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Validation of the pressure wave model in the aspect of special structures endurance

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Abstract. The article presents results of the research on the validation of impact wave in the aspect of special structures' endurance. Impact waves have been examined according to numerical and analytical methods. In order to verify the results obtained, a workstation for experimental research had been prepared. Before conducting the experiment, approximate pressure value had been checked empirically. The pressure had been measured using sensors for free wave measurements.

The topic of the paper is empirical examination of the phenomenon of explosion in the aspect of the free wave pressure. Due to a difficulty of the issue, it was necessary to conduct complex examination of the phenomenon for various charges. The paper examines cases of spherical charge explosions with a balanced mass of 1 kg TNT.

Key words: explosion, analytical methods, numerical methods.

1. Introduction

Numerical examination of short course phenomena is very complicated and requires much experience on scientists' part. In the case of explosion, the phenomenon is complex due to high energies and short duration. It is not uncommon for the scholars to have problems estimating the free wave pressure values. A proper selection of parameters describing the explosion's numerical model guarantees appropriate numerical analyses results.

The explosion is an abrupt and short-lasting phenomenon. An example of a fireball resulting from an explosion is presented in Fig. 1.

Fig. 1. An example of a fireball's appearance during TNT explosion

The essence of an explosion is the fact that the initial stage is characterized by a detonation (permeating of the detonation wave in the explosive material) which later transforms into spreading of the pressure wave in the air. It is worth remembering that a detonation wave spreads at the speed of 2–8 km/s.

There exist many approaches to simulating the explosion. Some authors present in their studies [1–3] the phenomenon of explosion as an appropriately selected pressure impulse. In another study [4], an approach of modeling the explosion through the explosive material's combustion has been presented. In the latter study, the explosion model has been limited to presenting the Euler method (air) in which initial, concentrated energy has been placed (shape of a ball). A similar approach to the problem gives fairly good solutions [5].

The aim of the following paper is to examine the modeling possibilities of the pressure wave triggered by an explosion of 1 kg spherical charge placed underground. In the first attempt, an approximate pressure value in the distance function to the charge given has been defined. This estimate is essential to the initial sensors calibration. Next, experimental examination has been performed.

The analyses of the aforementioned phenomenon have been conducted using numerical methods by means of MSC Dytran, as well as analytical ones. The presented research constitutes one of the stages of drawing up a concept of military and civilian facilities' protective structures. The results of the experiment will be used to calibrate the impulse-charged structure numerical models. For initial sensor calibration basic formulas, discussed below, have been used. The following study presents the results of pressure validation measurements for 1 kg TNT.

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2. Analytical approach to the problem

Available specialist literature allows with some approximation to analytically define dynamic constructions' charge values. Such approach has also very limited possibilities and allows only to casually estimate the pressure impulse affecting the discussed systems. Such solutions can usually be found efficient for idealized, unrealistic conditions, constituting an approximation mainly for model cases.

Modern literature abounds in formulas and simplifications of various kind, describing the spreading pressure impulse in the air. These practical descriptions have at their foundation basic laws of physics. Mathematical formulation of the problem in the case of one-dimensional ideal gas is described by equations which express basic laws of behavior in the nature [6]:

$$\text{mass:} \qquad \frac{\partial \ln \rho}{\partial t} + u\frac{\partial \ln \rho}{\partial r} + \frac{\partial u}{\partial r} + \frac{Nu}{r} = 0, \tag{1}$$

$$\text{momentum:} \qquad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \tag{2}$$

$$\text{energy:} \qquad \frac{\partial (p/\rho^{\gamma})}{\partial t} + u\frac{\partial (p/\partial^{\gamma})}{\partial r} = 0, \tag{3}$$

where p – pressure, ρ – density, u – mass flux.

The high pressure wave generated by the detonating explosive charge can be described with the following equation introduced by Brode in 1955 [7]:

$$\Delta p_{TNT} = \begin{cases} \dfrac{6.7}{\overline{R}^3} + 1\ bar \\ \qquad\qquad \Delta p_{TNT} > 10\ bar, \\ \dfrac{0.975}{\overline{R}} + \dfrac{1.455}{\overline{R}^2} + \dfrac{5.85}{\overline{R}^3} - 0.019\ bar \\ \qquad\qquad 0.1 < \Delta p_{TNT} < 10\ bar, \end{cases} \tag{4}$$

where $\overline{R} = R/\sqrt[3]{m_{TNT}}$ – reduced distance, R – the distance from the charge in m, m_{TNT} – equivalent explosive material mass expressed in kg.

The high pressure value of the high pressure free wave in the function of time can be described by the Friedlander relationship:

$$P_s(t) = P_{so}\left(1 - \frac{t - t_A}{t_o}\right)\exp\left(-\beta\frac{t - t_A}{t_o}\right). \tag{5}$$

A typical course of pressure wave generated by an explosive charge has been presented in Fig. 2.

