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THE APPLICATION OF CORELESS INDUCTORS FOR DISPLACEMENT MEASUREMENTS IN LABORATORY INVESTIGATIONS OF ROCK PROPERTIES

WYKORZYSTANIE BEZRDZENIOWYCH INDUKCYJNYCH CZUJNIKÓW PRZEMIESZCZEŃ W LABORATORYJNYCH BADANIACH WŁAŚCIWOŚCI SKAŁ

The paper presented the coreless inductive sensor, its construction and principle of operation. The impact of temperature on the outcome of a measurement performed with the inductor was discusses, together with the possibility of temperature compensation of the inductor's performance. Subsequently, the reasons for limited measurement accuracy and resolution were discussed, particularly under the variable pressure in the order of some hundreds MPa. Two types of such sensor were presented: a sensor for measuring linear strains, e.g. during compressibility measurements, and an sensor for measuring circumferential strains during triaxial compression tests. Additionally, the manners of fixing the sensor on rock samples were presented. Finally, some examples of the sensor application were shown, together with the results of measurements of deformations of rock samples – especially in cases when resistance gauges cannot be used, and the samples are subjected to a load in the uniaxial and triaxial system, under the hydrostatic pressure of up to 400 MPa and the normal one.

Keywords: inductive strain sensor, strain measurement in high pressure condition, compression test

W Pracowni Odkształceń Skał Instytutu Mechaniki Górotworu prowadzone są badania właściwości mechanicznych skał. Wymaga to precyzyjnego pomiaru odkształcenia, na ogół pod wysokim ciśnieniem hydrostatycznym, które symuluje warunki panujące w głębi górotworu. Ciśnienie hydrostatyczne (do 400MPa w aparacie GTA-10) i ograniczona do kilku milimetrów przestrzeń w komorze ciśnieniowej na zainstalowanie odpowiedniego przyrządu, a także spękania i kawerny w skałach powodują znaczne trudności pomiaru odkształcenia z wymaganą rozdzielczością (nawet 10^{-6}). Stosowanie tensometrów elektrooporowych naklejanych wprost na próbkę często jest zawodne, gdyż ciśnienie wgniata ścieżkę rezystancyjną w nierówności próbki, powodując jej przerwanie, a co gorsze, fałszuje wyniki pomiaru. Wypełnianie szczelin lub kawern różnymi podkładami jak klej epoksydowy, gips, jest problematyczne. W przypadku skał przewodzących (nasączonych solanką) istnieje ryzyko zwarcia ścieżki rezystancyjnej do podłoża. Często naklejenie tensometru jest niemożliwe w przypadku skał słabo zwięzłych (fliszowe). Inne metody pomiaru np. transformator różnicowy z ruchomym rdzeniem (LVDT) ma ograniczoną odporność

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na wysokie ciśnienie i temperaturę i zbyt duże rozmiary. Czujnik LDT (*Local Deformation Transducer*), czyli naklejony tensometr rezystancyjny na sprężystą taśmę stalową, ma ograniczony zakres pomiaru deformacji do kilku procent i małą czułość.

Opracowano nową metodę pomiaru odkształcenia opartą na jednowarstwowej, bezrdzeniowej cewce indukcyjnej, wykonanej z cienkiego sprężystego drutu (0,2 mm) i średnicy zwojów kilku milimetrów. Tak wykonany czujnik jest instalowany do zaczepów zamontowanych na badanej próbce (rys. 1 i 2). Odkształcenie próbki powoduje zmianę długości cewki (czujnika), a zatem jej indukcyjności. Czujnik stanowi indukcyjną część generatora *LC*, umieszczonego na zewnątrz komory. Zmiana indukcyjności skutkuje zmianą częstotliwości drgań, którą łatwo zmierzyć z dużą precyzją. Prostota czujnika gwarantuje jego dużą odporność na ciśnienie hydrostatyczne, temperaturę i udary mechaniczne.

Minimalizacja błędów spowodowanych zmiennym ciśnieniem i temperaturą realizowana jest dwoma sposobami. Po pierwsze, czujnik wykonano z wysokorezystywnego drutu, co skutkuje dużymi termicznymi zmianami jego rezystancji, które zmieniają częstotliwość drgań (poprawka częstotliwości w generatorze Colpitts'a (4) przeciwstawnie do wpływu temperatury na indukcyjność czujnika (rozszerzalność termiczna). Umożliwia to prawie całkowitą kompensację termiczną czujnika w kilkunastostopniowym zakresie (rys. 4). Drugim sposobem jest użycie czujnika referencyjnego wykonanego w identyczny sposób jak czujnik pomiarowy, który jest zamocowany na wsporniku o znanej ściśliwości i rozszerzalności termicznej (rys. 7). Zmiany częstotliwości z czujnika referencyjnego są poprawkami do wskazań czujnika pomiarowego. Oba czujniki są naprzemiennie podłączane do tego samego generatora poprzez elektroniczny przełącznik (rys. 5). Zastosowanie jednego generatora powoduje, że poprawki te umożliwiają również praktycznie całkowitą eliminację błędu pomiaru ze względu na zmiany temperatury otoczenia i napięcia zasilania na generator i częstościomierz.

Charakterystyka przetwornika *długość-częstotliwość* jest nieliniowa (rys. 3), co wynika z zależności między długością cewki czujnika, więc jej indukcyjnością, a częstotliwością rezonansową obwodu *LC* (1). Najdokładniej charakterystykę czujnika otrzymać można przez wzorcowanie. Uwzględnione są wtedy głównie pasożytnicze indukcyjności i pojemności połączeń, których wartości trudno obliczyć lub zmierzyć. W pomiarach należy dążyć, na ile to możliwe, do montowania krótkiego czujnika do długich próbek, w ten sposób zmiany długości badanego materiału będą większe, a krótszy czujnik dozna większego odkształcenia, więc czułość pomiaru będzie duża. Jednak zbyt krótki czujnik ma małą indukcyjność i wtedy jego czułość ograniczy indukcyjność połączeń (2).

