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JAKUB SIEMEK*, JERZY STOPA*

ANALYTICAL MODEL OF WATER FLOW IN COAL WITH ACTIVE MATRIX

ANALITYCZNY MODEL PRZEPŁYWU WODY W WĘGLU Z UWZGLĘDNIENIEM WPŁYWU MATRYCY WĘGLOWEJ

This paper presents new analytical model of gas-water flow in coal seams in one dimension with emphasis on interactions between water flowing in cleats and coal matrix. Coal as a flowing system, can be viewed as a solid organic material consisting of two flow subsystems: a microporous matrix and a system of interconnected macropores and fractures. Most of gas is accumulated in the microporous matrix, where the primary flow mechanism is diffusion. Fractures and cleats existing in coal play an important role as a transportation system for macro scale flow of water and gas governed by Darcy's law. The coal matrix can imbibe water under capillary forces leading to exchange of mass between fractures and coal matrix. In this paper new partial differential equation for water saturation in fractures has been formulated, respecting mass exchange between coal matrix and fractures. Exact analytical solution has been obtained using the method of characteristics. The final solution has very simple form that may be useful for practical engineering calculations. It was observed that the rate of exchange of mass between the fractures and the coal matrix is governed by an expression which is analogous to the Newton cooling law known from theory of heat exchange, but in present case the mass transfer coefficient depends not only on coal and fluid properties but also on time and position. The constant term of mass transfer coefficient depends on relation between micro porosity and macro porosity of coal, capillary forces, and microporous structure of coal matrix. This term can be expressed theoretically or obtained experimentally.

Keywords: two-phase flow, coal, imbibition, mathematical modeling, analytical solution

W artykule zaprezentowano nowy model matematyczny przepływu wody i gazu w jednowymiarowej warstwie węglowej z uwzględnieniem wymiany masy między systemem szczelin i matrycą węglową. Węgiel jako system przepływowy traktowany jest jako układ o podwójnej porowatości i przepuszczalności, składający się z mikroporowatej matrycy węglowej oraz z systemu szczelin, spękań i ewentualnie największych porów. Przepływowi w systemie szczelin towarzyszyć może wymiana masy z matrycą, której intensywność zależy m.in. od właściwości węgla i warunków panujących w układzie przepływowym. W szczególności matryca węglowa może pochłaniać wodę pod wpływem sił kapilarnych, co wpływa na przepływ w szczelinach. W artykule zostało zaproponowane równanie różniczkowe cząstkowe opisujące

* AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY, FACULTY OF DRILLING OIL AND GAS, AL. A. MICKIEWICZA 30, 30-059 KRAKOW, POLAND

nasycenie wodą w systemie szczelin z uwzględnieniem wymiany masy z matrycą pod wpływem sił kapilarnych. Podano dokładne rozwiązanie analityczne, które może być zastosowane w praktyce inżynierskiej. Zauważono, że szybkość wymiany masy między szczelinami i matrycą wyraża się formułą analogiczną do prawa stygnięcia Newtona, ale w analizowanym przypadku współczynnik wymiany masy zależy nie tylko od właściwości węgla i płynów ale również od położenia i czasu. Stały człon tego współczynnika może być obliczony teoretycznie lub wyznaczony eksperymentalnie.

Słowa kluczowe: przepływ dwu-fazowy, węgiel, siły kapilarne, model matematyczny, rozwiązanie analityczne

Nomenclature

A – coefficient defined by (12), $s^{-1/2}$,
C_L – leakoff coefficient, m $s^{-1/2}$,
D – diffusion coefficient, m^2/s,
k_i – effective permeability, m^2, $i = w$, g,
p_c – capillary pressure, bar,
p_i – pressure, bar, $i = w$, g,
Q_i – rate of mass exchange, kg/m^3s, $i = w$, g,
R – typical dimension, m,
S_i – saturation in fractures, $i = w$, g,
S_w^* – water saturation in matrix,
t – time, sec, days,
u – Darcy velocity, m/s,
V – velocity of water front, m/s,
x – position, m,
ϕ – “fractures” porosity,
ϕ^* – “matrix” porosity,
μ_i – viscosity, Pa s, $i - w$, g,
ρ_i – density, kg/m^3, $i - w$, g,

subscripts

sc – standard conditions,
g – gas,
w – water.

