boarding and alighting time with respect to the bus stop zone, has been linked with the parameter  $w_S$ . It presents the unitary extension of the boarding and alighting time  $t_w$  for farther zones in comparison with the first one:

$$w_S = \frac{\bar{t}_{wS} - \bar{t}_{w1}}{S} \quad (12)$$

where  $S$  is the zone number, and  $t_{wi}$  the average boarding and alighting time at the  $i$ -th zone. In the earlier identification research [9]  $w_S = 3s$ . Based on these measures, it has been discovered, that the value of this parameter depends on the geometric features of the stop (localisation of the shelter, available waiting areas) and that the extension of the boarding and alighting time for farther zones generally is not clearly visible. It has been assumed that default value of the parameter  $w_S$  is equal 0.

The major factor influencing the incurred time losses, is the localisation of the stop in terms of road and traffic lights placement. The case, in which the traffic lights are placed in front of the stop, has been omitted as a rarely encountered in Poland. On the ground of the individual measurement as well as [2], it has been found out, that the average time lag incurred after the boarding and alighting process, equals to the value between 5-8 s. As a special case, the most unfavourable zones have been chosen, situated at the bay along with the bordering lane earmarked for all vehicles. In the Table 1, the parameters of both theoretical distributions have been presented as well as probability value of non-occurrence of the time losses higher than 8 s.

Table 1

**Parameter values of the theoretical distribution of the time losses**

normal distribution for $t_i \leq 8$ s		exponential $t_i > 8$ s	theoretical distribution	
mean [s]	standard deviation [s]	mean [s]	mean [s]	$P(t_i \leq 8)$
1	2	3	4	5
4.7	1.9	14.9	7.9	0.69

The time needed to reach the boarding and alighting zones after queuing, is described by the linear regression equation presented in figure 4. – it emerges from following correlation; the growth of the distance causes the proportional extension of the ride time. Moreover, the buses moving from the second position in the queue and stopping at the second zone, need on the average more time to move by  $\tau = 2.0s$ . Bearing in mind this rule, the movement time describe the formula:

$$t_d = \gamma \cdot DL + 4.2 + (S - 1) \cdot \tau \quad (13)$$

where:  $t_d$  – movement time of the bus at the zone after queuing,

$DL$  – length of the movement distance from the queue [-],

$S$  – zone at which the bus stops (1, 2, 3) [-],

$\gamma$  – coefficient of the movement time – 0.24 [1/m],

$\tau$  – unitary extension of the movement time at the second and third zone – 2.0 [s].

In the case, where bus, occupying the second or third zone finishes the boarding and alighting process, which happens before the bus, occupying the lower zone moves away from the stop, than it incurs the time losses resulting from the blocking of the stop. Moving away together with the preceding bus is possible (we ignore the other causes of stopping the bus) and is followed with a time lag. It results from the rule, that some drivers don't close the vehicle doors after the process of boarding and alighting, until they have a possibility to move away from the stop. For the sample of 76 buses, the value of a departure delay while moving has been identified  $\eta = 1.9s$ .

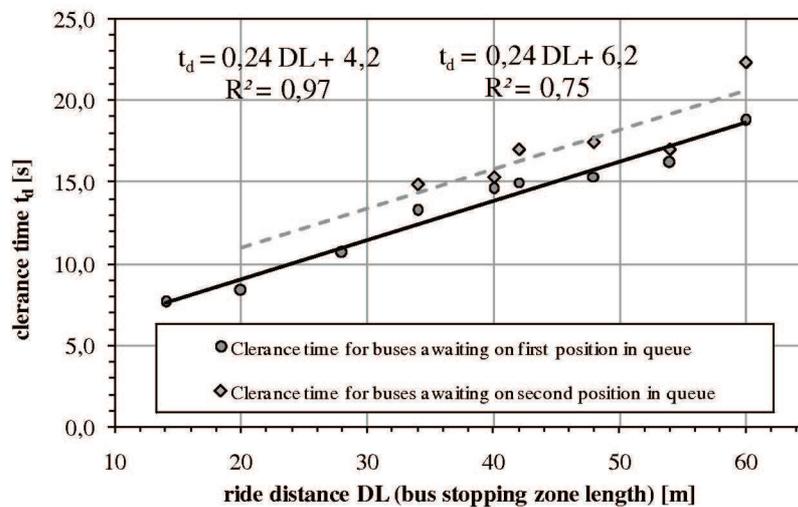


Fig. 4. The dependence of movement time at the boarding and alighting zone for queuing buses, from the length of the distance (the number of zones) based on the measurements performed at the stops in Krakow and Warsaw