The proportion coefficient allowing to use presented relationships in case of semtex equals 1.24, while for composition B it is 1.148. After reaching any object, the pressure wave generated by the explosive reflects off the object. The reflected pressure wave value can be defined according to the relationships provided by Rankin-Hugoniot. The value depends on the pressure present in the undisturbed medium, the relationship of specific heats and maximal dynamic pressure value. In case of perpendicular surface load, it assumes the maximal value described by the relationship:

$$p_r = 2p + (\gamma + 1)q, \tag{6}$$

$$q = \frac{5p^2}{2(p + 7p_0)}. \tag{7}$$

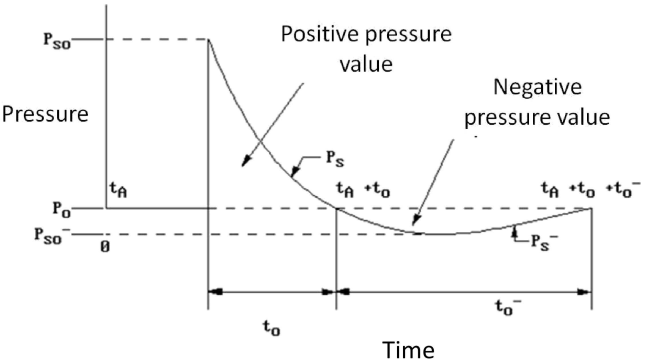


Fig. 2. The pressure wave impulse caused by the explosive detonation where: P_{so} – pressure in the undisturbed medium, t_o – duration of positive impulse phase, β – breakdown constant, t_A – explosive detonation initiation moment

For air ($\gamma = 1.4$) the reflected pressure wave value can be expressed by the relationship

$$p_r = 2p\frac{7p_0 + 4p}{7p_0 + p}, \tag{8}$$

which shows that for weak waves $p_r = 2p$, while for strong ones $p_r = 8p$. Literature descriptions show that in real situations one can even encounter reflected waves' reinforcements exceeding 10 (some sources suggest even 12).

In a general case the pressure value can be determined by the relationship [6]:

$$P(t) = P_r \cos^2\theta + P_i(1 + \cos^2\theta - 2\cos\theta), \tag{9}$$

where θ – wave's angle of incidence, ρ – reflected wave's pressure, p_i – falling wave's pressure (this relationship was drawn up in 1991 for the use of US Army [8]). Another approach to defining the impulse pressure is based on a premise that in the construction analysis it is assumed [9, 10] that the effect of the impact wave activity is dependent on the explosion's parameters:

$$Z = \frac{R}{\sqrt[3]{W}}, \tag{10}$$

where W – stands for the charge's mass, R – charge's distance to the examined object.

An approximate value of the maximal pressure in the impact wave front is calculated as a Z parameter function:

$$p = p\left(\frac{1}{Z}\right) = p\left(\frac{\sqrt[3]{W}}{R}\right). \tag{11}$$

It means, that the maximal pressure p triggered by a 100 g charge at a distance of 0.4 m estimated this way has the same value as the pressure triggered by a 100 kg charge at a distance of 4 m. In the latter case, however, the effect of the wave activity is much bigger because the impulse t_d duration is approximately 10 times longer [11].

In the initial calculations various function approximations are used (11). The simples pressure value estimation (based on immediate detonation) has the following form [11]:

$$p = \frac{1}{2\pi} Q \frac{1}{Z^3} = 0.159 \cdot Q \frac{1}{Z^3}, \tag{12}$$

where Q – unit internal energy of the explosive (for TNT 4.2 MJ/kg). More accurate approximation (similarly to 12, based on immediate detonation) could be obtained, using the formula [12]:

$$p = \frac{a_1}{Z} + \frac{a_2}{Z^2} + \frac{a_3}{Z^3}, \tag{13}$$

where $a_1 = 82400$, $a_2 = 264870$, $a_3 = 686700$ – parameter values calculated for TNT based on the assumption, that Z is expressed in $\frac{\text{m}}{\sqrt[3]{\text{kg}}}$, while the pressure value in Pa. The approximation (12) is practically the third, the most significant segment (13). Maximal pressure p values obtained from formula (13) are approximately 25% higher than the ones from formula (12). In reality, the form of function (11) is much more complex than formulas (12) and (13). The pressure equivalent to the effect of explosion's impact wave for the given parameter Z can be defined with more accuracy based on widely available charts or by using advanced numerical methods for solving complex explosion models taking into account the process of the explosion's initiation and impact wave propagation. Formulas (12, 13) allow for estimating the pressure of a wave spreading freely in the air. The pressure of an impact wave, reflecting off an obstacle is considerably higher (it grows from 2 to 8 times, compared to a free wave pressure) and can be estimated from the formula

$$p_o = p_c \left(2 + \frac{6p_c}{p_c + 7p_1}\right), \tag{14}$$

where p_o – reflected wave pressure, p_c – free wave pressure, p_1 – air pressure.

Another approach is to define a construction element's explosion aggregate load by defining a total impulse generated by the charge's explosion. The task boils down to integrating the unit impulse on the surface assuming explosion load [12]:

$$J + \int_F idF, \tag{15}$$

where i – is a unit impulse, F – is the obstacle surface.

Using analytical methods of defining charges triggered by explosion products' effects requires assuming a number of simplifications (similarly to estimating the reflected wave's pressure). The following assumptions, among others, are made:

- an explosion takes place immediately in a highly rare medium;
- the barrier is assumed to be ideal (it does not move and deform);
- the charge shape can be arbitrary;
- the flow around the obstacle is omitted.

The numerical pattern for the spherical charge placed over the circular plate has been presented in Fig. 3. Based on the literature, it has been stated that the total impulse generated by the explosion in this case equals [9]:

$$J = \pi \cdot A_0 \cdot C \cdot \sin^2 \alpha_0, \tag{16}$$

where J – total force impulse expressed in Ns, A_0 – a constant, characterizing the explosive (for TNT it is 387–410 m/s), C – charge mass kg.

