

PRONY'S METHOD WITH REDUCED SAMPLING – NUMERICAL ASPECTS

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Abstract

This paper presents a new modification of the least-squares Prony's method with reduced sampling, which allows for a significant reduction in the number of the analysed signal samples collected per unit time. The specific combination of non-uniform sampling with Prony's method enables sampling of the analysed signals at virtually any average frequency, regardless of the Nyquist frequency, maintaining high accuracy in parameter estimation of sinusoidal signal components. This property allows using the method in measuring devices, such as for electric power quality testing equipped with low power signal processors, which in turn contributes to reducing complexity of these devices. This paper presents research on a method for selecting a sampling frequency and an analysis window length for the presented method, which provide maximum estimation accuracy for Prony's model component parameters. This paper presents simulation tests performed in terms of the proposed method application for analysis of harmonics and interharmonics in electric power signals. Furthermore, the paper provides sensitivity analysis of the method, in terms of common interferences occurring in the actual measurement systems.

Keywords: Signal analysis, harmonic analysis, signal processing, power quality.

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1. Introduction

1.1. Introduction to Prony's method

Prony's methods allow for accurate estimation of parameters of the analysed signal [1–20]. They provide new measurement opportunities by identifying actual frequencies of the analysed signal components and extending the signal model with data on damping coefficients of particular components. They allow using much shorter analysis windows, compared to the Fourier analysis, which is important when examining fast-changing phenomena. They also designate parameters of the low-frequency signal components with incomplete times in the test analysis window. To perform precise measurements of harmonic components' parameters, they do not require as well, sampling frequency synchronization with harmonic frequencies being designated. In Prony's methods, it is also unnecessary to adjust the analysis window length to the full cycles of the estimated components.

The great versatility of Prony's method results in its application as an alternative to the commonly used Fourier transform, allowing analysis of a wider range of signals in measurement systems, such as electric power quality (analysis window length, sampling frequency), which are inaccessible by other methods in test conditions.

1.2. Reduced Prony's method

Reduced Prony's method [1] uses a model of complex exponentials, which arbitrarily assumed that the attenuation coefficients of components are zero and the estimated sinusoid frequency vector is known. Thus, the stage of Prony's method [21], during which the vector of complex roots describing frequencies and damping coefficients of components for the designated signal model is being simplified. This procedure gives unequivocal improvement in stability and significant reduction of computational tasks in the algorithm. However, the reduced method requires adoption or knowledge of signal components' frequencies. In the simplest case, frequency vector with constant frequency step (quantization in a frequency domain) is assumed, for which other parameters of the model are being calculated, similarly as is the case in the Fourier transform [22]. Furthermore, in the reduced Prony's method, damping coefficients of components are not calculated. Other properties of the reduced Prony's method are similar to those in the classic LS Prony's method.

Reduced method is applied, inter alia, in electric power quality measurement systems, where components of the analysed signals are not damped, and their frequencies are known [2–3].

1.3. Properties of the method

In research studies presented in [1], it can be observed that when reduced Prony's method is used, the maximum frequency of the analysed signal component shall be at least four times lower than the sampling frequency of the signal (despite the fact that it is possible to modify the algorithm and achieve the Nyquist sampling rate condition). Furthermore, it is known that defining precise requirements for the analysis window length is more complex, as it depends on the number of estimated components and their location on the frequency axis. In general, it can be conclude that the minimum signal analysis window length should be at least equal to the reciprocal of the minimum frequency difference between sinusoidal components being present in the test signal. By reducing the number of estimated components, the minimum length of analysis window can be significantly reduced. Furthermore, significant issue in Prony's method is the fact that location of the test sinusoidal component on the frequency axis does not have effects on the estimation accuracy of its parameters and on the required analysis window.

The rest of this paper is organized as follows. Section 2 presents an outline of the reduced Prony's method with reduced sampling. The test simulation environment for the new method is described in Section 3. The performance of the method can be appreciated from the simulation results of Section 4. First experiments the validity of the method is confirmed, and then – values of its parameters are investigated. New results of the paper are summarized in conclusions of Section 5.