In the simulation model, the input process diagram and service process diagram (Fig. 6) have been distinguished, together with the servicing of the queues (Fig. 5) and pillars concerning the service quality indicators. The random processes comprise the instants of arrival of buses at the stop, the boarding and alighting time, and the time loss incurred after the boarding and alighting process. The remaining parameters are declared default.

In order to obtain the random numbers, a generator of uniformly distributed random numbers has been applied. The required distributions have been obtained by implementing the inverse cumulative distribution function (exponential distribution), summing method (normal distribution) and Neumann method of elimination (Erlang distribution).

Table 2

Parameters used in service process diagram

par.	No.	variable	variable description
1	2	3	4
defined by user	1	$Q_a$	bus input volume [P/h]
	2	$k_1$	input flow Erlang distribution k-parameter
	3	$k_2$	boarding and alighting time Erlang k-parameter
	4	$T_w$	average boarding and alighting time
	5	$D_{ST}$	bus stop stopping length
	6	$S_{max}$	number of boarding and alighting zones
constant	7	$\gamma$	coefficient of the movement time - 0.24 [1/m]
	8	$\eta$	time lag for blocked bus departing from second and third stopping zone immediately after bus on lower zone - 1.9 [s]
	9	$\tau$	unitary extension of the movement time at the second and third zone - 2 [s]
	10	$J_{max}$	number of simulated bus stop service - 2500 [A]
variable	11	$i$	counter of generated bus
	12	$j$	counter of serviced bus
	13	$S$	counter of bus in system, stopped or queuing
	14	$D[i]$	length of i-numbered bus
	15	$T1(i)$	i-bus generation timing
	16	$T2(i)$	i-bus beginning of boarding and alighting process
	17	$T3(j)$	j-bus end of autobusu z przystanku (zwolnienie stanowiska)
	18	$T_w(j)$	boarding and alighting time (generated using Erlang distribution)
	19	$T_T(j)$	time lag incurred after the boarding and alighting process
	20	$P[j]$	variable used for the classification of bus waiting in the queue
	21	$OC[j]$	delay incurred by j-bus queuing to reach the stop
	22	$B[j]$	j-bus delay caused by bus blocking on second or third stopping zone
	24	$ST[j]$	boarding and alighting zone number where j-bus is serviced