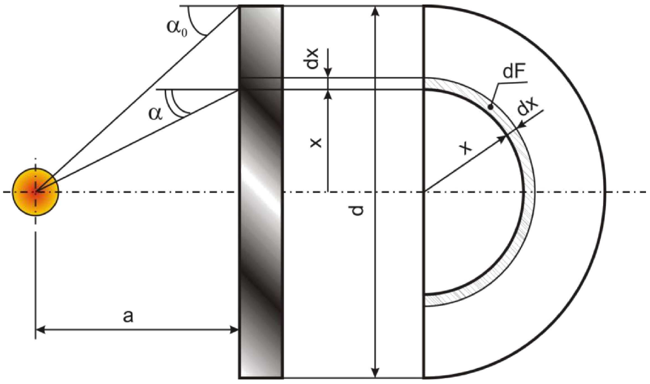


Fig. 3. A pattern for establishing the impulse affecting the circular plate

The remaining examples are examined in a similar way. There exists a general formula taking into account the maximal obstacle section, its slope and shape. The formula is as following:

$$J = k_f \cdot S \cdot i_0, \tag{17}$$

where S – is the surface of the maximal section to the gas stream direction, k_f – is the obstacle's crosswise shape coefficient, $k_f = 1$ – for the plate situated perpendicularly towards the stream, $k_f = \frac{1}{2}$ – for the plate situated diagonally towards the stream and for a sphere, $k_f = \frac{2}{3}$ – for a cylindrical obstacle, i_o – is a unit stream impulse.

Examining the coefficients k_f one may state, that obstacles in the form of a sphere and a deflector (letter V-shaped) assume the load two times smaller than a perpendicularly – situated plate (naturally, for the same conditions: explosive mass as well as the distance to the maximal section point). This phenomenon is used while designing structures exposed to mines and improvised explosives (IED).

After thorough analysis of the above mentioned formulas, analytical simulations using MATLAB software have been conducted. The achieved charts depicting pressure flows have been presented in Fig. 4.

Due to a big number of cases examined, estimated values of the pressure impact resulting from 1kg TNT explosion have been presented. Similarly to experimental research, the flows of pressure in four points situated at distances of 0.5, 1, 1.5, and 2 m have been examined. In this analysis, the variable was time. As mentioned previously, for the distance of 0.5 m maximal pressure value was 7 MPa. Due to the measurement range of the sensors used, it has been established that such a distance is too short (the pressure impulse might damage the sensor). The above mentioned formulas show, that pressure impulse values depend on the distance raised to the third power. For the sensor placed at the distance of 1 m the maximal pressure value is 1 MPa.

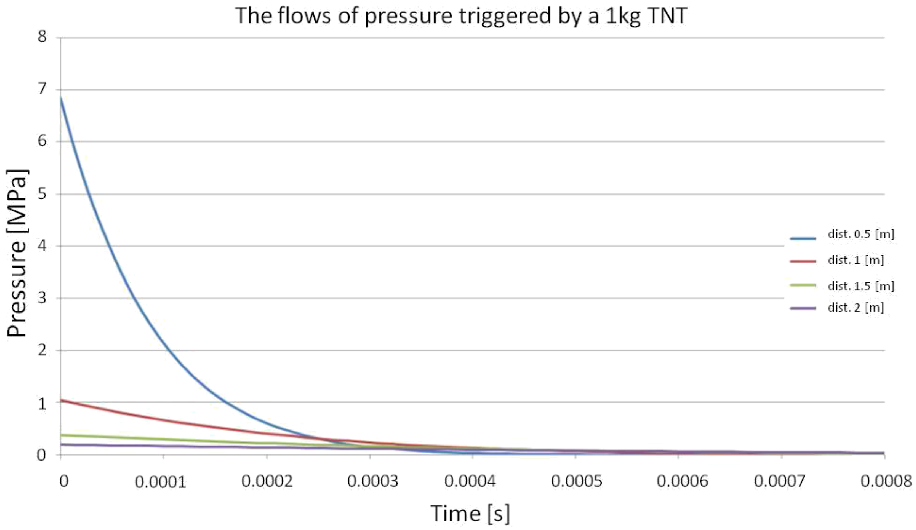


Fig. 4. The flows of pressure triggered by a 1kg TNT explosive detonation for various distances

3. Experimental approach to the problem

Experimental research has been conducted for the detonation of C4 in the shape of a sphere, being equivalent to 1 kg TNT charge. The charge was placed at the height of 1.6 m above the ground. The photograph of the charge is presented in Fig. 5.

Fig. 5. The view of the charge with its measurements

During experimental research the free wave pressures have been measured. For the measurement of the free and reflected wave pressure there were used measurement posts designed and manufactured in the Department of Mechanics and Applied Computer Science, which allow to measure the free wave pressure taking into account the distance and sensor height variations. While designing and manufacturing the station, possible changes in some inputs were taken into account, so that the experiment can be carried out for different sizes of the explosive. Such an approach allowed to eliminate the effect of the wave pressure being reflected off the ground. The view of the station is presented in Fig. 6.

Pressure sensors were directed at an angle. Such a location allows to eliminate the effect of the height of charge placement.

Fig. 6. The measurement system used during the research including the sensor placement method

For preconditioning and strengthening the signals from free wave pressure and acceleration transducers, a high-frequency amplifier has been used. The process of registration was carried out with the use of NI-USD model measurement card with a high-speed 16-bit analog-digital transducer (2 MHz sampling on each channel), a portable computer, as well as measurement card software. The measurement equipment used in the research is presented in Fig. 7.