2. Reduced Prony's method with reduced sampling

2.1. Origin of the method

The computational complexity of Prony's method was a major limitation in their practical implementation. Reduced Prony's method allowed reducing the computational complexity during estimation of Prony's model parameters, approaching the complexity level of the discrete Fourier transform (DFT) [22–24]. However, in measurements systems, such as in

electric power quality testing, fast Fourier transform (FFT) is still commonly used, where the computational complexity of the reduced Prony's method, when compared to the DFT, is unsatisfactory.

Implementation of reduced Prony's method in a measuring device, where many harmonic components are designated, requires a DSP system with higher computing power, which complicates the implementation of the system. Therefore, further research study of the authors was directed toward subsequent optimization of the reduced Prony's method, while maintaining its existing properties. The result is the reduced Prony's method with doubled sampled signal that is proposed in this paper.

Reduced Prony's method with reduced sampling is similar to the reduced Prony's method allowing additional reduction in computational tasks, when determining component parameters of the model. This allows a significant reduction in the average sampling frequency, which will be presented later in the paper, on the example of simulated signals.

2.2. Algorithm description

Reduced Prony's method with reduced sampling is continuation of the reduced Prony's method. The method is the result of research work related to the analysis of the relationship between the required parameters for a correct estimate of parameters using signal analysis window, and the minimum sampling rate of the signal. Research on relation between analysis window length and Prony's model estimation accuracy as a function of sampling frequency and the number of estimated components [1] made it possible to determine the analysed minimum window size of the method. On the other hand, possibility of modifying signal sampling frequencies are, in the best case, limited to the signal sampling condition defined by the Nyquist frequency. Failure to comply with the sampling condition caused overlapping frequency bands and the spectral aliasing distortion effect on a signal, preventing correction of the estimate of sinusoidal component parameters of the signal. In turn, too short window of analysis at a given sampling frequency can cause a lack of sufficient number of samples to designate all components of the exponential model.

By searching for further reduction of computational tasks in the reduced Prony's method, elimination method of analysed signal samples, being introduced into the reduced Prony's method was adopted, which lead to decrease of computational tasks. Research study has shown that there is a possibility to select parameters for the analysis, for which the correct results of the component parameters, estimated using the Prony's model with largely reduced number of signal samples and other pairs of signal samples, in accordance with Fig. 1 are obtained.

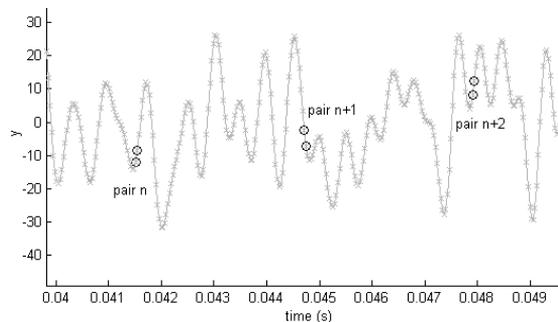


Fig. 1. Illustration of the idea for analysis of double sampled signal by the reduced Prony's method.

Based on the presented method for eliminating sampled signals, a sampling scheme was developed, named by the authors as double sampling, wherein the signal being analysed may be interpreted as a signal sampled by using two sampling systems, which sample the same frequency but with a time-delay relative to each other. An embodiment of the reduced Prony's method with reduced sampling was presented in Fig. 2.

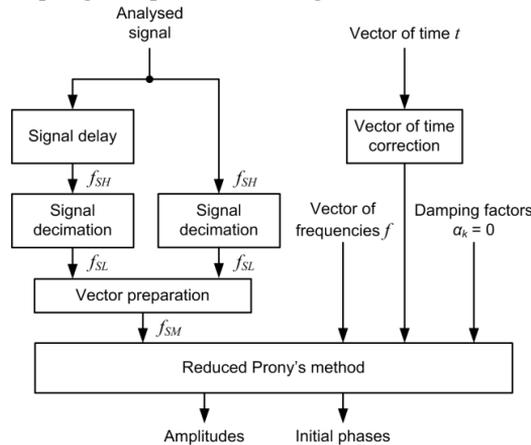


Fig. 2. An embodiment of the reduced Prony's method with reduced sampling (f_{SH} – signal sampling frequency prior to decimation process, f_{SL} – signal sampling frequency post decimation).