1. Introduction

Coal seams are dual porosity reservoirs that consist of porous matrix and cleat (fracture) network. Most coal seams are saturated with water which may exist in both the matrix and the cleats. The water in the matrix is usually considered as a factor that affects the gas transport in coal seams through changing the gas effective diffusivity. The water in the cleats affects the gas flow through the relative permeability effect (Chen et al., 2013).

The importance of these phenomena yields both from safety reasons (water inflow into coal mines) and increasing role of unconventional energy sources, such as coalbed methane. One promising method of recovery, currently being in testing phase, (Stopa et al., 2000; Saghafi et al.,

2007) is enhanced coal-bed methane (ECBM) recovery by use of CO_2. During primary operations, coal is dewatered to initiate and accelerate methane desorption. However, the re-saturation of coal with water following CO_2 enhanced recovery operations is critical to the long-term, secure retention of CO_2 in the subsurface (Chaturvedi, 2009).

In many paper published in past, the importance of understanding the role of matrix for flow phenomena in coal is emphasized (Nikoosokhan et al., 2014), but very limited number of papers reports research on interactions between water flowing in cleats and coal matrix.

Hence, the present paper is directed towards understanding water transport in coal and the role of wettability in flow properties.

2. Previous work

Naturally occurring methane within coal seams is usually produced using methods and technologies adopted from conventional oil and gas industry. In particular, this refers to the mathematical modeling of reservoirs which is the common technique used for the optimization of natural gas production systems. Unlike conventional reservoirs, flow phenomena in coal seams are quite complex, mainly due to the heterogeneous nature of coal and possible impact of double porosity nature of coal (Remner et al., 1986; Siemek et al., 1995).

Many commercial and research models have been developed to simulate the coalbed methane (CBM) recovery processes (Siemek & Stopa, 1998.). The existing models are classified as: empirical, analytical and numerical models. These models have taken into account many of the important factors including; the dual porosity nature of coal beds, multiphase flow of gas and water in the fracture system, coal matrix shrinkage due to gas desorption and other. The imbibition of water by coal and impact of this phenomenon on flow is usually not included in flow models, however the kinetics of the process in static conditions have been studied by many investigators. Imbibition of water by coal matrix can be modeled as a diffusion-like process under capillary forces (Stopa, 1990). It is known from the petroleum engineering literature (Economides & Nolte, 1989) that water flowing through the fracture leaks off to the formation with Darcy velocity u at the surface of fracture:

$$u(t) = \frac{C_L}{\sqrt{t}} \tag{1}$$

where t is the time of contact, $t > 0$ and C_L is empirical constant called "leakoff coefficient" which depends on rock and fluid properties, and on difference between pressure in fracture and in formation.

As the Darcy velocity represents volume of water entering the formation per unit area perpendicular to the flow direction, per time, than the mass of water imbibed by coal can be calculated by integration of eq. (1). On the other side, laboratory experiments on coal samples reported in the literature (Chaturvedi, 2009; Stopa, 1990) show that for small time, the mass of water imbibed by coal is proportional to square root of t. This corresponds to eq. (1) and moreover, this make it possible to obtain the C_L experimentally (Stopa, 1990). The leakoff coefficient C_L can be also obtained theoretically for simple geometries assuming diffusion like process under capillary forces as a driving force for water imbibition. Analytical solution of the resulting diffusion equation for spherical coal grains was presented in previous paper (Stopa, 1990, 1996). Relating the mass of water imbibed by coal with change of the water saturation within coal grains, it has been found that, the following approximate formula for average saturation of coal matrix with surrounding water may be used for small time:

$$\left\langle S_w^* - S_{w0}^* \right\rangle \approx 6(S_w - S_{w0}^*)\sqrt{\frac{D}{\pi R^2}t} \tag{2}$$

where: S_w, S_w^* are water saturations in cleats and in matrix respectively, S_{w0}^* is initial matrix saturation with water, < > denotes averaging on the volume of matrix block, D is "diffusion coefficient", R is specific length of coal matrix that may be interpreted as typical radius of coal grains, t is time.