Fig. 7. Measurement equipment used in the research: 1 – LTT 500 measurement amplifier, 2 – NI-USB 6833 measurement card, 3 – computer with measurement software, 4 – laboratory power pack

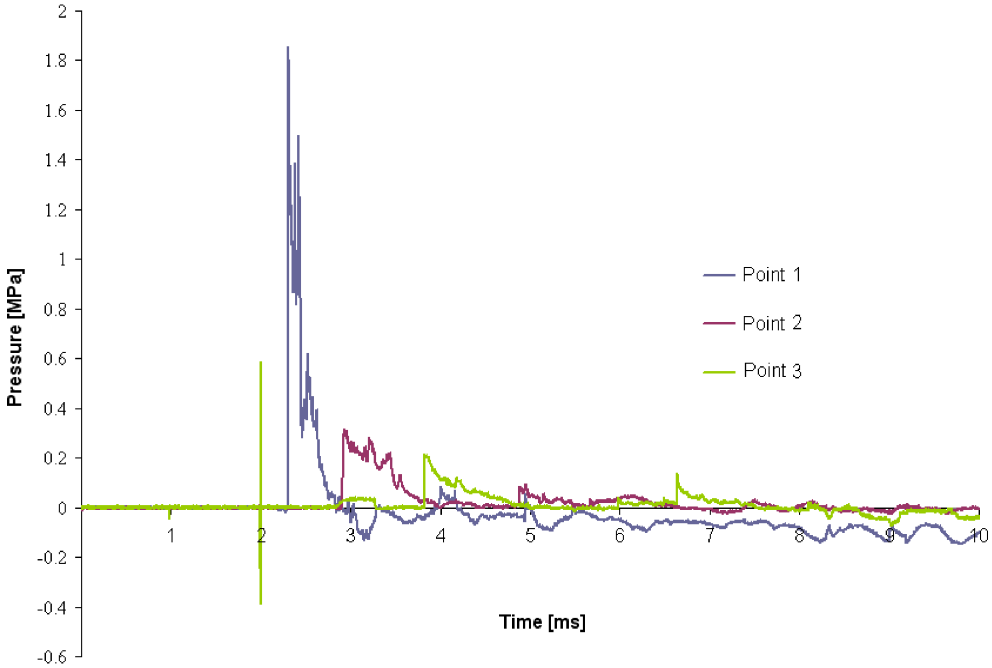


Fig. 8. The changes in free wave pressure – 1 kg TNT charge

The pressure of the free wave was measured by a specialist sensor of 143.3 mV/MPa sensitivity and 6.894 MPa range. The first sensor, closest to the explosive, was placed at a distance of 0.7 m from the explosive axis.

After carrying out the examination at the station, the obtained routes, free wave pressures and accelerations underwent scaling in order to create size changes charts in the function of time. In Figs. 8 and 9 particular routes for 1 kg TNT charge have been presented.

In the case of a 100 g charge, the values of free wave pressure equaled 1.8 MPa (as presented in Fig. 8).

4. Numerical approach to the problem. Numerical impact wave simulation

For numerical simulation examination, the Euler space, which reflects the experiment, was assumed. For space modeling cubic elements were used. Such a choice of element shape results from the need to adjust the elements of Euler's grid to finished elements defining the examined object. Spherical impact wave spreading in the grid of cubic elements undergoes minor deformation. With big changes (gradients) of pressure the size of the grid elements has big influence on calculating the pressure values. This is why Euler elements are very susceptible to grid parameter changes. On the other hand, thickening of the grid elements requires (during the calculating stage) bigger and bigger external and operational memory. The size of the elements is decided based on comparing the pressure values obtained numerically and estimated analytically or experimentally. From the comparison of the numerical and analytical results, it has been established that grid parameters need to be adjusted to every change of the charge size.

Because the influence of the grid shape on the pressure values at the front of impact wave is relatively small, the basis for further analysis was cubic, but adequately dense, the Euler element grid. The Euler space had a free outflow defined by the Flow function, allowing pressure wave movement out of the model. Through appropriate initial boundary conditions a ground was simulated.

The following assumptions regarding impact wave have been made: TNT charge with 1kg mass placed at the height of 1.6 m. The explosive density assumed for the analysis was $\rho = 1520$ kg/m^3 and the internal energy $Q = 1520$ J/kg [13]. Based on the mass, spherical charge's geometrical parameters were calculated.