Samples for the reduced Prony's method with reduced sampling are collected pairwise (Fig. 1). The time interval between samples in a pair is determined by the condition for signal sampling and is fixed through input signal time delay through the “Signal delay” block (Fig. 2). Delay value in the “Delay” block is set according to inverted proportion $1/f_{SH}$. The next sample pair is collected after a longer time interval which may be described using inverted proportion $1/f_{SL}$. Therefore, data vector preparation in the “Vector preparation” block for further analysis under the reduced Prony's method relies on successive supplementation of the data vector with above-described sample pairs. The length of thus prepared data vector per time unit is reduced in comparison to earlier versions of Prony's method, which results from reduced average signal sampling frequency. In the “Vector of time correction” block (Fig. 2), time samples are converted in such a way as to localize properly in time the individual samples of the analysed signal as prepared in the “Vector preparation” block.

Long time interval between pairs of samples allowed meeting the minimum requirement for the length of analysis window to ensure adequate resolution of the method. On the other hand, it also allowed for a significant reduction in the number of data being collected per time unit, while maintaining the accuracy of the estimation method. Implementation of the algorithm with reduced Prony's method remained unchanged [1]. However, proper vector of discrete time samples shall be introduced to the method, describing location on the time axis of subsequent samples of the analysed signal, in accordance with the principle of pairing samples shown in Fig. 1.

By meeting the above assumptions, it is possible to select other method parameters, such as: number of estimated components, signal sampling frequency, distance - on time axis - between pairs of signal samples. Therefore, high estimation accuracy of parameters in the exponent model for all the requested signal components is obtained, while the average sampling frequency f_{SM} of the analysed signal is even several hundred times lower than the maximum frequency f_{MAX} of the estimated component. Wherein, the average sampling

frequency in the reduced Prony's method with reduced sampling can be expressed by equation (1),

$$f_{SM} = 2 \cdot f_{SL} \quad , \text{ for } f_{SL} = f_{SH} / elim \quad , \quad (1)$$

where: f_{SH} – signal sampling frequency prior to decimation process, f_{SL} – signal sampling frequency post decimation, $elim$ – a multiple of signal decimation from blocks in Fig. 2.

3. Simulations

3.1. Measurement system and implementation method of simulation measurements

For simulation measurements of the reduced least-squares method with reduced sampling, the schema shown in Fig. 3 was applied.

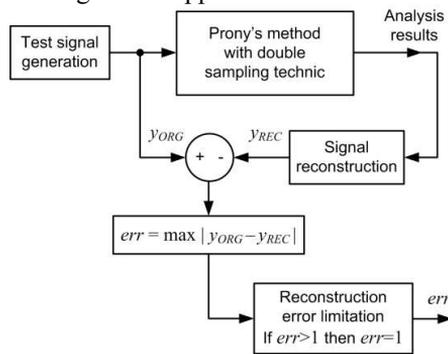


Fig. 3. Block diagram of a system for testing electrical signal processing systems using Prony's method with reduced sampling.

During the first step, test signal was generated, which was analysed in the subsequent step by using the proposed method. In the next step, results of the analysis in the form of parameters describing the Prony's model were used to reconstruct the signal that was previously analysed. The reconstructed signal was compared to the original signal in the last step. Model with maximum value difference between original and reconstructed signal samples was limited to a predetermined threshold value, which facilitated the subsequent analysis of the results, and served as err parameter for summary assessment of accuracy of determining the Prony's model parameters.

In the simulation stage, additional parameters were applied that described estimation accuracy of the analysed signal model. These parameters are e_{AMP} and e_{FI} , defined by equation (2),

$$e_{AMP} = \max|\mathbf{a}' - \mathbf{a}|, \quad e_{FI} = \max|\boldsymbol{\theta}' - \boldsymbol{\theta}|, \quad (2)$$

where: \mathbf{a}' – vector of given sine components amplitudes of the generated test signal, \mathbf{a} – vector of designated amplitudes, $\boldsymbol{\theta}'$ – vector of given initial phases of sine components of the generated test signal, $\boldsymbol{\theta}$ – vector of designated initial phases.