Coal as a flowing system, can be viewed as a solid organic material consisting of two flow subsystems: a microporous matrix and a system of interconnected macropores and fractures (Remner et al., 1986; Siemek et al., 1995). Most gas occurs within the microporous matrix, where the primary flow mechanism is diffusion. Fractures and cleats penetrating the coal matrix have limited storage capacity, but they play an important role as a transportation system for water and gas. In the majority of cases a mathematical model describing a two-phase gas-water flow in coal, the so-called Warren-Root model, is operational (Remner et al., 1986, Siemek et al., 1995). In the Warren-Root model gas is treated as a homogeneous fluid, the properties of which depend only on pressure. In this model a coal bed is characterized by two porosity systems, i.e. system of fractures and the matrix. The first of them is water and gas permeable and the flow is ruled by the Darcy's filtration law. It is assumed that in the second case no macroscopic flow takes place. Only gas exchange between the matrix and the system of fractures may take place. The model is based on mass balance equations for gas and water in fractures.

In this paper a new model incorporating imbibition of water by coal is introduced and rigorous analytical solution based on method of characteristics is presented.

3. Mathematical formulation

Mathematical model of two phase gas-water flow in one dimensional coal seam is:

$$\frac{\partial}{\partial t}(\phi S_w \rho_w) + \frac{\partial}{\partial x}(u_w \rho_w) + Q_w = 0 \tag{3}$$

$$\frac{\partial}{\partial t}(\phi S_g \rho_g) + \frac{\partial}{\partial x}(u_g \rho_g) + Q_g = 0 \tag{4}$$

$$u_i = -\frac{k_i}{\mu_i}\left(\frac{\partial p_i}{\partial x} + \rho_i g \sin\alpha\right), \quad i = w, g \tag{5}$$

$$S_w + S_g = 1 \tag{6}$$

$$p_g + p_w = p_c(S_w) \tag{7}$$

Here, the first two equations yield from conservation of mass, next is Darcy law, and then balance of saturations and capillary pressure equation. Mathematical analysis of the above system of equations has been presented by Siemek and Stopa (1998). It has been shown that rearranging equations (3) to (7), yields following equation for water saturation in fractures (Stopa, 1996).

$$\frac{\partial S_w}{\partial t} + V(S_w) \cdot \frac{\partial S_w}{\partial x} = -\frac{Q_w}{\phi \rho_w} \tag{8}$$

It has been shown that water entering the coal seam displaces gas from fractures forming the shock front of water saturation, moving with constant velocity V depending on saturation on the front. In general, V may be found by application of weak solutions theory to the equation (8) as has been discussed in (Siemek & Stopa, 1998). Assuming V = const, eq. (8) represents principle of mass conservation for water phase.

If no sources or sinks exist in the flowing system, then Q_w and Q_g may represent mass exchange between fractures and matrix of water (drainage or imbibition) and gas (sorption or desorption) respectively. In most papers Q_w is ignored as the model is usually used for simulation of degassing process. However if water is entering the coal seam, then coal can imbibe water and Q_w should not be set to 0. Physically, term Q_w represents the volume of water imbibed by unit of bulk volume, in unit time. If the coefficient of "fracture porosity" equals to ϕ, then the coal substance volume in a geometrical unit of bed is $1 - \phi$. Assuming that Ω is the bulk volume, thus the volume of coal matrix within the bulk volume is $(1 - \phi)\Omega$. If the coefficient of "matrix micro porosity" equals to ϕ^*, then the volume of micropores within Ω is: $(1 - \phi)\phi^*\Omega$. Thus the rate of water mass imbibed by unit volume is:

$$Q_w = \rho_w (1-\phi)\phi^* \frac{\partial}{\partial t} \left\langle S_w^* - S_{w0}^* \right\rangle \tag{9}$$

Assuming, that imbibition process at certain x_0 starts at $t_0 = x/V$, then for $t > t_0$ function S_w is differentiable at x_0. Thus for $t > t_0$, and for small time interval, S_w may be assumed constant in time. Under this assumption considering (1), and using (2) and (9) yields:

$$Q_w \approx 3\rho_w (1-\phi)\phi^* (S_w - S_{w0}^*) \sqrt{\frac{D}{\pi R^2}} \frac{1}{\sqrt{t}} \tag{10}$$