Opracowano dwa podstawowe typy takiego czujnika. Pierwszy, do pomiaru odkształceń liniowych, np. do pomiaru ściśliwości (rys. 2 i 6), o prostej cewce, który jest mocowany do próbki za pośrednictwem zaczepów przytwierdzonych do niej. W ten sposób czujnik nie kontaktuje się bezpośrednio z powierzchnią próbki, i odkształca się bez tarcia, co umożliwia precyzyjny pomiar, szczególnie przy obciążaniu cyklicznym. Bazę pomiarową można dostosowywać do długości próbki, mocując czujnik do zaczepów poprzez łączniki, uzyskując globalny pomiar odkształceń. Czujnik mierzy zmiany długości z rozdzielczością poniżej 1 μm, przy maksymalnych odkształceniach czujnika o kilkadziesiąt procent. Przykładowe pomiary przedstawiają rysunki 8 i 9. Na rys. 10 pokazano wyniki testu pomiaru ściśliwości stali, przy użyciu czujnika referencyjnego. W trzech cyklach obciążania, podczas których zmiany temperatury wywołane sprężaniem i rozprężaniem cieczy (do 350 MPa) sięgały kilkunastu °C. Histereza i rozrzut pomiaru w kolejnych cyklach wynosiły najwyżej kilka mikrometrów przy rozdzielczości około 0.2 μm. Czujnik stosowany jest również w pomiarach poza komorą ciśnieniową. Np. fotografia (rys. 11) przedstawia czujnik przy pomiarze ugięcia próbki drewna pobranego w kopalni soli Wieliczka. Fotografia na rys. 13 przedstawia stanowisko do pomiaru deformacji osiowych i obwodowych brykietu węglowego podczas testu jednoosiowego ściskania.

Drugi typ czujnika, do pomiaru dużych odkształceń obwodowych (kilkadziesiąt procent) w teście konwencjonalnego trójosiowego ściskania, w którym próbka jest jednocześnie ściskana ciśnieniem hydrostatycznym (okólnym) a następnie obciążana osiowo tłokiem prasy poruszającym się wewnątrz komory ciśnieniowej. W ciśnieniu hydrostatycznym setek MPa na ogół skały zachowują się plastycznie i w teście tym siła działająca osiowo na cylindryczną próbkę powoduje odkształcenie jej nawet o kilkadziesiąt procent, do postaci beczki. Pomiar odkształceń obwodowych jest realizowany czujnikiem indukcyjnym uformowanym na kształt torusa, przez spięcie jego końców izolacyjną płytką (rys. 1). Czujnik na próbce utrzymywany jest dzięki sile sprężystości jego zwojów. Na rys. 14. pokazano efekty trójosiowego testu: odkształcenie osiowe ε_1 (pomiar ruchu tłoka prasy, na zewnątrz komory) i poprzeczne ε_3 (czujnikiem toroidalnym) oraz zmianę objętości ΔV, walcowej próbki dolomitu. Jeśli nie są mierzone deformacje poprzeczne, to aktualny przekrój próbki wyliczany jest na podstawie odkształcenia osiowego, przy założeniu stałości objętości próbki (v = const. = 0,5). Uproszczenie to daje w miarę zadawalające wartości

naprężenia do granicy wytrzymałości materiału, a po jej przekroczeniu zawyża naprężenia (cienka przerywana linia).

Podsumowując, można stwierdzić, że przedstawione czujniki odkształceń współpracujące z generatorem *LC* rozwiązały problem pomiaru odkształceń skał porowatych, słabo zwięzłych lub przewodzących, szczególnie w badaniach ciśnieniowych. Mają wysoką czułość oraz bardzo szeroki zakres pomiaru, od mikronów do centymetrów. Prostota i mały koszt wykonania, odporność na udary mechaniczne i łatwość mocowania do badanego obiektu czyni je atrakcyjnym narzędziem pomiarowym. Zbędny jest przetwornik analog/cyfra. Możliwość kompensacji termicznej czujnika i zastosowanie czujnika referencyjnego umożliwia pomiar w zmiennym ciśnieniu (GPa) i temperaturze (kilkaset stopni) oraz pozwoliło praktycznie wyeliminować wpływ zmian temperatury otoczenia i napięcia zasilania na generator i częstościomierz, umożliwiając długotrwałe, nawet wielodniowe pomiary. Osiągana rozdzielczość pomiaru jest poniżej 1 μm, przy dokładności około 1%. Maksymalne ciśnienie hydrostatyczne, przy którym wykonano pomiary odkształcenia omawianym czujnikiem wynosiło 1,4 GPa w aparacie GCA-30. Trudno określić maksymalną wartość ciśnienia uniemożliwiającą pomiar takim czujnikiem. Na pewno, przy zastosowaniu czujnika referencyjnego, są to setki a nawet tysiące GPa.

Słowa kluczowe: indukcyjny czujnik przemieszczeń, wysokie ciśnienia, test trójosiowego ściskania, ściśliwość

1. Introduction

In the Laboratory of Rock Deformation of the Strata Mechanics Research Institute, mechanical properties of rocks are investigated. Most often, the investigations involve determining material constants, such as Young's modulus, Poisson's ratio, compressibility, and compressive strength. This calls for a precise strain measurement, usually under high hydrostatic pressure, which is supposed to simulate the conditions in the depths of the rock mass. The measurements are usually performed on cylindrical rock samples, in the pressure cell of the GTA-10 device (Długosz et al., 1981). The pressure medium is kerosene or silicone oil, and the samples are protected against the contact with medium by means of latex or silicone jackets. The high value of the hydrostatic pressure (up to 400 MPa), the limited space for the installation of an adequate measuring device in the pressure cell (several millimiters), and the specific structure of investigated rocks (cracks, caverns), result in considerable difficulties when it comes to measuring the strain with the required resolution (in some cases, even 10^{-6}). Resistance gauges fixed directly on samples are usually fallible, as the pressure thrusts the resistive track into the roughness of the sample surface, which results in the breaking up of the track, or – which is even worse – yields false measurement results. Filling the cracks and caverns under the gauge with various substances (glues, gypsum, cement, etc.) is not always effective; moreover, it locally changes some properties of the investigated rock. In the case of electric conductive rock (e.g. those through which salt is carried), or rocks saturated with brine, there is a risk of electrical short-circuits of the gauge. Often, glueing the gauge to the rock sample is simply impossible – this happens when we deal with rocks of low firmness (flysch rocks). These problems have already been discussed in the works by Hakami, Alm, Stephansson (1987); Linton, McVay, Bloomquist (1988).

The aforementioned problems led to the construction of a strain sensor in the form of a one-ply, coreless inductor made of a thin wire (0.2 mm of diameter), with a coil diameter of several millimeters. Such an inductor (sensor) is attached to the catches fixed on the investigated sample and placed – together with the sample – inside the pressure cell (Figs 1, 2, and 7). There is no direct contact between the inductor and the sample surface, which eliminates the problems discussed above. The type of catches and the manner in which they are fixed on the sample is selected depending on given limitations, such as the firmness of the sample, the type of the pore fluid, etc. (Figs 2 and 6). The strain of the sample results in the change in the length of the inductor, and thereby in its inductance. The sensor constitutes the inductive part of the *LC* oscillator, which is placed outside the cell. The change in inductance results in the change in oscillation frequency. The latter can be easily measured with high accuracy, and registered. The simple construction of the sensor guarantees its high resistance to the hydrostatic pressure, temperature, and mechanical shocks.

