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STUDIES OF A MIXING PROCESS INDUCED BY A ROTATING MAGNETIC FIELD WITH THE APPLICATION OF MAGNETIC PARTICLES

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We demonstrate in this study that a rotating magnetic field (RMF) and spinning magnetic particles using this kind of magnetic field give rise to a motion mechanism capable of triggering mixing effect in liquids. In this experimental work two mixing mechanisms were used, magnetohydrodynamics due to the Lorentz force and mixing due to magnetic particles under the action of RMF, acted upon by the Kelvin force. To evidence these mechanisms, we report mixing time measured during the neutralization process (weak acid-strong base) under the action of RMF with and without magnetic particles. The efficiency of the mixing process was enhanced by a maximum of 6.5% and 12.8% owing to the application of RMF and the synergistic effect of magnetic field and magnetic particles, respectively.

Keywords: mixing process, alternating magnetic field, magnetic particles, mixing time, Navier–Stokes equation

1. INTRODUCTION

Mixing is a process that reduces inhomogeneity, including its form, scale, and intensity (Bałdyga and Pohorecki, 2013). Previous studies have reported that mixing occurring in chemical engineering processes is the subject of experimental investigations (Baldyga and Bourne, 1988; Baldyga and Bourne, 1992).

In recent years, there has been a growing interest in the use of the magnetic field (MF) in practical applications (Moffatt, 1965; Moffatt, 1990; Molokov et al., 2007). Studies have shown that an alternating magnetic field (AMF) can intensify chemical engineering processes instead of mechanical mixing (Hristov, 2002). This kind of magnetic field (MF) acted as a Hristov's non-intrusive mixing device that could be engineered to provide effective mixing processes (Moffatt, 1991).

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A large and growing body of literature has investigated the application of electromagnetic mixing in the casting of molten metals to improve the homogeneity of the cast products (Davidson, 1999). It has been suggested that the chaotic mixing induced by electromagnetically controlled thermal convection might be effectively applied to improve the homogenization of glass melt (Gopalakrishnan and Thess, 2010). Another research has indicated that the non-uniform MF also positively impacts the heat and mass transfer during the mixing and evaporation of liquid oxygen and nitrogen (Bao et al., 2019). Furthermore, Krakov (2020) found that various MFs, including a rotating field, significantly influenced the mixing of miscible liquids. Similarly, magnetohydrodynamic forces can be applied to mix magnetic and non-magnetic fluids in a micromixer device (Ergin et al., 2015). Chen and Zhang (2019) analyzed the data from 95 references and showed the progress in the development of magnetically actuated micromixers. Finally, it has been demonstrated that the application of MF allowed to improve the mixing process inside a Y-shaped micromixer (Nouri et al., 2017). Generally, magnetic mixing showed great potential for high mixing efficiency (Shanko et al., 2017).

The MF, as a manifestation of the electromagnetic field, is the space surrounding a magnet or a material in which the electric current flows. It is caused by moving electrical charges, and it is characterized by the movement through the charged particles on which the force is acting. The MFs can be divided into two main types: direct current (DC) MF (DCMF), and alternating current (AC) magnetic field (ACMF). DCMF does not change with time or change very slowly. Such fields do not have a frequency (MF vector is constant in time and space). An example of MF of the constant current is the static magnetic field (SMF). In contrast to DCMF, ACMF varies with frequency. An example of this type of MF is a pulsating magnetic field (PMF) characterized by an external MF vector that changes as a sine-wave with time at each point of space. The vector of this type of field pulsates with the frequency of the current flowing through the coil. Alternatively, the superposition of three 120° out of phase PMFs is causing a rotating MF (RMF). This field has a constant intensity over time while it changes its direction continuously at any point of the domain (RMF is variable in space). Such a field is created, for example, in the stator and rotor windings due to the supply of windings and makes possible the engine's rotation. The rotating magnetic field is characterized by the fact that its axis rotates relatively to the reference system (relative to the stator), and its return remains constant along the axis (Hajiani and Larachi, 2013; Hajiani and Larachi, 2014).