3.2. Test signals

Implementation of this method was analysed on the example of multiple test signals. Whereas, the paper presents four characteristic cases discussing the potential for application of the method in the field of power quality testing - estimation of harmonics or interharmonics in electric power signals. These signals are marked as #1, #2, #3 and #4.

The first three (#1,2,3) test signals aim at showing the method correctness (they are performed for carefully selected values of the method parameters), and the last one (#4) addresses investigation of method's features and finding optimal values of its parameters.

The #1 test signal represented the sum of 40 sinusoidal components, which are successive multiples of the fundamental component at the frequency of 50 Hz: $f = [50, 100, 150, \dots, 2000]$ Hz ($\Delta f = 50$ Hz). It was assumed that amplitudes of all components have the value of one $a_k = 1$ and initial phases equal to zero $\theta_k = 0$. Signal length being subject to the analysis was set to $T_O = 0.2$ s – this is the length of 10 periods of the fundamental harmonic, according to the standard [25].

The #2 test signal consisted of 501 sinusoids at frequencies of $f = [5, 10, 15, \dots, 2505]$ Hz ($\Delta f = 5$ Hz), with random initial phases and random amplitudes (normal distribution, $\mu = 0$, $\sigma^2 = 1$). For the initial phase, randomly selected values were multiplied by 180° , and for the random parameters of amplitude, their absolute value was designated. Signal length being analysed was set to the value of $T_O = 1.633$ s.

The #3 test signal was formed by adding to the #1 test signal, normal white noise ($\mu = 0$, $\sigma^2 = 1$) of adjusted level SNR [2].

The #4 test signal is a modification of the #1 test signal, where the unit value of amplitude was replaced with a module of randomly selected values, in accordance with normal distribution, $\mu = 0$, $\sigma^2 = 1$, as well as, the initial phases were randomly selected using the same distribution and multiplied by the value of 180° . The signal duration was $T_O = 0.2$ s or the parameter was set, depending on the performed analysis. The number of sinusoidal components forming a test signal in the f vector was $K = 40$, the same as for the #1 test signal, or was set as parameter, depending on the simulation being performed.

Signal sampling frequencies f_{SH} for all the test signals were determined according to specific conditions for processing the proposed method.

The first two test signals were used to illustrate signal reconstruction error modeled for in prior selected parameters of the proposed method. The #3 test signal was applied to illustrate the sensitivity of the proposed method to noise occurring in the analysed signal. In turn, the #4 test signal was implemented to demonstrate the methodology for selecting parameters of the method, in order to minimize errors in modeling the analysed signal in application to the analysis of harmonics of an electric power signal, according to [25] and [26].

4. Results

4.1. Case studies for selected parameters of the method

The first series of research tests indicates results for already determined parameters of the proposed method. The method for selecting operating conditions for the algorithm is presented in section 4.3 of the paragraph.

Results of the conducted analyses of #1 test signal are presented in Fig. 4. Top figure shows the entire waveform of the #1 test signal, while the lower figure presents computed absolute errors of its reconstruction.

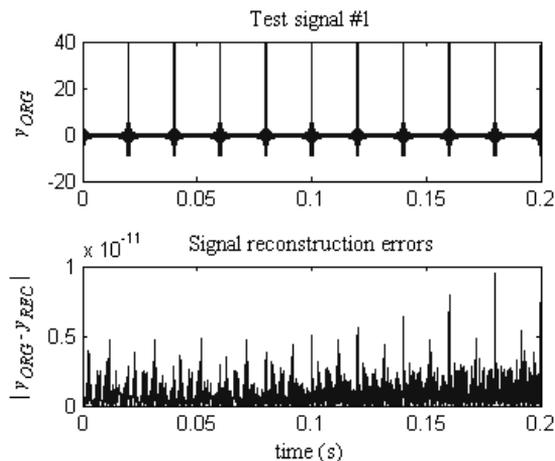


Fig. 4. Graph of #1 test signal and its reconstruction errors by using the reduced Prony's method with reduced sampling.