It is assumed that initially the system is in equilibrium, thus $S_{w0}^* = S_{w0}$ for $t = 0$. Assuming V = constant as known, t should be replaced by $t - x/V$ because (2) had been obtained with initial condition at $t = 0$. Thus the mass balance equation for water (8) becomes:

$$\frac{\partial S_w}{\partial t} + V \frac{\partial S_w}{\partial x} = -\frac{A}{\sqrt{t - \frac{x}{V}}} (S_w - S_{w0}) \tag{11}$$

The parameter A can be viewed as empirical constant, however from (2):

$$A = 3 \frac{(1-\phi)\phi^*}{\phi} \sqrt{\frac{D}{\pi R^2}} \tag{12}$$

The right hand site of Eq. (11) is an analog to the Newton cooling law known from theory of heat transfer, but in present case the transfer coefficient depends on time and position.

Eq. (11) should be solved with initial and boundary conditions:

$$S_w(x,t) = S_{w0} \quad \text{for} \quad x > 0, t = 0 \tag{13}$$

$$S_w(x,t) = S_{w1} \quad \text{for} \quad x = 0, t > 0 \tag{14}$$

4. Solution using the method of characterisctics

Equation (11) is a first-order and quasi-linear partial differential equation. The wide discussion of such type of equations may be found in mathematical literature e.g. (Aris & Amundson, 1973), but for coefficient on right hand site depended on single variable x or t. The solution of similar equation related to heat transfer in artesian layer was also presented by Kaminski, Kordas and Siemek (1976).

Equation (11) with conditions (13) and (14) can be solved using well known method of characteristics. Assuming constant velocity V, the characteristic family of Eq. (11) is described by the following system of equations:

$$\frac{dS_w}{dt} = -\frac{A}{\sqrt{t - \frac{x}{V}}}\left(S_w - S_{w0}\right) \tag{15}$$

and:

$$\frac{dx}{dt} = V \tag{16}$$

Dividing (15) by (16) yields:

$$\frac{dS_w}{dx} = -\frac{A}{V\sqrt{t - \frac{x}{V}}}\left(S_w - S_{w0}\right) \tag{17}$$

To solve the problem (11), (13), (14) one should observe that the characteristic curve in the space (x, t, S_w), passing through a point $(0, 0, S_{w1})$, divides the characteristic curves family into two groups, which should be considered separately. The first group refers to the initial condition (13) and presents a family of characteristic curves starting from points $(x_0, 0, S_w)$ for $S_w > 0$, where x_0 is a certain point on the x axis. In this case $x = x_0 + Vt$, where x_0 should be treated as a variable parameter, and $x > Vt$. As $\sqrt{t - x/V} = \sqrt{-x_0/V}$, the only possible solution is $S_w = S_{w0} = \text{const}$.

The second group of characteristics refers to the boundary condition (14) and consists of curves starting from points $(0, t_0, S_{w1})$, for a certain time $t_0 > 0$. In this case the t_0 formally should be treated as variable parameter. Now, $x < Vt$, and $x = V(t - t_0)$. Solving (17) with $t - x/V = t_0$ and using (14) yields exact solution for $x < Vt$. Finally, the solution is composed of two functions:

$$S_w(x,t) = S_{w0} \quad \text{if} \quad x \geq Vt \tag{18}$$

$$S_w(x,t) = S_{w0} + (S_{w1} - S_{w0})\exp\left(-\frac{Ax}{V\sqrt{t-\frac{x}{V}}}\right) \quad \text{if} \quad x < Vt \tag{19}$$

For $t > 0$, position of water front can by defined as $x_w = Vt$. It may be deduced from eq. (12) and eq. (19), that for $A > 0$ the saturation behind water front depends on properties of coal and water velocity. Mass transfer coefficient between fractures and coal matrix depends on diffusion coefficient and specific dimension of coal matrix blocks or grains. These parameters varies strongly depending on coal seam structure and consequently the coefficient A in eq. (19) may differ by orders of magnitude for different coals.