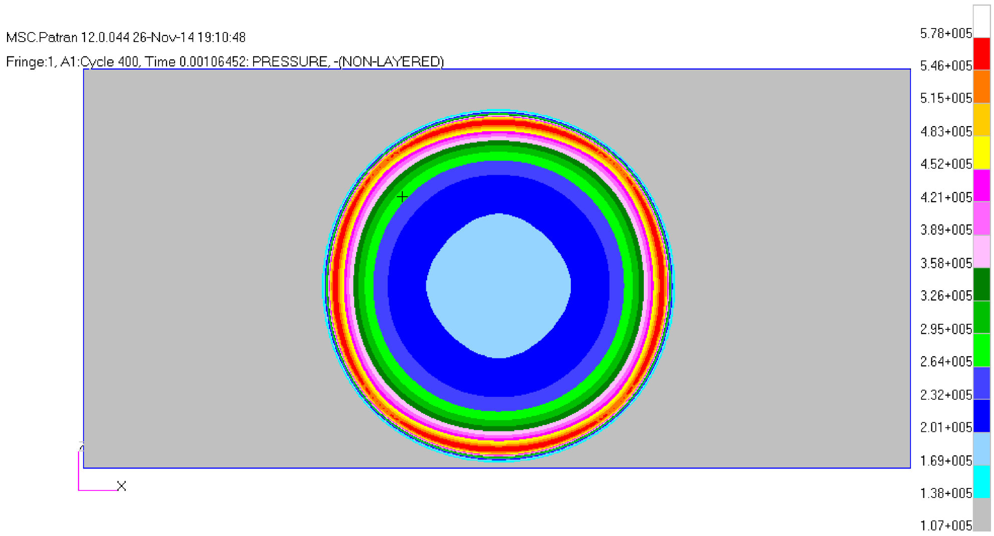
In order to obtain impact wave pressure spread generated by the explosion an element grid of the Eulerian Solid type was defined. The simplest ideal gas state equation was assumed, both for the description of the gas medium (air) and for detonation products. In this model defined are: density of the medium (for air $\rho = 1.29$ kg/m^3), isentropic exponent ($\kappa = 1.4$) and internal energy ($Q = 193800$ J/kg).

In Fig. 9a-j subsequent phases of pressure wave and disturbed medium's speed spreading in the air for 1 kg charge have been presented.

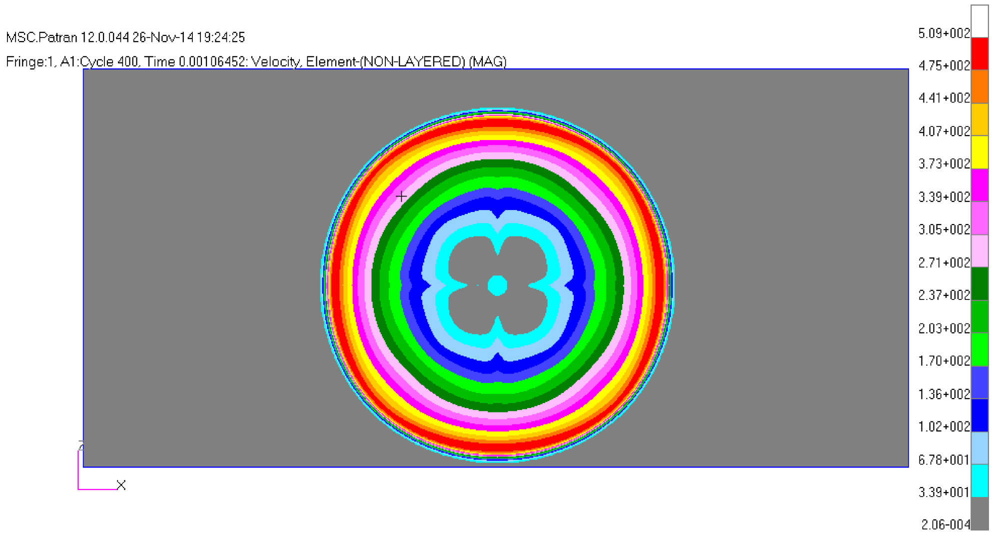
In Fig. 9b the pressure wave reaching the ground is presented. The pressure impulse's reaching took place for the time of 0.00106 s.

The maximal pressure value for this time is ca. 0.6 MPa, and maximal medium speed approaches 500 m/s. Pressure impulse's reflection off the ground is presented in Fig. 9 c-d. For the time of 0.0016 s it has been stated that the pressure value grows twice, which is a result of being reflected off a stiff obstacle. The value equals to 1.33 MPa. Maximal speed value in the reflection zone was 285 m/s. The velocity vector value dropped by nearly a half. It is worth noticing that due to the reflection, vector's direction changed.