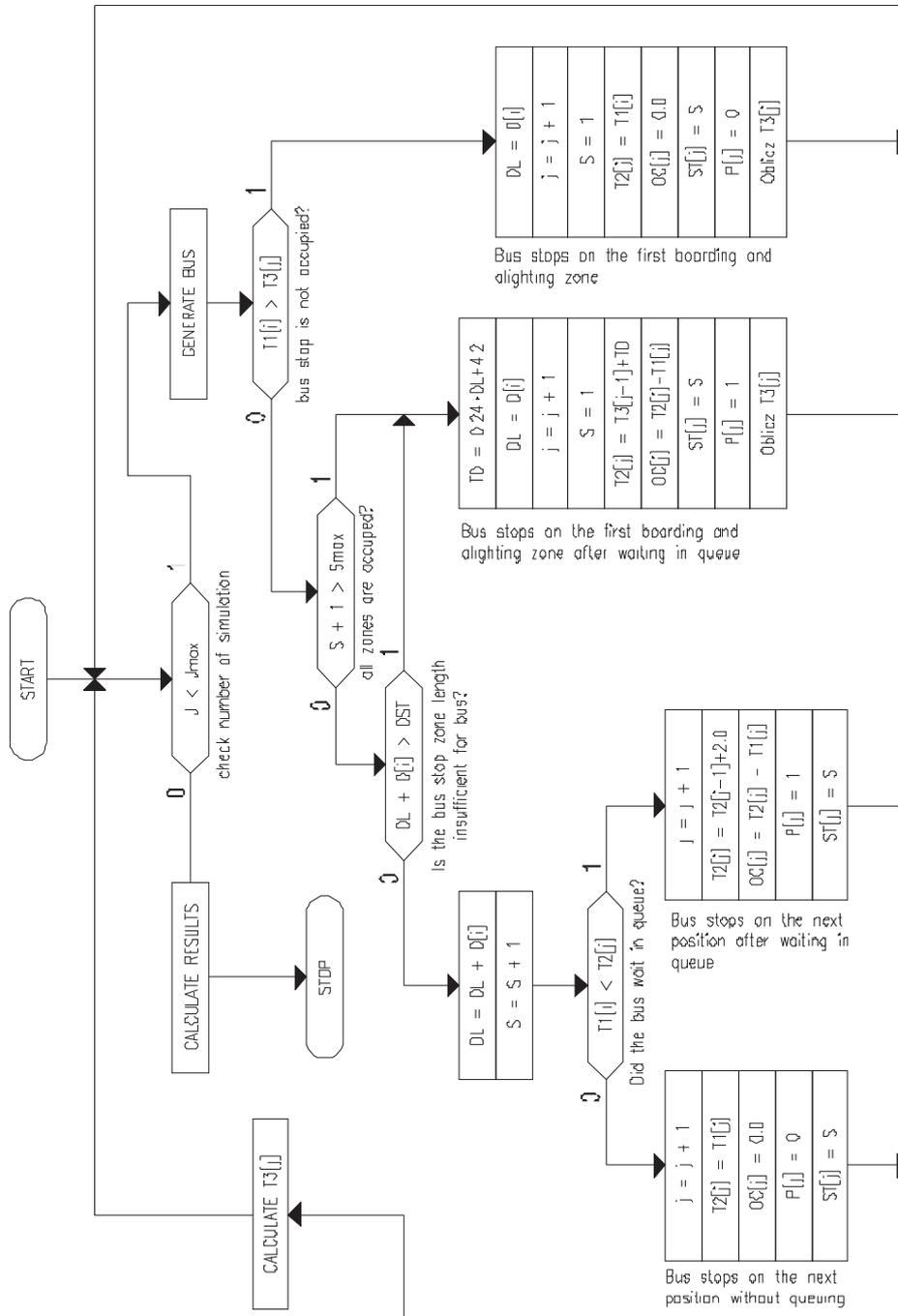


Fig. 5. Service process diagram with servicing of the queues

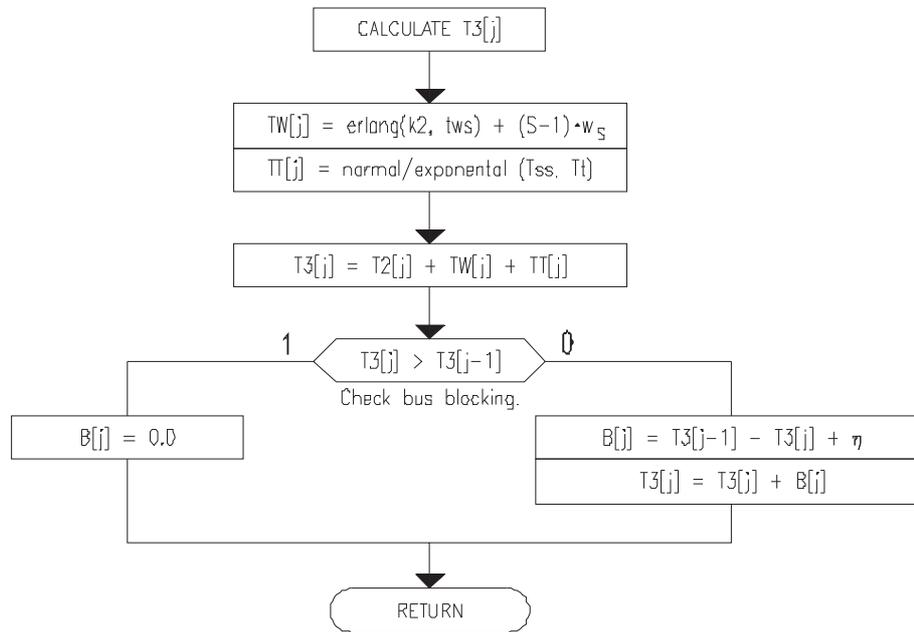


Fig. 6. Service process diagram

## 4. Model Verification

The basic parameters, that constitute an assessment background for the consistency of the simulation results with the actual performance, are the delays incurred by buses and a probability of queuing. The results, obtained from the simulation, have been compared with the measurements conducted at the stops in Krakow and Warsaw, presented in Table 3.

A compilation of the simulation results with the measurements (Table 4) imply the relatively good consistency. In case of time losses, the average discrepancies are equal to 0.30 s/A. The relative differences (concerning the values of the incurred time losses) oscillate around 0.12. However, the most consistent outcomes have been recorded for the bus stops at "Rondo Barei" and "Plac Inwalidów" in Krakow. These were the bus stops, where the biggest measurement data has been performed.

By contrast, the biggest discrepancies have been observed at the bus stops "Konopnickiej" in Krakow and "Dworzec Wileński 01" in Warsaw. It is hard to find an objective explanation for the first one. However, the difference for the second stop, might be caused by the simplification related with the presence of traffic lights. The bus flow has been operated from the three inflows of the intersection for the separate traffic phases, despite its increased intensity. What is more, the lack of traffic lights from the front side of the stop has been assumed.

Table 3

**The stops, at which the measurements of the time losses have been performed**

No.	city, a bus stop, (direction)	Number boarding and alighting zones	Influence of traffic lights	Sample size