In the presented simulation, the parameter of decimation in frequency, shown in decimation blocks in Fig. 2, was set to $elim = 100$. In this way, the determined initial frequency of signal sampling $f_{SH} = 24.5$ kHz was reduced 100 times, up to the value of $f_{SL} = 245$ Hz. At such frequency, subsequent pairs of samples were collected (as shown in Fig. 1) for the analysis by reduced Prony's method with reduced sampling. Therefore, the average sampling frequency amounted to $f_{SM} = 490$ Hz.

The maximum frequency of the #1 test signal that was analysed was $f_{MAX} = 2000$ Hz, at average sampling frequency of the method of only $f_{SM} = 490$ Hz. Therefore, average sampling frequency was four times lower than f_{MAX} , and more than eight times lower than the Nyquist frequency at uniform sampling. Absolute reconstruction errors of #1 test signal were calculated as shown in Fig. 3, and amounted to $err = 9.5 \cdot 10^{-12}$, so they were comparable with the reduced Prony's method for signals without noise [2].

Another simulation shows research tests performed for signals of more complex structure - for the #2 test signal. Results are presented in Fig. 5. In this case, for the selected parameters of the algorithm, absolute reconstruction errors for #2 test signal were $err = 3.0 \cdot 10^{-10}$.

Selected for this case parameters of the reduced Prony's method with reduced sampling are listed below:

- initial sampling frequency of a signal: $f_{SH} = 31.3125$ kHz,
- subsampling: $elim = 100$ ($f_{SL} = 313.125$ Hz),
- average sampling frequency: $f_{SM} = 626.25$ Hz.

The analysed maximum frequency of #2 test signal was $f_{MAX} = 2505$ Hz, while average sampling frequency for double sampled signal amounted to $f_{SM} = 626.25$ Hz, so it was up to four times lower than the one, and eight times lower than the Nyquist frequency.

However, it is possible to perform such selection of parameters of the method, so even greater frequency ratios with high estimation accuracy will be obtained.

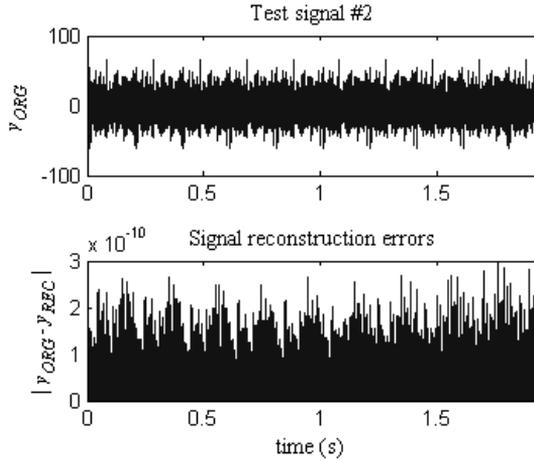


Fig. 5. Graph of #2 test signal and its reconstruction errors by using the reduced Prony's method with reduced sampling.

4.2. Noise effects on estimation accuracy of Prony's model for selected parameters of the method

Further research study concerned analysis of noise effect on signal reconstruction error and estimation accuracy of the component parameters of the model. Simulation tests were performed for #3 test signal. In the signal, white noise is present with normal distribution of ($\mu = 0, \sigma^2 = 1$) and adjustable signal to noise ratio *SNR*. Results of the analysis are shown in Fig. 6.

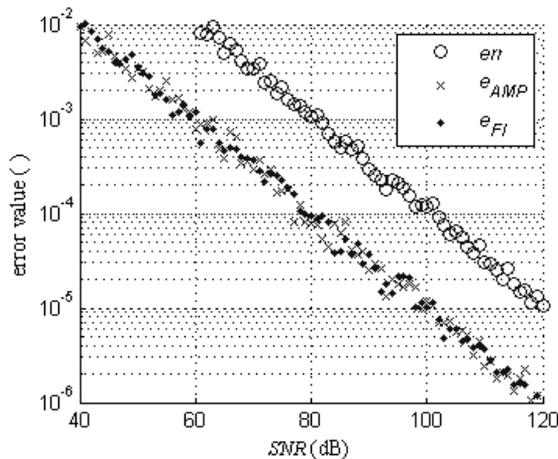


Fig. 6. Dependency between reconstruction and estimation errors of the model parameters and the noise level contained in the analysed signal.