a) 0.00106 s



b) 0.00106 s



c) 0.001651 s

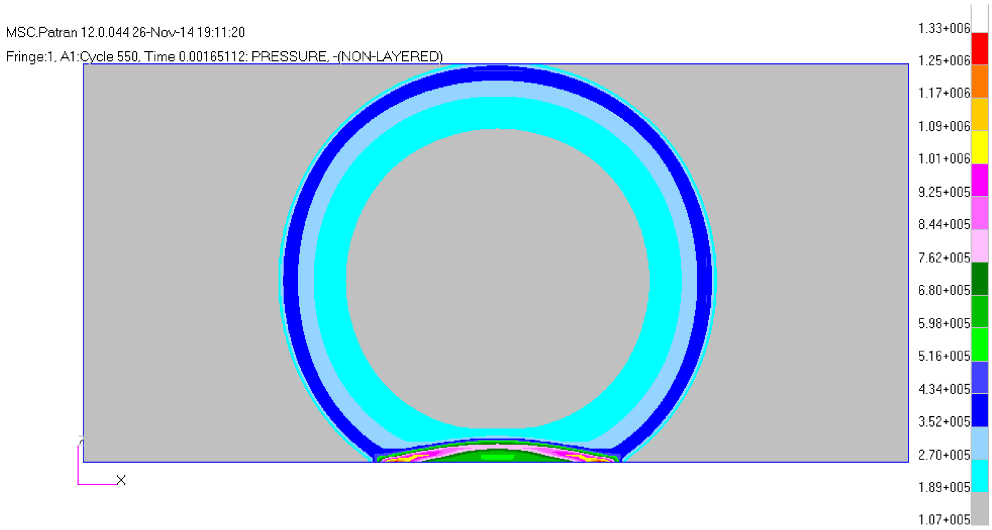
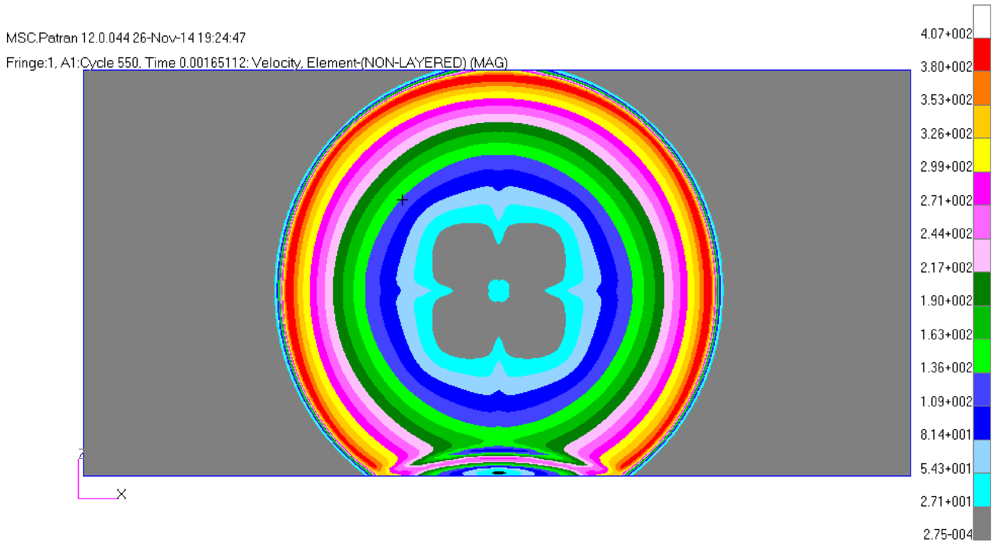
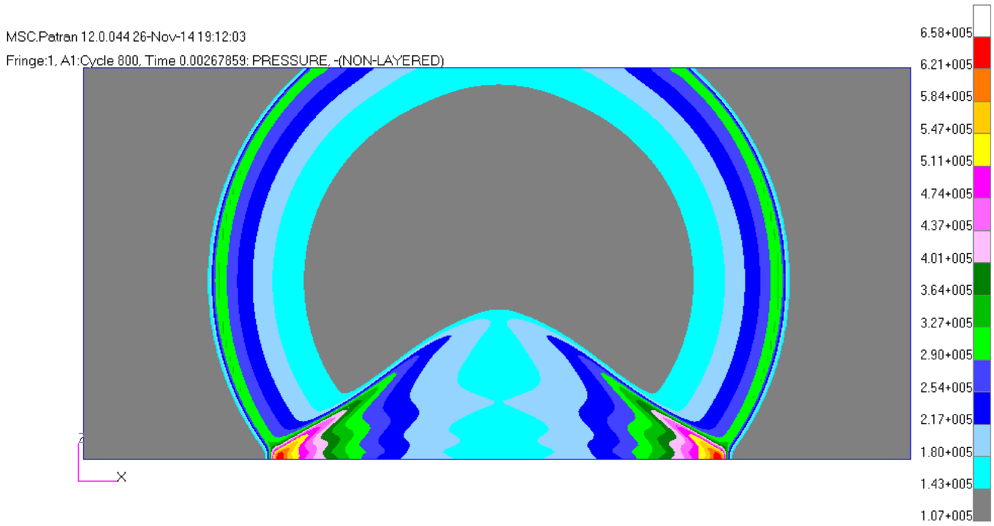