1	2	3	4	5
1	Kraków, Uniwersytet Ekon. (R. Mogiłskie)	1	no	102
2	Kraków, Konopnickiej (Most Grunwaldzki)	1	yes	269
3	Kraków, Plac Inwalidów (AGH)	2	yes	449
4	Kraków, Rondo Barei (Park Wodny)	2	no	520
5	Kraków, Rondo Barei (Park Wodny)	2	yes	198
6	Kraków, Wiślicka (Okulickiego)	2	no	201
7	Warszawa, Plac Konstytucji (Politechnika)	2	yes	102
8	Warszawa, Dworzec Wileński (Dw. Wschodni)	2	no	211
9	Bielsko-Biała, Dworzec 3 Maja (Piastowska)	3	no	112
10	Warszawa, Centrum 06 (Dw. Centralny)	3	no	461

Table 4

**The comparison of the simulation results with the results of empirical measurements**

No of s.	bus stop (no. boarding and alighting zones)	Traffic intensity $Q$ [A/h]	Average boarding and alighting time $t_w$ [s]	Bus delays $d_a$ [s/A]		Probability of queuing [-]	
				measure.	sim.	measure.	sim.
1	2	3	4	5	6	7	8
1	Uniwersytet Ekon. (1)	41	13.2	7.9	8.6	0.31	0.27
2	Konopnickiej (1)	36	10.6	<u>2.8</u>	<u>5.4</u>	<u>0.13</u>	<u>0.20</u>
3	Plac Inwalidów (2)	32	21.1	2.9	3.1	0.08	0.09
4	Rondo Barei (2)	40	11.3	1.2	1.3	0.05	0.04
5	Rondo Barei (2)	40	12.3	1.7	2.0	0.07	0.06
6	Wiślicka (2)	65	10.6	2.2	2.7	0.08	0.11
7	Plac Konstytucji (2)	51	10.5	1.6	1.7	0.07	0.07
8	Dworzec Wileński (2)	58	18.6	<u>1.8</u>	<u>4.8</u>	<u>0.06</u>	<u>0.14</u>
9	Dworzec ul. 3 Maja (3)	56	16.5	1.8	1.4	0.04	0.05
10	Centrum 06 (3)	86	14.0	3.1	2.9	0.11	0.09

The presented models of the traffic lights influence, lead to the time losses, whereas the simulation results significantly exceed the measurement data. The analogical case, yet smaller in its scale, shows the “Wiślicka” stop (toward Krakow city centre), where differences comprise 0.5 s/A (although here, the statistical test does not allow for rejecting the t-test hypothesis).