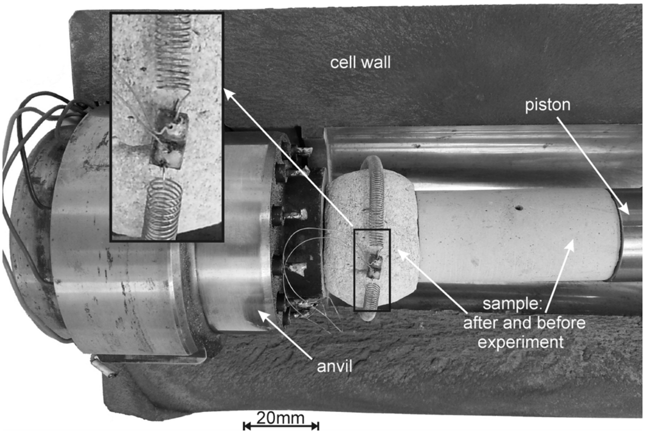

Fig. 1. A toroidal sensor on a limestone sample, placed in the intersection of a pressure cell; the sample presented before and after a triaxial compression test (a photomontage)

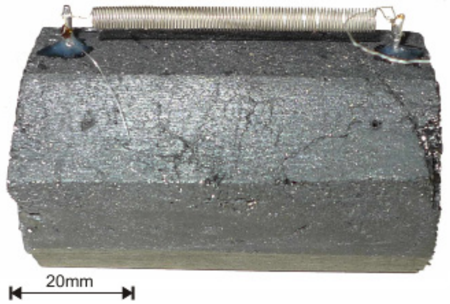

Fig. 2. An inductor fixed on a coal sample

Two basic types of the sensor in question were developed. The first one is supposed to measure large circumferential strains (several dozen percent) in the triaxial compression test, where a sample is simultaneously exposed to the hydrostatic pressure and subjected to the load in the form of the axial force exerted by the piston of the press, which moves inside the pressure cell (Nowakowski & Nurkowski, 1995). The other type of the sensor is supposed to measure linear strains, e.g. during compressibility tests, when a sample is exposed only to the hydrostatic pressure – or to perform measurements outside the pressure cell, e.g. in the INSTRON 6500 stiff, servo-controlled press Rock Testing System (Nurkowski, 2003).

The main factors responsible for errors in strain measurements, especially in a pressure cell, are: the impact on the resonant frequency of changes in the values of electrical capacity of pressure seals (connecting the sensor with the oscillator) as effect a change in pressure, and the impact of temperature on the sensor inductance. The influence of these two sources of errors can be minimized in two ways. Firstly, the reduction of the impact of temperature and pressure upon the measurement result was achieved by means of a reference sensor, constructed in the same way as the measuring sensor. The reference sensor is installed on a support of known mechanical properties (compressibility and thermal expansion). On the basis of frequency changes from the circuit of the reference sensor, the corrections of the readings of the measuring sensor are calculated. Secondly, the measuring sensor itself was subjected to temperature compensation by the proper selection of the material used to construct the sensor and the selection of parameters of the resonant circuit.

Such a method makes it possible to conduct only several measurements per second (at the maximum). This limitation of the measurement dynamics is due to the necessity to perform the frequency measurement with high resolution (10^{-6}), from the circuits of both sensors, which requires a considerable amount of time, needed to count the impulses. One might consider a modified method of strain measurements, performed with an inductor which is part of the oscillator's circuit. The method eliminates the usage of a frequency meter, which reduces the speed of the measurement. Instead of a frequency meter, one would have to use a frequency-voltage converter, just like the one used to detect the radio signal, with frequency modulation (FM) – and then a fast analog-digital converter. The estimations show that, in such a situation, it would be possible to conduct several hundred measurements per second. The constraint of such a method is the tempo of the propagation of disturbances along the coil (i.e. the impulse that changes the coil length), which is limited by the frequency of the coil's natural axial oscillations.

When compared with other methods, the presented sensor constitutes a competitive solution as far as measuring rock deformations is concerned, especially when the measurements are performed under high pressure. Apart from controversial resistance gauges, an option worth considering is the application of a sensor whose performance is based on the principle of a differential transformer with a movable core – the so-called Linear Variable Differential Transformer (LVDT). However, LVDT is characterized by limited resistance to high pressure and temperature; it is also relatively large in size. Some remarks concerning its application in a high pressure environment can be found in Scholey et al. (1995).

It seems that the only solution alternative to the inductor described in the present paper, for pressure measurements of rock deformations, could be the LDT (*Local Deformation Transducer*), whose performance is based on the principle of an elastically deformable steel tape, with resistance gauge glued to it. However, this device is characterized by a limited range of deformation measurements (up to several percent) and relatively low sensitivity, as the deformation of the gauge – in relation to the sample deformations – is significantly reduced on the tape (Hoque et al., 1997).

Miniaturization of the measuring sensor is necessary as regards the GTA-10 device, as its cell pressure has a diameter of 50 mm, only, which enforces the use of samples whose diameter is up to 30-40 mm, so that the coreless inductive sensor could also be installed. Using the LDT or LVDT sensors would limit the size of samples even more. Theoretically, a bigger cell could be built, but there will always be a need to compromise one of the following factors: its dimensions, price, weight, and strength (the maximum achievable pressure). In addition, there is the usual tendency to investigate possibly largest rock samples, which is due to their heterogeneity (the scale issue). This necessitates the maximum usage of the cell volume, which should be occupied by measuring instruments to the smallest extent possible.

The coreless inductive sensor, discussed above, have been successfully used for many years now. Some other effects of their application, not presented in this paper, can be found in the works by: Nowakowski, Młynarczuk (2012), who discussed the impact of the thermal shock on the properties of granites (compressibility), and Nowakowski et al. (2003), where the impact of temperature on the properties of sandstone was examined.

The paper briefly discusses the principle of operation of the inductive sensor together with the resonant circuit, as well as its temperature compensation. The idea of a measurement performed with the aid of a reference sensor was also shown. Additionally, the metrological properties of a toroidal sensor and a linear sensor were presented, with some examples concerning their usage and the effects of measurements.