Fig. 9a-c

d) 0.001651 s



e) 0.002678 s



f) 0.002678 s

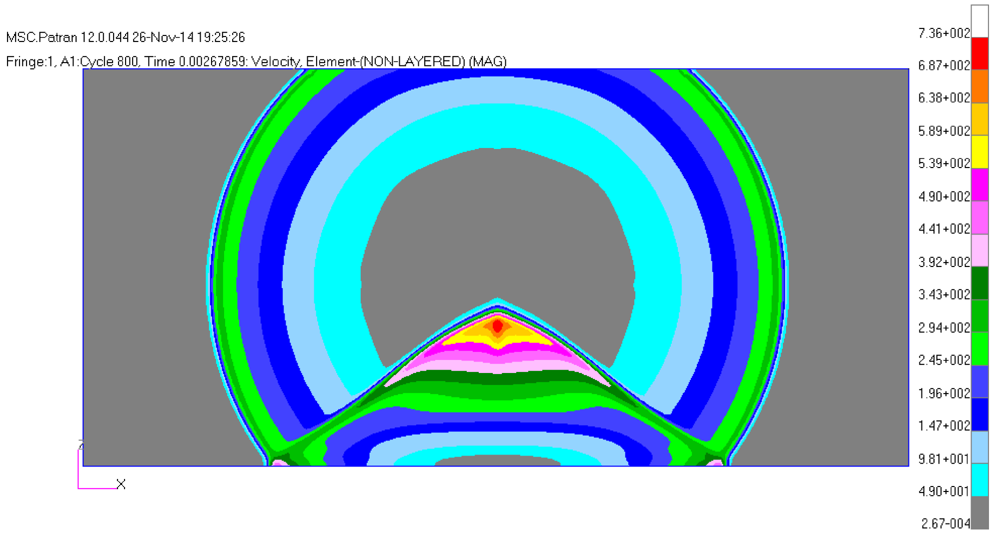
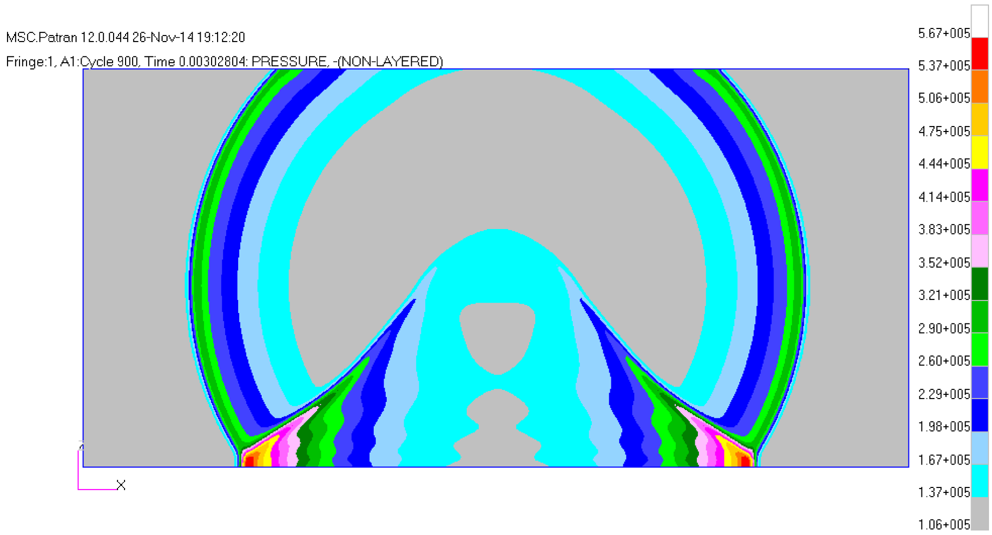
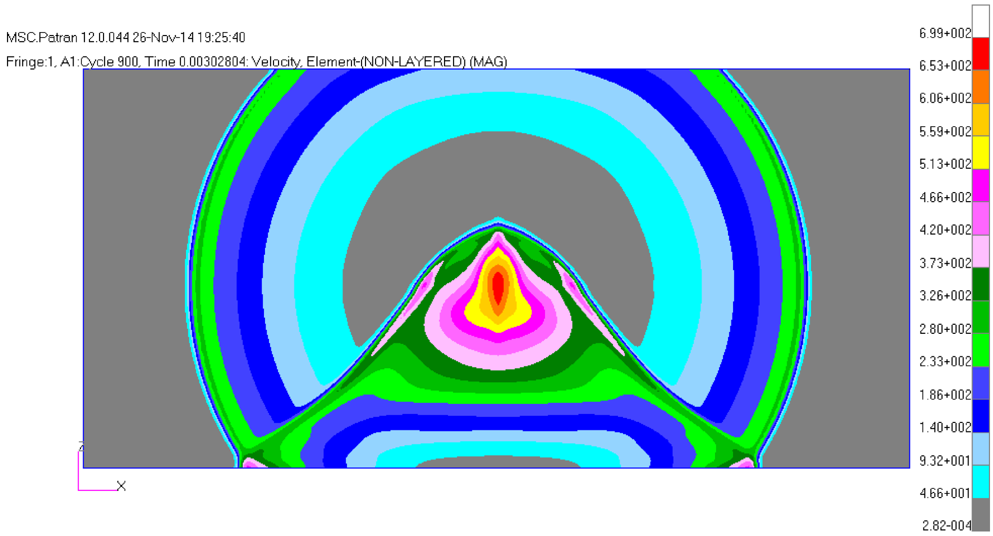


Fig. 9d-f

g) 0.00302 s



h) 0.00302 s



i) 0.00423 s

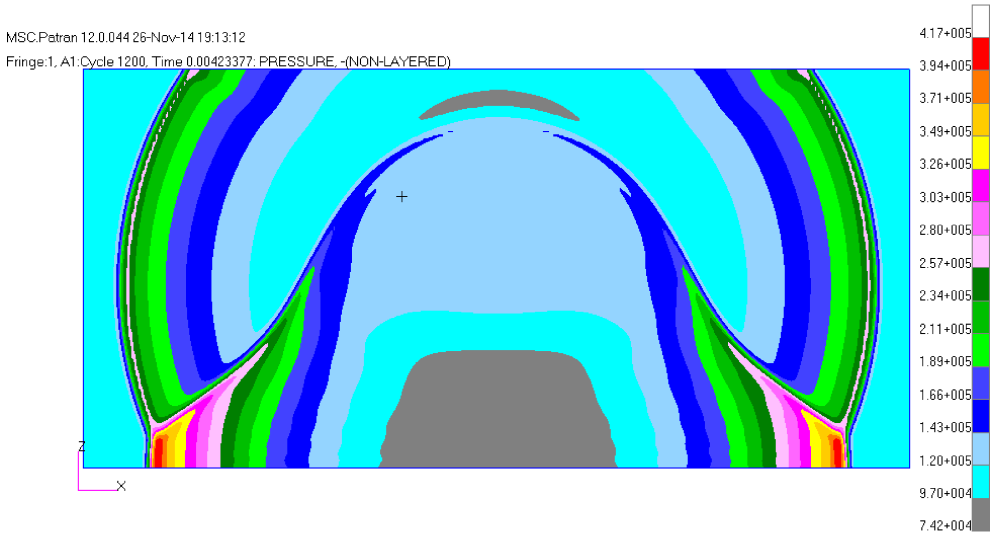


Fig. 9g-i

j) 0.00423 s

Fig. 9. Subsequent pressure wave phases freely spreading in the air. In figures 9g-j further pressure drop can be observed. It is worth noticing that the gas medium's velocity value increased. It causes the appearance of "peaks" of velocity and pressure, presented in Figs. 10 and 11

In Fig. 10 a pressure impulse obtained in a numerical way is presented.