The obtained solution is illustrated by numerical example using the data presented in Table 1.

TABLE 1

Numerical data

Parameters	Value
Flow velocity, m/s, (m/day)	1.157E-5, (1)
Fractures porosity	0.08
Matrix porosity	0.3
Initial water saturation, S_{w0}	0.2
Maximal water saturation, S_{w1}	0.9
Case no.; D/R^2, (1/s)	1.; 5.0E-9 2.; 5.0E-7 3.; 5.0E-5

Results obtained by use of (18), (19) are presented graphically in figures 1 to 3. Fig. 1 represents the case of smallest A where the coal matrix reaches the full saturation and the water advances in the fractures with variable saturation. In Fig. 2, the capacity of coal matrix is higher and mass exchange is faster, but matrix still can reach its ultimate saturation with water. Consequently, the water front advances slowly in the fractures system. Fig. 3 refers to situation where the matrix capacity for water is large and rate of imbibition is high, and consequently the saturation in fractures varies very slowly. For the ultimate case $A = 0$, which represents hydrophobic coal with no imbibition effect, the solution (18), (19) represents a stepwise function moving with constant velocity V. In the last case the proposed model simplifies to so called Buckley-Leverett equation, well known from petroleum engineering literature.

4. Conclusions

1. Coal as a flowing system, can be viewed as a solid organic material consisting of two flow subsystems: a microporous matrix and a system of interconnected macropores and fractures. Most of gas is accumulated in the microporous matrix, where the primary flow mechanism is

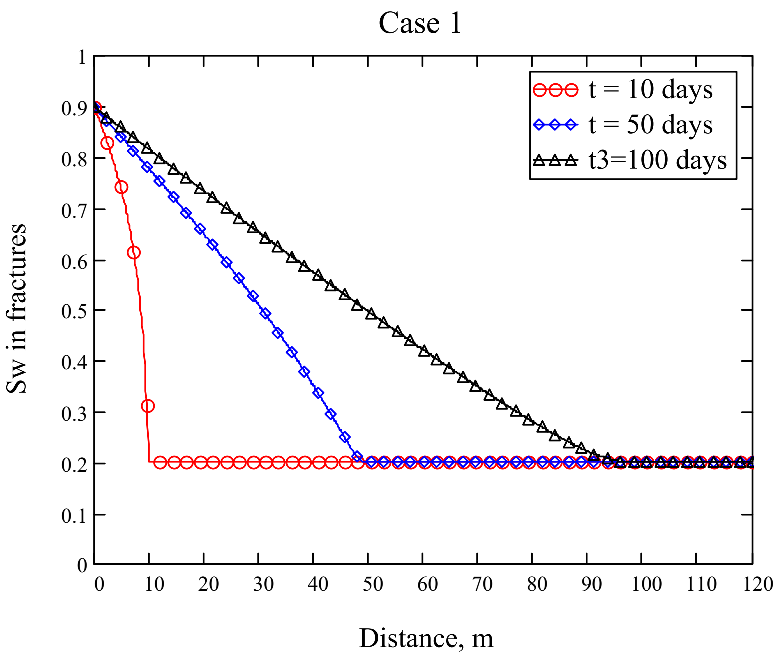

Fig. 1. Water saturation in fractures, Case 1, $A = 4.129\text{E}{-4}\ \text{s}^{-0.5}$