2. The principle of operation of the inductor for deformation measurements, the factors interfering with the measurement and the ways of reducing them

The sensor – which has the form of a one-ply, coreless induction coil – is fixed by means of catcher to the rock sample. Therefore, it can be subjected to deformation together with the sample. As a result, the length of the sensor, and thereby its inductance, change (Fig. 2). If the sensor is connected to the *LC* resonant circuit of the electrical oscillator, becoming its induction component, we will obtain a length-to-frequency converter. The inductance of the sensor is low (several mH), as there is no magnetic core, and the number of turns is limited to several dozens. Such a solution has numerous advantages. It is very easy to build such a sensor. It is made of a thin (0.2 mm) steel spring wire, which is why it is highly resistant to mechanical shocks and stretching (even between 50 and 80 percent). The oscillator with which the sensor cooperates is quite simple and can be easily miniaturized. It is also very easy to measure and register frequency, for example with computer system. An analog-digital converter is unnecessary. Using the oscillator makes it possible to detect sensor inductance changes in the order of 10^{-12} H (1 pH), which gives us the resolution of the measurement of the changes in the sensor length (and the sample length as well) of ca. 0.1 μm. It is possible to measure changes in inductance directly, instead of measuring frequency (this would mean eliminating the oscillator) – however, the resolution of instruments used for measuring such low inductance values is insufficient. To the best of the author's knowledge, the instrument with the highest resolution of the inductance measurements – 10^{-10} H (100 pH) – is offered by a company called *GW Instek* (Precision LCR Meter 8101G). This is 100 times worse than in the case of the frequency method.

The characteristics of the length-to-frequency converter are non-linear (Fig. 3), and they result from the relationship between the length of the coil (sensor), i.e. the coil's inductance, and the resonant frequency of the *LC* circuit. Excluding the loss resistance, this can be depicted in the following way:

$$\begin{aligned} f &= \frac{1}{2\pi\sqrt{C_R L_R}} = \frac{1}{2\pi\sqrt{C_R}}\frac{1}{\sqrt{L_s + L_p}} = \frac{1}{2\pi\sqrt{C_R}}\frac{1}{\sqrt{\dfrac{\mu z^2 S}{l_s} + L_p}} \\ \text{for} \quad L_p &= 0 \quad f = \frac{\sqrt{l_s}}{2\pi\sqrt{C_R \mu z^2 S}} = \frac{\sqrt{l_s}}{\pi z D\sqrt{\pi\mu C_R}} = k\sqrt{l_s} \end{aligned} \tag{1}$$

where:

f — frequency oscillaction.
L_R, C_R — total inductance and capacitance of the resonant circuit,
L_s — inductance of the sensor,
L_p — parasitic inductance of the connection: sensor – resonant circuit,
l_s — ength of the sensor,
μ — magnetic permeability,
z — number of turns of the sensor,
S — cross-sectional area of the coil (sensor),
D — diameter of the coil (sensor).

The most accurate sensor characteristics can be obtained in the course of calibration. Then, all the factors influencing the frequency of oscillations are taken into account – mostly, the parasitic inductance and capacitance existing between connections, whose values are difficult to calculate or measure with sufficient accuracy. The effects of calibrating a sensor deformed from the initial length of 12 mm to the length of 15 mm (25%) is presented in Figure 3. The approximation of its characteristics using a second degree function gives us the coefficient of

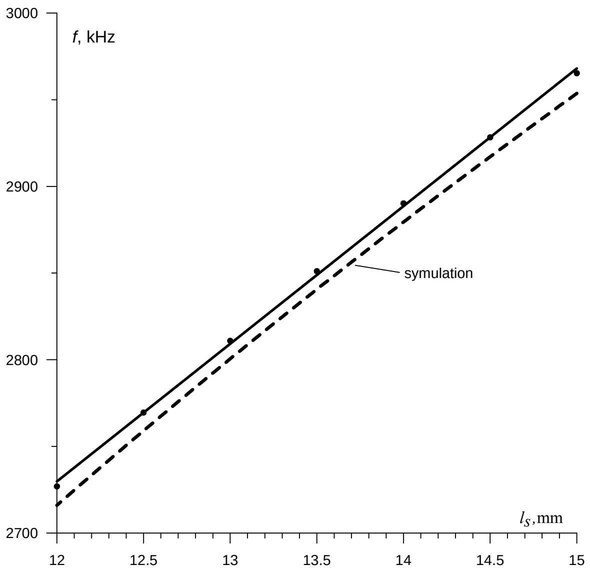

Fig. 3. Characteristics of the sensor, linear approximation

determination $R^2 = 0.999999$ (for a linear approximation $R^2 = 0.999412$). The changes in length were measured with a micrometer screw gauge integrated with a stiff frame. One end of the sensor is fixed to the frame, and the other – to the movable, rotational spindle of the micrometer by means of a follower that moves in the hole in the frame. Additional springs hooked to the follower cause the sensor to move reciprocally, eliminate clearances, and prevent the transmission of the rotation of the screw on the sensor (Nurkowski, 2013).

During the calibration and measurement processes, the sensor should be installed in such a way as to make its axis parallel to the direction of the deformation. Experience shows that an angular error accompanying fixing the sensor cannot be greater than 2° (0,0349066 rad). Calculations demonstrate that the relative value of measurement uncertainty resulting from that error should be ca. $\delta = 10^{-4}$. However, the main limitation to the measurement accuracy is the calibration error caused by different values of inductance of connections between the sensor and the oscillator during the calibration and measurement processes ($\delta = 10^{-3}$).

For deformations below 1%, the characteristics can be described – with sufficient accuracy – as linear, with the directional index equal to the derivative of the function approximating the characteristics, for a given sensor length. The derivative describes the sensor sensitivity s as the quotient of frequency changes and sensor length changes. With relative frequency changes taken into account, the relative sensitivity s_R obtained from the equation (1) will be described by means of the following formula:

$$s_R = \frac{df}{dl_s} \cdot \frac{1}{f} = \frac{1}{2l_s\left(1 + \dfrac{L_p}{L_s}\right)} \quad \text{for} \quad L_p = 0 \quad s_R = \frac{1}{2l_s} \tag{2}$$

As demonstrated by the equation above, the short sensor (small l_s) has high sensitivity. This results from the fact that, for the same change in the sample length (measuring base), a shorter sensor will undergo a bigger deformation. Therefore, during the measurement process, one should aim – as much as it is possible – to install a short sensor for long samples. However, too short a sensor has small inductance, which is why its sensitivity is reduced by the inductance of connections. Due to that fact, the inductance of the sensor should be over ten times greater than the inductance of connections (L_p). The sensor is usually installed via ceramic insulators in the form of resistors characterized by a high resistance (over 3 MΩ).