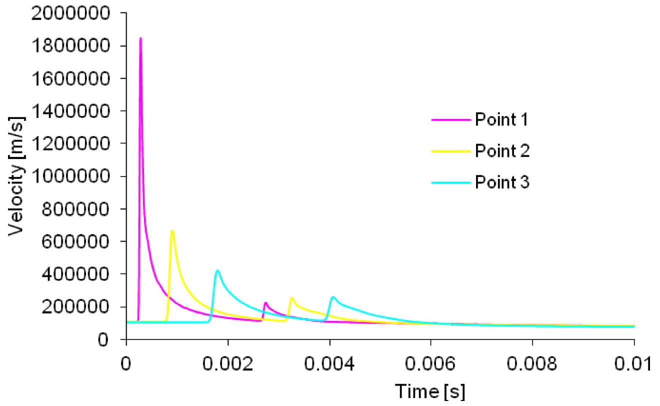


Fig. 10. Pressure impulse chart obtained by numerical calculations

In the same Euler cells in which pressure had been measured, the velocity of gas spreading was checked, which is presented in Fig. 11.

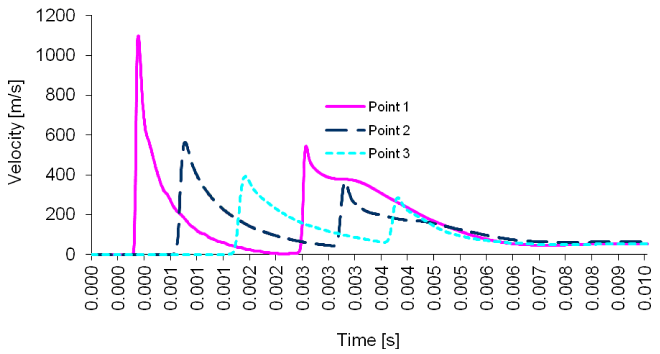


Fig. 11. Gas velocity chart obtained by numerical calculations

Examining pressure charts it is worth paying attention to the appearance of characteristic peaks which resulted from the reflection of the pressure wave front off a stiff ground. Velocity values are quite considerable and maximally equal to 1100 m/s, which means that the explosion practically caused the pressure to move with supersonic speed. It is caused by the fact that the wave spread in an almost ideal medium.

5. Conclusions

The source of the impact wave was a 1 kg TNT charge placed at the height of 1.6 m. The impulse load caused by the explosion is characterized by short duration and high amplitude. The duration of such a pressure impulse is one, or even two orders shorter than the impact duration and equals a couple of tenths of a millisecond. While conducting the experiment, free wave measurements were performed. The pressure measurement was performed using pressure sensors fixed to appropriate stands. The pressure measurement was repeated a few times in order to eliminate any haphazard measurement mistakes. For the case in study, high concordance between results. The research presented constitute a phase of a bigger study on the concept of protective structures exposed to an impact wave. Computer simulation techniques play an increasingly important role in analyzing collision- and explosion related phenomena. Such techniques allow for significant cost reduction and research effectiveness increase by providing data which cannot be measured experimentally. The calibration and verification of the calculation models based on experimental data and analytical research is of considerable importance. Numerical models checked for given structures states will allow conduct analysis for other states. In the discussed case MES model calibration was performed by measuring the free pressure on a realistic object. During research remembering the so-called scale effect is crucial.

REFERENCES

[1] W.E. Baker, *Explosions in Air*, University of Texas Press, Austin, 1973.

[2] R. Krzewiński and R. Rekrucki, *Construction Works with the Use of Explosives*, Polcen, Warszawa, 2005, (in Polish).

[3] S. Dobrociński, *Stability of Solutions of Structure Percussive Resistance*, AMW, Gdynia, 2000, (in Polish).

[4] *Sygn. Eng.*. 572/94, *Mine Works and Destruction*, WP General Staff, Warszawa, 1994 (in Polish).

[5] *MSC Dytran Example Problems*, MSC, Warszawa, 2002, (in Polish).

[6] К.П. Станюкович, "Физика взрыва", Наука, Москва, 1975.

[7] H.L. Brode, "Blast wave from a spherical charge". *Phys. Fluids* 2, 217–229 (1959).

[8] *MSC Dytran Example Problems*, MSC, Warszawa, 2005, (in Polish).

[9] R. Krzewiński, *Explosion Dynamics. Part I. Methods of Load Determination*, WAT Publishing House, Warszawa, 1982, (in Polish).

[10] G.I. Pokrowskij, *Osnowy Rasczeta Zariadow*, MON, Moskwa, 1945, (in Russian).

[11] *MSC Dytran Example Problem Manual. Version 3.0*, MSC, Warszawa, 1996.

[12] E. Włodarczyk, *Introduction to Explosion Mechanics*, PWN, Warszawa, 1994 (in Polish).