By comparing the obtained characteristics with the results of research studies performed for the reduced Prony's method [1] or variable-frequency Prony's method [2], deterioration in the estimation accuracy of the components' parameters, and associated with it, accuracy in signal reconstruction can be observed. However, the effect of this phenomenon due to

implementation of the specific sampled signal and the reduced Prony's method is not large enough to disqualify measurement advantages of the presented reduced Prony's method with reduced sampling.

4.3. Property analysis of the method for variable parameters of the analysis - determination of parameter selection method

The presented analysis method allows precise determination of component parameters of sinusoidal signals ($err \approx 10^{-10}$) at average sampling rate much lower than the maximum frequency of the components present in a signal. However, it is recommended to select parameters of Prony's method with reduced sampling. Proper estimation of parameters of Prony's model is determined by selection of the following parameters:

- f_{SH} frequency,
- coefficient of decimation in frequency $elim$,
- length of analysis window T_O ,
- a number of estimated K components,
- distance between particular adjacent frequencies Δf in the adopted f frequency vector ($\Delta f = f_{k+1} - f_k$ for $k = 1, 2, \dots, K - 1$).

This paragraph presents research study that allows proper selection of parameters of the presented method. The research study was illustrated by the case of the #4 test signal.

Figs. 7-13 present selected results of the performed simulations. Simulations in Figs. 7 and 8 allow performing selection of frequency division parameter $elim$ and signal sampling frequency f_{SH} . A characteristic phenomenon observed in all the performed simulations is the presence of a single major area, in which are solutions with an error larger than $err = 10^{-9}$, defined as incorrect solutions. Additionally, there are a series of small areas and narrow strips of incorrect solutions. Besides these areas, the presented method ensures high estimation accuracy in component parameters - light area in graphs. Selection of the method parameters shall be performed using set-ups of parameters from light areas in the graphs.

Further simulations shown in Figs. 9-13 present the simulation results for set as parameters values of K or T_O parameters. Again, bright colours indicated areas where $err < 10^{-9}$, defined as area of proper estimation of component parameters of Prony's model.

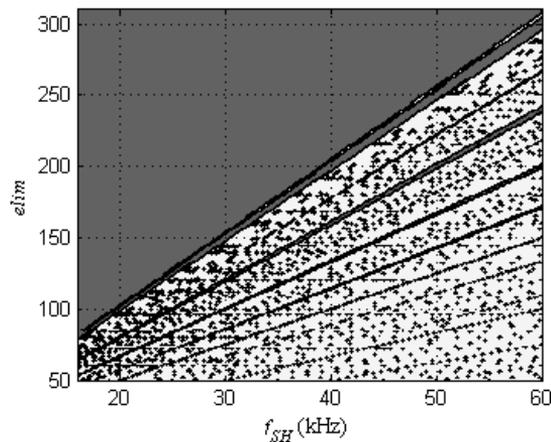


Fig. 7. Method for selecting the parameter of frequency division $elim$ and frequency f_{SH} – bright colour indicates the areas of proper estimation of component parameters of Prony's model, where $err < 10^{-9}$, ($K = 40$, $T_O = 0.2s$).

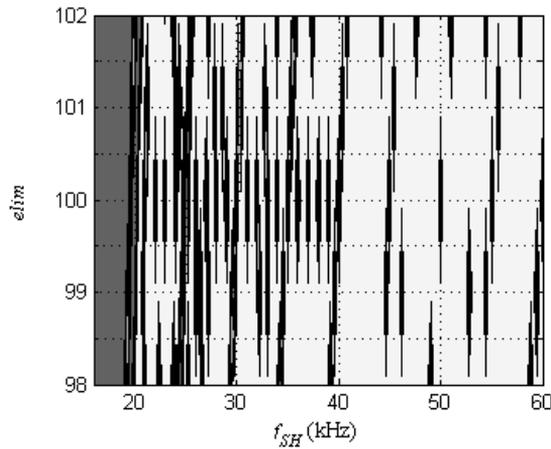


Fig. 8. Enlarged area of simulation presented in Fig. 7 for $elim = 100$.