The main factors responsible for the error of the strain measurement in the pressure cell are: the impact of changes in the values of the capacities of electrical pressure seals (through which the sensor is connected to the oscillator) on the resonant frequency (when pressure changes occur), and the impact of temperature changes on the sensor inductance. During the process of measuring compressibility, the pressure of the liquid in the cell of the GTA-10 device changes (in a cyclical fashion) from 0 to 400 MPa, at the maximum, which causes changes in the temperature of the liquid, to a value of between ten and twenty degrees. Due to the thermal expansion, an increase in temperature results in an increase in the length of the coil wire, out of which the sensor was made. Consequently, its inductance increases as well (cf. (1)). Finally, this leads to a decrease in the oscillation frequency, according to the formula:

$$\frac{\Delta f_T(\alpha_T)}{f} = \sqrt{\frac{L\,C}{L_T C}} = \frac{D}{D_T} - 1 = -\frac{\alpha_T \Delta T}{1 + \alpha_T \Delta T} \approx -\alpha_T \Delta T \tag{3}$$

where:

$f_T(\alpha_T)$ — the frequency of oscillations due to the thermal expansion α_T of the coil wire,
D_T — the coil diameter D_T under temperature T.

An increase in temperature results also in an increase in the resistance of the coil, which, in the Colpitts oscillator with divided capacitance, causes an increase in frequency (formula 4), which is an effect opposite to the impact of thermal expansion (Pawłowski, 1980):

$$\frac{\Delta f_T(r)}{f} = \sqrt{\frac{C_2}{C_1 + C_2} G\Delta r} \approx \frac{1}{2}\frac{C_2}{C_1 + C_2} G\Delta r \approx \frac{1}{2}\frac{C_2}{C_1 + C_2} G\alpha_r \Delta T \tag{4}$$

where:

C_1, C_2 — the capacity of the capacitors in the resonant circuit (Fig. 5),
r — coil resistance for alternating current,
G — conductivity of the output oscillator,
α_r — the thermal coefficient of the coil resistance.

In this way, it is possible to thermally compensate a sensor, partly or even totally, in a certain range of temperature changes. The last formula is approximate in its nature, and it is not possible to use it in order to determine the temperature-frequency characteristics of the sensor The last formula is approximate and not much thermal performance of the sensor (temperature-frequency) can not be set on the basis of, nor can optimize the parameters of the resonant circuit to compensate for the thermal sensor. The actual thermal characteristics of the sensor are parabolic. Figure 4 presents such a set of features, up to the temperature of 200°C. It is the result of currently performed tests investigating the possibility of measuring strains with an sensor under such high temperature values. A proper selection of the parameters of the resonant circuit makes

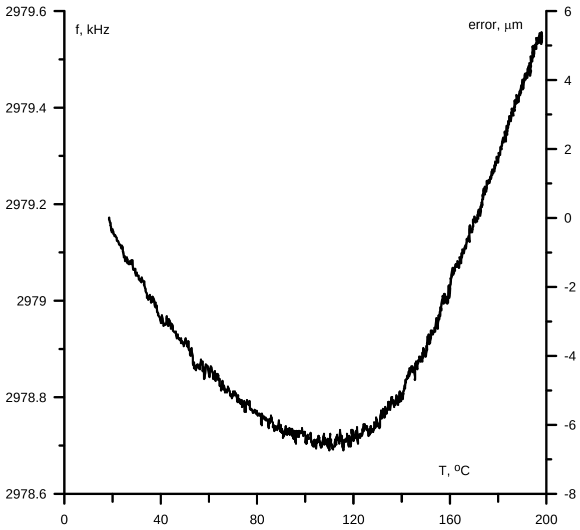

Fig. 4. Thermal characteristics of the sensor (changes of frequency caused by temperature and the corresponding measurement error in µm)

it possible to reach the thermal stabilization of the sensor for any range of temperature values – in this case, halfway through their range of changes.

Another way of reducing the impact of temperature and pressure changes upon the measurement result is using a reference sensor, constructed in the same way as the measuring sensor. The reference sensor is placed on a support of known mechanical properties (compressibility and thermal expansion). Changes in pressure and temperature have an almost identical effect on both sensors (i.e. the measuring and the reference one), as well as on electrical pressure seals in their circuits. On the basis of the changes of frequency from the reference sensor, corrections of the readings of the measuring sensor are calculated (Nurkowski, 2008). Both sensors are periodically (alternatively) connected to the same oscillator by means of an electronic switch (Fig. 5). Therefore, the corrections enable almost complete elimination of the measurement error due to the instability caused by the changes in the temperature of the environment and the voltage of the power supply of the frequency meter and the oscillator. This makes it possible to perform long-term measurements, for many hours or even days. If the reference material has a similar compressibility to the investigated material, the measurement errors connected with the sensor calibration can be reduced to a large extent, which is due to the very idea of comparative measurements.

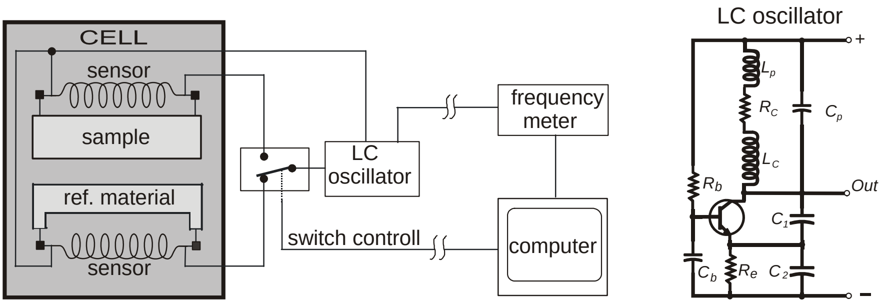

Fig. 5. The idea of strain measurement using the reference sensor

Changes in the parameters of the measurement environment, such as temperature, pressure, or dielectric permittivity, will result in a measurement error whose value is going to be proportional to the asymmetry of the parameters of the *RLC* circuit in the measuring and the reference track. Therefore, it is necessary to ensure the greatest electrical similarity possible for both tracks particularly as far as the sensor inductances are concerned. It is also essential that the capacities of the connections should be small when compared to the constant capacities of the resonant circuit and the inductances of the connections should be small in relation to the inductances of the sensors.