While comparing the time losses, the t-test for two means has been used, aiming at assessing the relevance of differences. Where, for the comparison of probability of queuing, the significance test for two structure indices. With the exception of these

two cases (underlined in Tab. 4), the differences appear to be statistically irrelevant adopting a level of significance  $\alpha = 0.05$ .

Finally, apart from these two exceptions, the simulation seems to work in accordance with the measurement data. For the case, where the stop has only one boarding and alighting zone, the simulation results are compliant with the formula (7), whereas the probability of queuing is higher for the simulation, than obtained from the formula (9). It might result from the simulation features, where each bus incurring a minimal time loss, is counted as a queuing bus.

## 5. Presentation of the Simulation Results

The designed for this purpose computer program, has been used, to perform the simulation series on a various data. The stop with an exponential input flow of articulated buses (share of the articulated bus  $u_p = 100\%$ ), unobstructed with the traffic lights operations, has been used as a fundamental case of analysis.

The  $\delta$  parameter has been introduced, aiming at assessing the time losses incurred by queuing buses. It has been defined as a ratio of the average time losses incurred by buses to service time:

$$\delta = \frac{\bar{d}_a}{t_z} \quad (14)$$

where:  $\bar{d}_a$  – average delays incurred by buses queuing to reach the stop [s],  
 $t_z$  – the average service time [s].

Based on parameter  $\delta$ , we can derive the formulas for average delays of buses  $d_{ai}$  (for correlation  $\delta = f(\sigma)$  a very high determination coefficient  $R^2$  can be obtained) with respect to assumed number of boarding and alighting zones:

$$\begin{aligned} d_{a1} &= t_z \cdot \left[ 0.148 e^{3.22 \frac{Q_a \cdot t_z}{3600}} - 0.148 \right] R^2 = 0.99 \\ d_{a2} &= t_z \cdot \left[ 0.023 e^{3.50 \frac{Q_a \cdot t_z}{3600}} - 0.023 \right] R^2 = 0.99 \\ d_{a3} &= t_z \cdot \left[ 0.007 e^{3.49 \frac{Q_a \cdot t_z}{3600}} - 0.007 \right] R^2 = 0.97 \end{aligned} \quad (15)$$

While considering the time components of the process of reaching the zone, the formulas (9) take form:

$$\begin{aligned} d_{a1} &= (t_w + 17) \cdot \left[ 0.148 e^{3.22 \frac{Q_a \cdot (t_w + 17)}{3600}} - 0.148 \right] \\ d_{a2} &= (t_w + 17) \cdot \left[ 0.023 e^{3.50 \frac{Q_a \cdot (t_w + 17)}{3600}} - 0.023 \right] \\ d_{a3} &= (t_w + 17) \cdot \left[ 0.007 e^{3.49 \frac{Q_a \cdot (t_w + 17)}{3600}} - 0.007 \right] \end{aligned} \quad (16)$$

in which, the average boarding and alighting time  $t_w$  and traffic intensity of buses  $Q_a$  are basic variables.