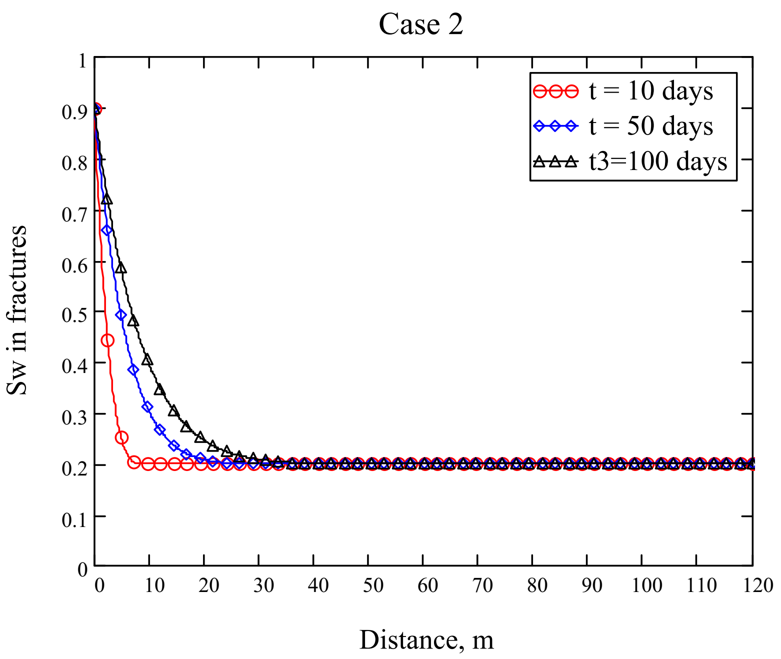

Fig. 2. Water saturation in fractures, Case 2, $A = 4.129\text{E}{-3}\ \text{s}^{-0.5}$

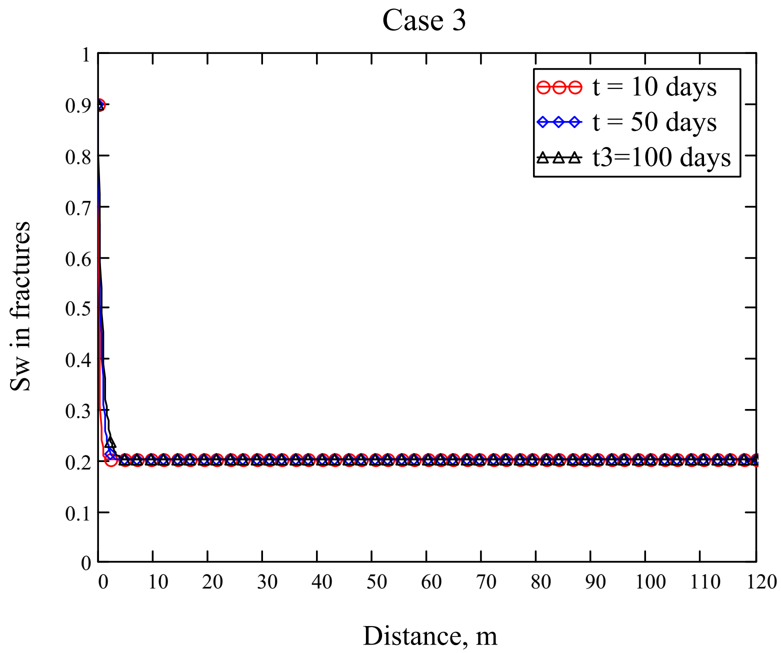

Fig. 3. Water saturation in fractures, Case 3, $A = 4.129\text{E}{-2}\ s^{-0.5}$

diffusion. Fractures and cleats existing in coal play an important role as a transportation system for macro scale flow of water and gas governed by Darcy's law.

2. The dual porosity nature of coal influences mass transfer in two ways. First, gas is desorbing from matrix to fractures if the pressure is lowering. Second, microporous coal matrix can imbibe water under capillary forces leading to exchange of mass between fractures and coal matrix. In this paper new mathematical model for water saturation in fractures, respecting mass exchange between coal matrix and fractures has been proposed. Exact analytical solution has been obtained that may be useful for engineering calculations.

3. The rate of exchange of mass between the fractures and the coal matrix is represented by right hand site of eq. (11) which is analogous to the Newton cooling law known from theory of heat exchange, but in present case the mass transfer coefficient depends not only on coal and fluid properties but also on time and position. The constant term of mass transfer coefficient represented by parameter A, depends on relation between micro porosity (ϕ^*) and macro porosity (ϕ), capillary forces represented by "diffusion" coefficient D and microporous structure of coal matrix represented by typical size of grain R and matrix porosity ϕ^*. This term implies that for water wet coal, a larger matrix porosity ϕ^* provides more access of the water to flow from the open fracture into the coal matrix.

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