The maximum hydrostatic pressure value for which the strain measurement with the discussed sensor was performed was 1.4 GPa, in the GCA-30 apparatus. With such high pressure values, there are numerous factors that change the *R*, *L*, and *C* values (and thereby the frequency of the oscillation) in a significant way. This concerns mostly the capacities of the pressure seals, sensor resistance, and the dielectric permittivity of the pressure liquid. It is only because of the application of the reference sensor that a measurement of the required accuracy becomes possible with such considerable pressure changes. However, this should not be treated as an inconvenience

– for example, using a resistance gauge fixed on a sample necessitates using a second strain gauge fixed on a reference material (a steel plate) and combined into the branch of the bridge circuit. In this way, the impact of pressure and temperature changes on the gauge resistance is compensated, and – in the case of the compressibility measurement – the obtained result is a relative one, and refers to steel compressibility. It is hard to determine the maximum value of the hydrostatic pressure that would make a measurement with such a sensor impossible. Certainly, in a situation when a reference sensor is used, these could be several hundred, or even several thousand GPa.

Two types of sensors were developed – one for measuring linear strains, with a linear coil; the other for measuring circumferential strains of a cylindrical rock sample, torus-shaped.

It can be concluded that the accuracy of a measurement performed with a linear sensor, inside a pressure cell, is limited to ca. 1 percent, mainly because of an error connected with calibrating and fixing the sensor, as well as because of imperfect compensation of the impact exerted by pressure and temperature changes, caused by the asymmetry of the parameters of the measuring and reference track. Under normal conditions, this accuracy is higher, and is ca. 0.5 percent. The resolution of the measurement is a fraction of a micrometer, and is limited by the short-time instability of the generation, both in the circuit of the measuring sensor and the circuit of the reference sensor.

3. The sensor for measuring linear strains

The sensor for measuring linear strains, with a linear coil, is fixed to a rock sample by means of strikers attached to it. This ensures that the sensor is not in direct contact with the surface of the sample, and the changes in the sample length cause the length of the sensor to change, too, but without friction. Due to that, a precise measurement is possible, especially with a cyclic load. The sensor measures the length changes with high resolution, below 1 μm, and a possible deformation of the sensor is several dozen percent. The sensor is usually shorter than the sample, so – in order to obtain the longest measurement base possible, which reduces measurement errors – it is fixed to the sample via a special connecting bar in the form of a thin (ca. 0.8 mm in diameter) metal rod of proper length. Contrary to a local measurement performed with a resistance gauge, on a small base of dozen or so millimiters, in the spot where it was fixed, the inductor makes it possible to measure both global and local strains (minimum measuring base of the inductive sensor is approximately 10 mm). This is significant in the case of heterogeneous rocks. Between the sensor and the connector is soldered the ceramic insulator in the form of a high resistance resistor (over 3MΩ). The resistor represents the SMD (Surface Mounted Devices) type, and its dimensions are 3.2 mm × 1.6 mm (Fig. 6). Due to that fact, it is possible to electrically isolate the sensor from the sample, if the sample comes from an electrically conductive rocks (salt, certain types of coal, or samples saturated with brine or water). Catches can be fixed on samples in various ways, depending on the needs resulting from the rock properties (firmness, porosity, the degree of saturation with a liquid) and on the measurement environment (normal conditions, pressure cell). The photograph in Figure 6 shows various ways of fixing the sensor – from left to right: 1. to the rings encircling the sample, which are tightened by additional tension springs; 2. to the screws driven into the fixed anvils (a fragment of the latex cover is visible); 3. to the anvil equipped with a with a screwed pin (at the bottom) and to the rod soldered to the anvil (at the top); 4. to supports fixed to the sample bases.

Fig. 6. Various ways of fixing the sensor to rock samples; in the corner, the magnified isolator is visible

Usually, during measurements performed inside a pressure cell, additional metal anvils, whose diameter is the same as the diameter of the sample and whose height is ca. 10 mm, are fixed to the bases of the cylindrical sample. The side surface of such a system is then shielded with latex covers, which protect the sample from a contact with a pressure medium (such as kerosene or silicone). To the anvils, strikers of the sensor – in the form of driveable screws or clamping rings – are fixed. In the case of samples characterized by large cracks, their surface is first made even with some glue or gypsum, so that the liquid under pressure would not thrust the covers into a crack or a cavern, causing them to break.

In the case of rocks characterized by low firmnes and high porosity, such as coal (Nowakowski et al., 2009), the catches have a form of steel rods, several millimiters in length and ca. 1 millimiter in diameter. The rods are glued into the holes drilled on the ends of the sample. The whole set is protected from a contact with pressure liquid with a thick layer of a silicone paste. The sensor is soldered on the rod ends (cf. Fig. 2). Such a sample, in the air-dry state and ready for a compressibility measurement, is shown in Figure 7. The sample has a polygonal shape; it was made from a fragment of a coal block, out of which it was cut with a saw blade (due to low mechanical strength of coal, coring by means of a bit resulted in the disintegration of the sample).

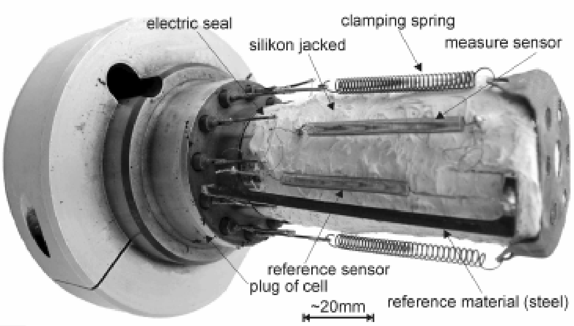

Fig. 7. A coal sample (cf. Fig. 2) in the air-dry state, prepared for a compressibility measurement. The sample covered with a silicone paste isolates it from the pressure liquid

Figures 8 and 9 present sample results of measurements of coal and salt compressibility. These are linear compressibility ε, measured along the direction determined by the sensor axis. In the case of isotropic materials, the volumetric compressibility can be calculated using three times the value of the linear compressibility, with quite good approximation.