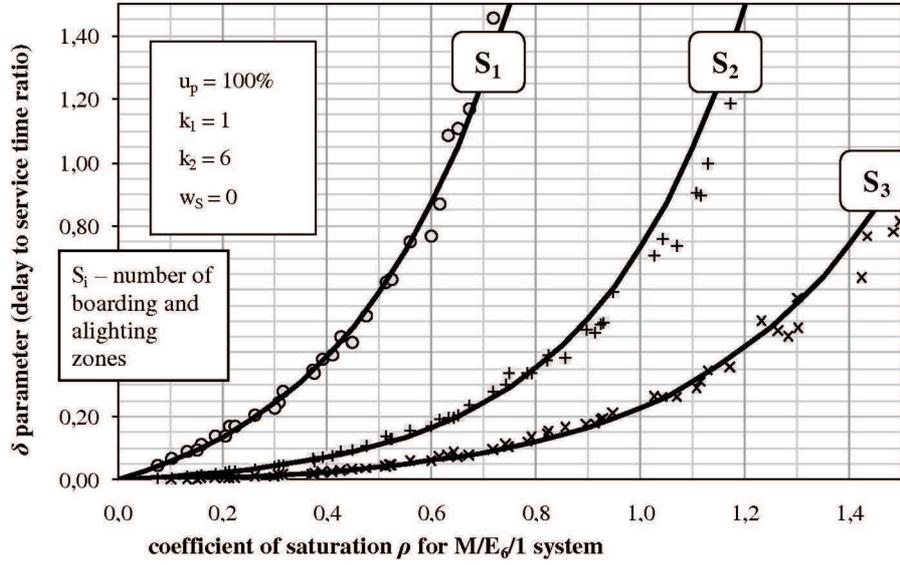


Fig. 7. Correlation of the parameter  $\delta$  and coefficient of saturation  $\rho$

In the situation, where the number of boarding and alighting zones is dependent from the kerbside the average delays incurred by queuing buses might be calculated as a weighted average between the time losses  $d_a$ , defined for the fixed number of the zones. The weights comprise the probabilities, that the stop may be used by the certain number of buses. It depends on the length of the stop as well as the length of buses and its input contribution. The formula below, presents the analytical determination of weighting coefficients, for the frequently encountered case of occurring the two types of bus services, articulated and traditional.

Treating the arrival of the articulated bus at another zone as a success, and as a failure the arrival of traditional bus, the probability of standing a higher number of buses ( $S_{max}$ ) is equal to binomial cumulative distribution function:

$$P_{S_{max}} = \sum_{k=0}^{k_{dop}} \binom{S_{max}}{k} \cdot u_p^k \cdot (1 - u_p)^{S_{max}-k} \quad (17)$$

$P_{S_{max}}$  – probability that the stop can be used by the higher number of buses,

$S_{max}$  – higher number of zones ( $S_2$  or  $S_3$ ),

$k$  – number of articulated buses standing at the stop,

$k_{dop}$  – acceptable number of articulated buses allowing for simultaneous presence of higher number of other buses.

Assuming, that the boarding and alighting zone is longer than bus of about 2m, the values  $P_{Smax}$  classified in terms of the length of the bus bay  $l$  and the share of articulated buses in the input flow, have been compiled in Table 5.

Table 5  
Probability that the stop can be used by the higher number of buses, depending on the length of the bus bay  $l$  and share of articulated buses in the input flow

Combination of bus stop occupation	$l$ [m]	$S_{max}$ [-]	$k_{dop}$ [-]	$P_{Smax}$				
				20%	35%	50%	65%	80%
	20 - 34	2	0	0.64	0.42	0.25	0.12	0.04
	34 - 40	2	1	0.96	0.88	0.75	0.58	0.36
	40 - 42	2	2	1.00				
	42 - 48	3	0	0.51	0.27	0.13	0.04	0.01
	40 - 54	3	1	0.90	0.72	0.50	0.28	0.10
	54 - 60	3	2	0.99	0.96	0.88	0.73	0.49
	> 60	3	3	1.00				

The delays incurred by buses describe the formula:

$$d_a = P_{Smax} \cdot d_{Smax} + (1 - P_{Smax}) \cdot d_{Smin} \quad (18)$$

whereas, the equivalent number of boarding and alighting zones  $S_e$  describes the formula:

$$S_e = S_{min} + P_{Smax} \quad (19)$$

The presented calculation method matches the results obtained from the simulation and measurement.

In order to assess the influence of the traffic lights on a value of delays, the following  $f_m$  coefficient has been defined:

$$f_m = \frac{\bar{d}_{aSGN}}{\bar{d}_a} \quad (20)$$

where:  $d_{aSGN}$  means the average time loses of buses with traffic lights influence.

For both cases, the coefficient  $f_m$  can be described using the equation:

$$f_m = 1.2 - (0.017T - 0.8) \cdot \ln \lambda_G - 0.004 TR^2 = 0.73 \quad (21)$$

where:  $T$  – duration of the traffic light cycle,

$\lambda_G$  – share of the effective green interval in the cycle G/T.