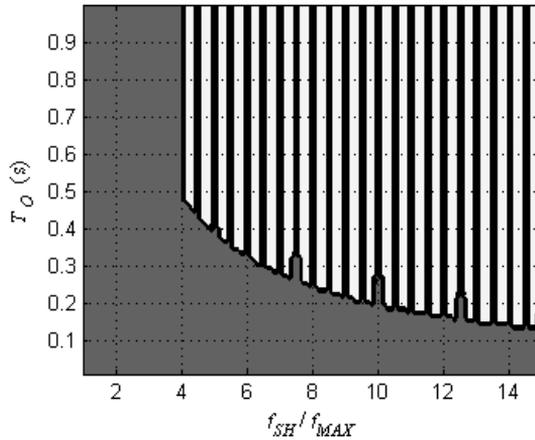


Fig. 9. Method for selecting frequency parameter f_{SH} , according to the maximum frequency f_{MAX} of the estimated component and length of analysis window T_O with set parameter to $elim = 100$ and $K = 40$.

Based on simulation shown in Fig. 9, it can be concluded that, among others, in order to receive correct results of Prony's model estimation, the frequency f_{SH} shall be four times higher than the maximum frequency of the estimated component f_{MAX} . The condition follows the applied mathematical tool of the reduced Prony's method [1].

Fig. 10 indicates the dependence, which shows that the area of correct estimation of the harmonic components' parameters can be received for shorter lengths of analysis windows, when increasing f_{SH} frequencies, and thus for constant $elim$ coefficient, when increasing the average sampling frequencies f_{SM} . An interesting phenomenon is reduction in density of the vertical bars for unstable solutions above a certain frequency threshold - here - for the frequency of $f_{SH} = 40$ kHz.

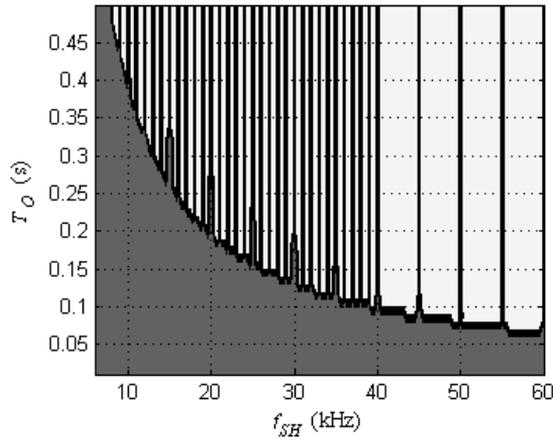


Fig. 10. Method for selecting frequency parameter f_{SH} , according to the length of analysis window T_O with set parameter to $elim = 100$ and $K = 40$.

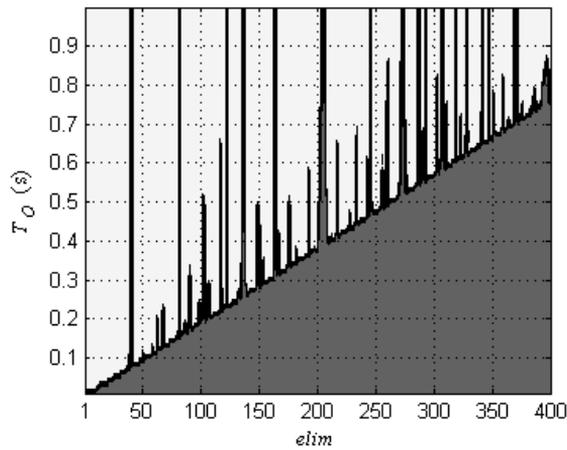


Fig. 11. Method for selecting the length of analysis window T_O , according to $elim$ parameter with the set parameter to $f_{SH} = 20.5$ kHz and $K = 40$.

Fig. 11 presents behaviour of the algorithm during changes in the $elim$ parameter and length of analysis window T_O . Once again, there is one large area of incorrect solutions, probably due to insufficient number of collected measurement data to estimate parameters of Prony's model, and narrow vertical bars of incorrect solutions. Some of the narrow belts of incorrect solutions in Fig. 11, end in a way which transforms them into areas of correct solutions for the required length of analysis windows T_O .