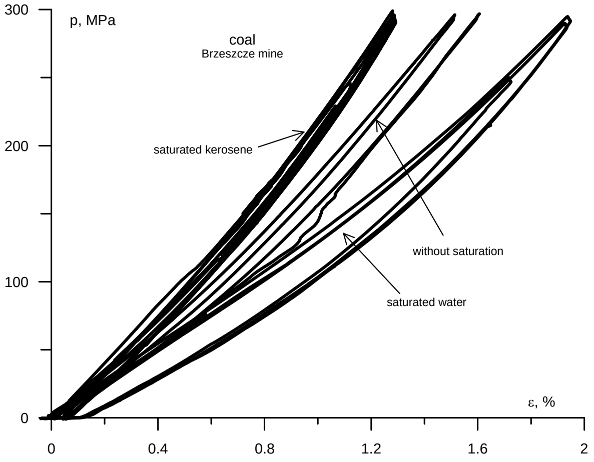

Fig. 8. The compressibility of coal (mine Brzeszcze), an inductive sensor

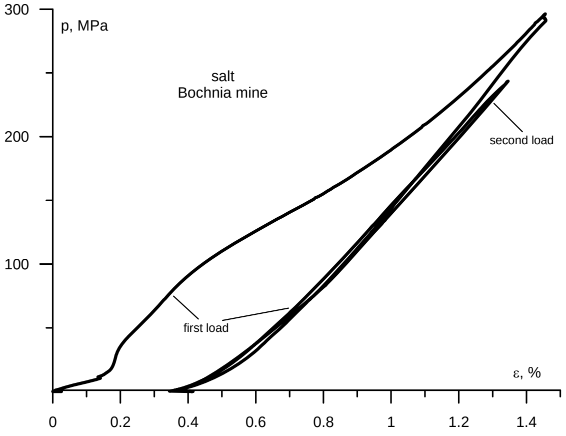

Fig. 9. The compressibility of salt (mine Bochnia)

The measurements of coal compressibility were performed for samples saturated with kerosene, water, and for a sample in the air-dry state; for several load cycles. As far as the obtained results are concerned, one can conclude that – according to predictions – coal saturated with kerosene is less compressible due to the fact that its cracks were filled in, providing a support for its skeleton. An interesting effect occurred after saturating coal with water, as compressibility and hysteresis were greater than in the case of the dry sample and the sample saturated with kerosene (Nowakowski et al., 2012). It is hard to explain this fact; some complicated sorption phenomena have to be taken into account – different for water and for kerosene, which is due to their polarity (coal hydrophobicity) and non-polarity. In the case of salt compressibility, it is worth noticing that, in the first load cycle, the cracks close, which is the reason for a high degree of hysteresis. In the second cycle, hysteresis is virtually unnoticeable.

Figure 10 presents the measurement possibilities of the sensor placed inside the pressure cell. It shows the results of the test involving the measurement of steel compressibility using a reference sensor. In the three load cycles – during which the compression and expansion of the liquid caused the temperature to change by over ten degrees Celsius – hysteresis and strain measurement dispersion in consecutive cycles were, at the most, a few micrometers in value, for the resolution of ca. 0.2 μm. For the hydrostatic pressure values of 150 MPa and 300 MPa, a drop in temperature was forced, with the pressure staying constant. This was done in order to observe the impact of temperature upon the strain measurement. This impact is actually so small that it is almost undetectable on the diagram.

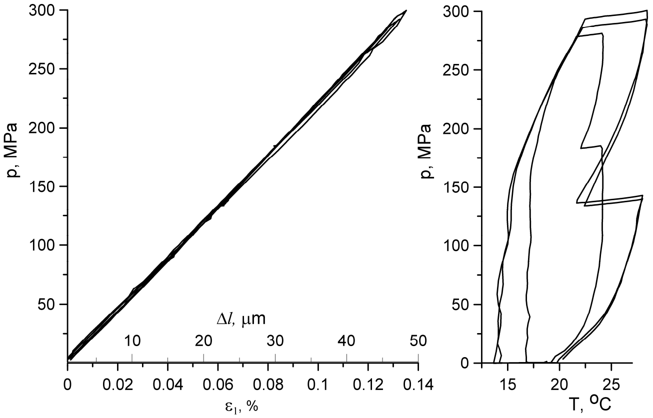

Fig.10. A test measurement of steel compressibility (left) in three load cycles and changes in the temperature of the pressure liquid during its compression and decompression (right)

The sensor is also used in tests outside a pressure cell: the photograph in Figure 11 presents the sensor used in the deflection measurement in the bending test of wood samples collected in the Wieliczka salt mine (Pieprzyk, 2012). A sample result of the measurement is shown in Figure 12. Such a manner of installation results both from the anisotropy of wood and the free positioning of the sample, which causes buckling of the sensor ends in extremely large sample deformations of up to 30 mm.

The photograph in Figure 13 shows a station for measuring axial and circumferential deformations of a coal briquette during a uniaxial compression test, performed in order to determine Young's modulus and Poisson's ratio. The sensors were fixed on metal rods glued into the sample.

At present, some attempts are being made to adapt the sensor to measurements carried out in high temperature, reaching the value of 200°C. The sensor itself, made of spring steel, is resistant

Fig. 11. A stand for measuring wood strength (normal conditions)

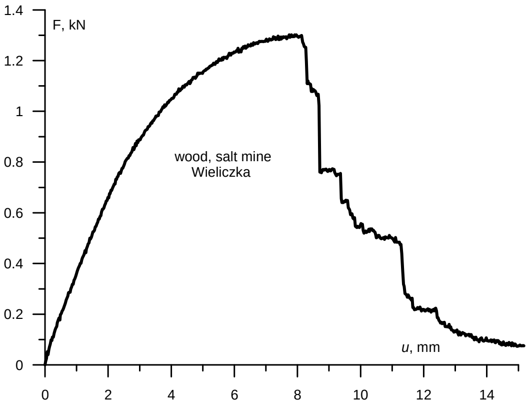

Fig. 12. Strength (force F vs deflection u) of wood from the lining of the excavation in the Wieliczka salt mine

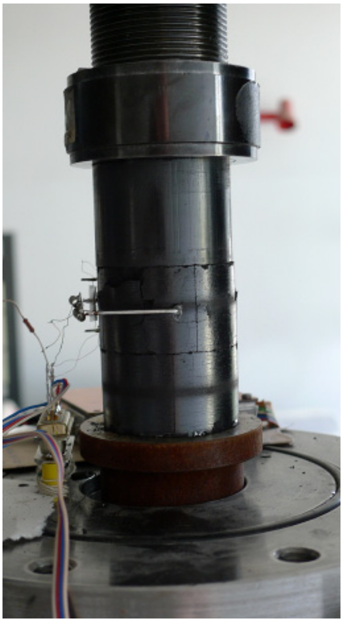

Fig. 13. Measuring transverse and longitudinal deformations of a coal briquette

to far higher temperature values, but the problem concerns the fixing of the measurement sensor to the sample and the fixing of the reference sensor, as well as establishing electrical connections. So far, soldering with a tin-lead alloy has been used, which let the system withstand the temperature of up to 170°C. Initial tests showed that lead-free soldering (95% Sn) will make it possible to reach the value of 200°C, for the proper mechanical strength of the connections. The thermal characteristics of the sensor in this respect are shown in Figure 4. The visible frequency changes (ca. 1 KHz for the whole range of temperature changes) translate into a measurement error of ca. ±6 μm (the right axis of the diagram). The correction of this error, performed due to the reference sensor, will reduce this error to the value of ca. 1 μm. The thermal characteristics take the shape of a parabola. If the parameters of the resonant circuit are properly selected, it is possible to thermally stabilize the sensor for any temperature range – in this case, halfway through the range of changes. In the vicinity of the stabilization point, within the range of a dozen or so °C, the thermal measurement error is almost negligible; however, a reference sensor is indispensable when it comes to measurements performed under changing pressure.