A low value of the determination coefficient  $R^2$  results from the interaction between the particular parameters, influencing the delays incurred by buses. For some proportions of a different traffic intensity of buses as well as the cycle duration

and green light interval, the implication is that traffic lights might lower the time losses of buses, organizing them in a number that corresponds the available zones of the stop.

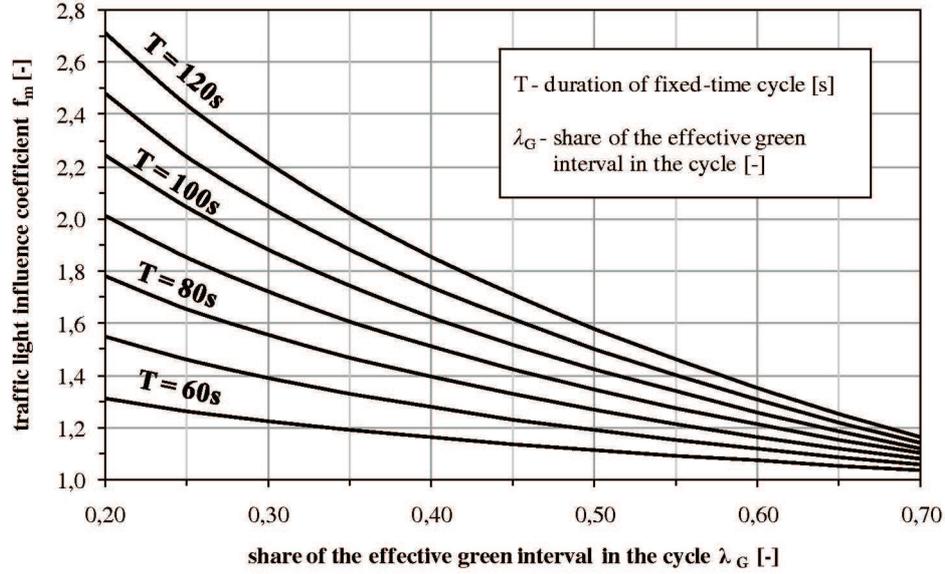


Fig. 8. Correlation of coefficient  $f_m$  and signalization parameters

The improvement of regularity of bus rides significantly reduce the time losses, incurred by buses due to queuing. However, it is only possible for a small volumes of buses. The intensity equal to 40 A/h has been assumed as a critical value. The coefficient  $f_k$  reducing the time losses on the advantage of enhanced regularity of bus rides has been defined:

$$f_k = \frac{\bar{d}_{aki}}{d_{k1}} \quad (21)$$

where:  $d_{aki}$  – average time loss for input flow described by any  $k$ -parametr,

$d_{k1}$  – average time loss for exponential input flow,

in which  $k$  is a parameter of the Erlang distribution of input flow (represents the regularity of bus arrivals) where the coefficient can be described by the formula:

$$f_k = 0.75k^{-2.1} e^{(0.4k+1)\rho} \quad (22)$$

where  $\rho$  is a coefficient of the saturation of system (formula 8). For the stop with only one boarding and alighting zone, the derivation of the correlations is shown in the Figure 7. In addition, for the bus stops with two or three zones, the influence of the regularity of bus rides on time losses incurred by buses, is comparable.

A slightly increased improvement of the regularity results from the limiting of the bus – to – bus blocking at the stops.