Subsequent Figs. 12 and 13 present the method for selecting analysis window and frequency f_{SH} for varying number of K components being estimated.

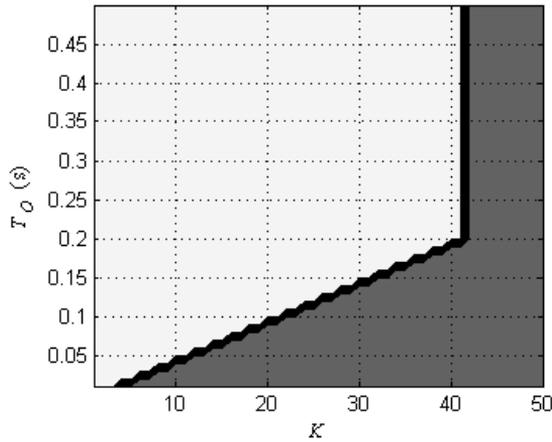


Fig. 12. Method for selecting the length of analysis window T_O , according to the number of estimated K harmonic components with the parameter set to $f_{SH} = 20.5$ kHz and $elim = 100$.

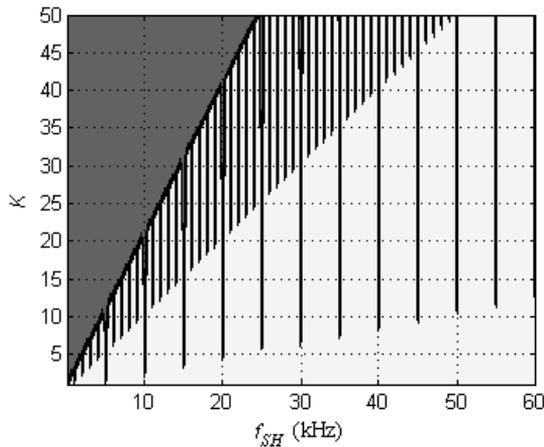


Fig. 13. Method for selecting the f_{SH} frequency parameter, according to the number of K harmonic components being estimated with the parameter set to $elim = 100$ and $T_O = 0.2$ s.

5. Conclusions

Application of the reduced Prony's method with reduced sampling allowed signal sampling with an average frequency significantly lower than the Nyquist frequency, while maintaining the possibility of very accurate reconstruction of the analysed signal, and thus accurate determination of parameters of all the estimated components.

For a certain set-ups of parameters, it was however observed that results of the analyses are burdened with a significant modeling error. This paper presents methodology for the method parameter selection, which allowed for obtaining high estimation accuracy in parameters of the Prony's model.

Designated conditions that must be met to obtain successful implementation of the method are a series of simple dependencies mentioned below:

- analysis window with length not shorter than the inverse of the smallest frequency difference in the adopted vector of requested frequencies - analogy to the reduced Prony's method.
- time reversal and shifting between sampling waveforms shall be at least four times smaller than the length of the maximum frequency component in the adopted vector of the required frequencies - adopted and modified property of reduced Prony's method,
- a number of signal samples after subsampling process shall be at least twice the number of the estimated components.

The proposed method provided an opportunity for n-multiple decrease (or increase) in the sampling frequency with n-multiple increase (or decrease) in the length of the analysis window, that is: $f_{SM} \cdot T_O = \text{const}$. This property allowed for selection of sampling frequencies in reduced Prony's method with reduced sampling, regardless of the Nyquist frequency. In extreme cases, the sampling frequency can be up to several hundred times lower than the Nyquist frequency.

The above described property enabled practical application of the reduced Prony's method with reduced sampling in embedded-type measurement devices (with low processing power), for testing high-frequency components using analog-to-digital converters with significantly lower limits of sampling frequencies. Application of the method also made it possible to remove, in some measurement systems, buffering of measurements in high-speed memories, which contributed to a cost reduction during construction of these type devices.

An example application of the method in an actual measurement system will be the subject of subsequent publication.

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