4. The toroidal sensor for measuring circumferential strains

Under the hydrostatic pressure of several hundred MPa, a rock begins to behave ductilely when subjected to the triaxial test. The force which is acting axially on a cylindrical sample causes its deformation – by several dozen percent, making it assume the shape of a barrel (this particular shape is the result of large friction between the sample fronts and the anvil and piston; this makes it impossible for a sample to deform in a free manner). The measurement of large transverse strains is difficult; in addition, on the sample surface, there appears a network of cracks, and the free space between the sample and the cell wall is reduced to several millimeters during the deformation process. Obtaining information as to the progress of transverse strains is vital for two reasons. First of all, on the basis of this information one can calculate the change in the transverse area as the sample gets deformed, which makes the process of calculating the stress more accurate. Secondly, it makes it possible to calculate the changes in the sample volume.

The measurement is performed with an sensor in the shape of a torus (Fig. 1). Its ends are linked with a small isolation plate (visible under magnification). To the ends, two thin wires are soldered, which connect the sensor to pressure electrical seals of such a length that it makes the free deformation of the sensor and the sample possible. The sensor is placed in the center of the sample, directly upon it or on the covers sheltering the sample. The sensor holds onto the sample thanks to force of spring turns. In the triaxial load test, after the sample has been placed inside the cell, the confining pressure is first increased. The pressure presses the latex cover to the surface of the sample; then, after the pressure and temperature have been stabilized, the sample is loaded with a piston, and the transverse strains are measured. Thus, the presence of the covers does not cause any measurement errors that would be of essence. In the triaxial test, the toroidal sensor gives us information about maximum transverse strains as it is placed in the center of the sample, and the sample changes its shape from a cylindrical to a barrel-like one. Numerous measurements performed on such deformed samples show that the average change in the cross-section equals 0.9 of the maximum change measured in the center of the sample. This average value, calculated in such a manner, is used to calculate the stress within the sample.

Figure 14 provides an example of usage of the circumferential sensor. It shows the effects of the triaxial load test performed on a cylindrical dolomite sample, under the confining pressure of $\sigma_3 = 300$ MPa. The sample is axially compressed by a piston of the press, which results in the strain e_1. The strain is calculated by measuring the movement of the piston outside the pressure cell. The process is accompanied by the emergence of transverse strains ε_3, measured with a circumferential sensor put onto the sample. Measuring these strains makes it possible to calculate the actual changes in the cross-section of the sample, as well as the actual stress in the sample. It should be taken into account that the vertical axis shows the value of the so-called differential strain $\sigma_1 - \sigma_3$ (the difference is the pressure force of the piston minus the force caused by the confining pressure). If the transverse deformations are not measured, the current cross-section of the sample is calculated on the basis of the axial strain, with the assumption that the volume of the sample during the whole experiment is constant (i.e. that the transverse strain index v = const. = 0,5). Such a simplification gives us quite satisfactory stress values up to the strength limit of the material. Over strength limit of the material, the calculated stress values are greater than the proper ones, which is illustrated with the thin chain-dashed line in Figure 14. The figure also shows the measured transverse strains ε_3 and the change in the sample volume ΔV calculated on their basis.

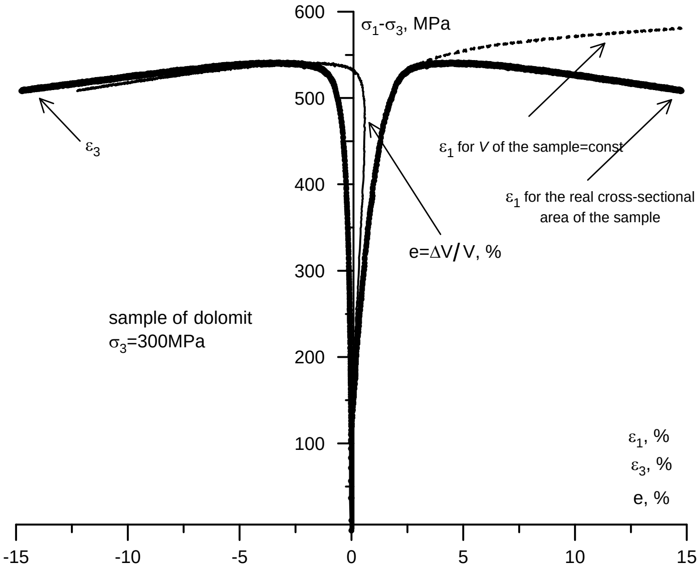

Fig. 14. Differential stress $\sigma_1 - \sigma_3$ vs. axial strain ε_1, transverse strain ε_3 and volume change e, a cylindrical sample of dolomite in triaxial load test

5. Summary

Inductive coreless sensor for measuring strains, cooperating with the *LC* oscillator, solved the problem of the laboratory measurement of strains of porous rocks, rocks characterized by low firmness, and of electrically conductive rocks, especially in the tests carried out in high confining pressure conditions. Use of such a sensor eliminated the risk connected with resistance sensors gluing directly onto an investigated sample. It seems that the discussed inductors have an advantage over the *LDT* sensors. They are characterized by high sensitivity and a very wide measurement range, from microns to centimeters. The simplicity and low cost of constructing the sensor, coupled with resistance to mechanical shocks, plus the fact that it is really easy to fix them on any investigated object – all this makes the sensor in question an appealing measuring device. The fact that the measuring sensor can be subjected to temperature compensation, as well as the comparative measurement method involving the usage of a reference sensor, make it possible to perform a measurement under changeable pressure (GPa) and temperature (even several hundred degrees Celsius). The application of such a measurement method virtually eliminated the impact of environment temperature and the impact of the power supply voltage on the oscillator and frequency meter, creating an opportunity for long-term measurements, extended over a considerable number of days. The obtained resolution of the measurement is below 1 μm, with the accuracy of ca. 1%. The limited accuracy is mostly due to the errors connected with the uncertainty of the calibration of the sensor and fixing it on the investigated objects, as well as with the limited possibility of compensating the pressure and temperature impact by means of a reference sensor.

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