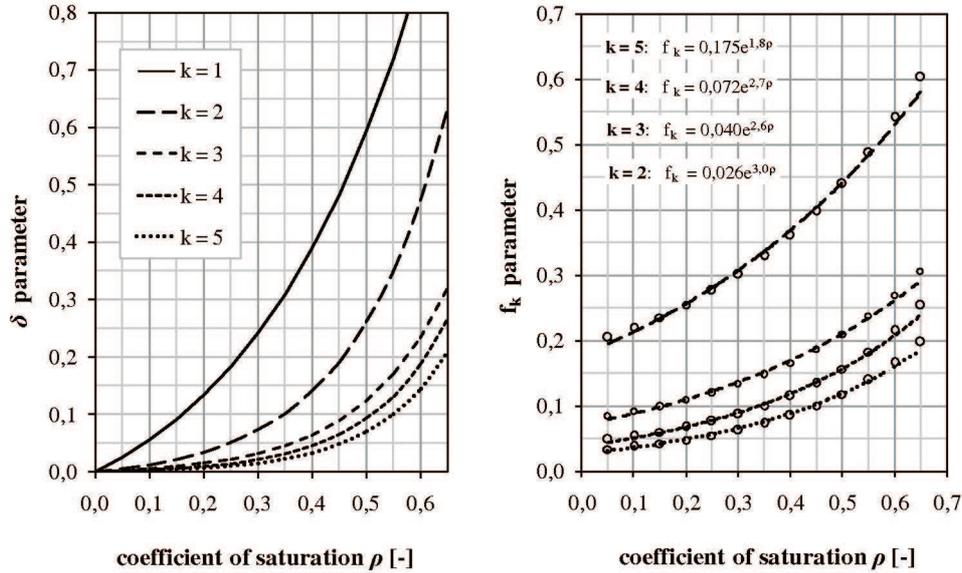


Fig. 9. Correlation of coefficient  $f_k$  and saturation coefficient  $\rho$  and parameter  $k$  of the input flow

In practice, there might be a case, where a boarding and alighting time increases at farther zones. It is due to the fact, that passengers have to get there from the front side. Assuming  $w_s = 3,0$ , coefficient reducing the time losses with regards to a fundamental case  $f_w$  comprise:

for the stop with two zones  $S = 2, f_w = 1.05$

for the stop with three zones  $S = 3, f_w = 1.18$

As a result, the time losses incurred by the buses at stops describe the formula:

$$d = d_a \cdot f_m \cdot f_k \cdot f_w \quad (23)$$

The remaining variables have a minor impact on the time losses incurred by queuing buses.

## 6. Conclusion

The delays incurred by queuing buses might constitute a parameter of service quality measurement at bus stops, aiming at determining the design capacity of a bus stop and defining a number of boarding and alighting zones.

From the point of view of mass service theory, the bus stop is a complex system, where the input flow comprise buses, whereas a service time, constitute a

occupation time at the boarding and alighting zone. The analytical solutions exist only for simple cases of a bus stop with only one boarding and alighting zone. The identification of a function describing the delays for a higher number of zones, requires modelling of the stop, using a computer simulation.

The simulation model has been verified, based on performed measurements of the time losses incurred by queuing buses. In most cases, the statistical tests confirmed the compilation of the simulation results with the measurements.

The created model has been used to estimate the time losses incurred by buses. On the basis of simulation results, we can conclude that:

- The basic parameters, having impact on the amount of time losses incurred by buses are: the traffic intensity of buses, the service time (dwell time), the number of boarding and alighting zones as well as signalization program at the intersection in front of the stop.
- Increasing the number of boarding and alighting zones from 1 to 2, allows to reduce the time losses incurred by buses from 78% to 83%. Adding another zone will reduce the time loss of another 12% – 15%.
- The traffic lights situated in front of the stop, causes the increase in time losses incurred by buses due to filtration of the input flow. The basic variables, that decide about its size are; the share of a green interval in a cycle  $\lambda$  and a duration of a traffic light cycle. For both cases, where a bus uses a bus lane or shares the lane, the results obtained from the simulation are compliant. It gives a possibility to conceptualise the simulation results as intermediate instances of traffic intensity at intersection, with relatively good approximation. Traffic lights may increase the loss of buses more than twice for long cycles and low share of a green interval.
- The improvement of regularity of bus rides is a highly effective method of limiting the delays incurred by queuing buses. For bus stops with only one boarding and alighting zone, the reduction might comprise from 40% to 90%.
- The randomness of a boarding and alighting time is reflected for a higher traffic intensity level at the stop.
- The extension of a boarding and alighting time for farther zones causes the increase in time losses of about 5% for stops with two zones, and 15% – 20% for stops with three zones.
- The impact of other variables on time losses is of minor importance.

The presented simulation model allow for determining the design capacity of a stop, based on acceptable value of time losses  $d$ , and its share as to the service time  $\delta$ .

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