The RMF may be used to augment the process intensity instead of mechanical mixing (Rakoczy and Masiuk, 2011). For example, a mixer equipped with an RMF generator provided the intensification of the gas–liquid mass transfer process (Rakoczy et al., 2018). The RMF may be used to generate and control the hydrodynamic states for the magnetic disperse systems (Boroun and Larachi, 2016). The application of magnetic particles (or electrically conducting particles) as active micro-stirrers may be strongly influenced by the mixing process with external MF. The nature of mechanical interactions between magnetically excited particles and the liquid is dependent on the applied MF (Hajiani and Larachi, 2013). The movement of magnetic particles under the action of RMF has led to intensive magnetic stirring and improved the mixing process (Hristov, 1998; Reichert et al., 2004). The magnetic particles, functionalized with bio-specific surface coatings, are commonly applied in many biochemical assays for the capture, transport and mixing of the biological materials or targets (Gao, 2013).

Although many experimental successes have been achieved to improve the mixing process with external MFs, only a few experimental investigations on these processes under the RMF have been available so far, e.g., the mixing process in the batch system (Rakoczy and Masiuk, 2011) and the mixing process in the flow system (Rakoczy et al., 2014). It is partially due to the difficulties of direct measurement of parameters describing the mixing process. Additionally, the application of magnetic particles as well as the interactions between these particles and the used magnetic field make the quantitative analysis more difficult.

To provide a more in-depth insight into the mixing problems under the action of RMF, some mathematical modelling with the application of the fluid mechanic relationships or equations have been made, in which the non-dimensional numbers from magnetohydrodynamics (MHD) were taken into consideration.

The main aim of the current study was to determine the influence of the RMF and magnetic particles on the mixing process. In the present report, the application of RMF on the mixing process is described using non-dimensional numbers based on fluid mechanics equations. The first part of the paper describes the theoretical background of the mixing process under the RMF action. In particular, the synergic effect of the RMF and magnetic particles in the mixing process is discussed. The second part presents the used experimental procedures. Finally, the third part contains the discussion associated with the influence of RMF and magnetic particles on the mixing process, using mathematical relationships with non-dimensional numbers.

2. THEORETICAL

The fluid motion may be described by the Navier–Stokes equation

$$\frac{\partial \rho \boldsymbol{w}}{\partial \tau} + \nabla \cdot (\rho \boldsymbol{w}\boldsymbol{w}) = \rho \boldsymbol{f} - \nabla p + \mu \Delta \boldsymbol{w} \tag{1}$$

The flow under the action of RMF may be defined by including the Lorentz force $\boldsymbol{f}_L$ in the Navier–Stokes equation

$$\frac{\partial \rho \boldsymbol{w}}{\partial \tau} + \nabla \cdot (\rho \boldsymbol{w}\boldsymbol{w}) = \rho \boldsymbol{f} - \nabla p + \mu \Delta \boldsymbol{w} + \boldsymbol{f}_L \tag{2}$$

For the constant density ρ (incompressible fluids) the terms ∇p and $\rho \boldsymbol{f}$ can be combined if $\boldsymbol{f}$ is expressed as the gradient of a potential. Given that the appearing dynamic pressure is small, the term ∇p may be omitted. In this case, Eq. (2) can be rewritten as follows

$$\frac{\partial \boldsymbol{w}}{\partial \tau} + (\boldsymbol{w} \cdot \nabla)\, \boldsymbol{w} = \nu \Delta \boldsymbol{w} + \frac{\boldsymbol{f}_L}{\rho} \tag{3}$$

The Lorentz force, $\boldsymbol{f}_L$, may be defined as follows

$$\boldsymbol{f}_L = \boldsymbol{J} \times \boldsymbol{B} \tag{4}$$

The current density $\boldsymbol{J}$ is determined by means of the Ohm's law

$$\boldsymbol{J} = \sigma_e \left[\boldsymbol{E} + (\boldsymbol{w} \times \boldsymbol{B})\right] \tag{5}$$

Some simplifications of Ohm's law are allowed for magnetic stirring using the RMF. For simplicity, we suppose that the field rotation is low and in the fluid under the action of RMF, no electrostatic field is observed. Then, the electric field $\boldsymbol{E}$ may be omitted in Eq. (5) and the Lorentz force for this case may be defined as follows (Davidson, 2001)

$$\boldsymbol{f}_L = \left[\sigma_e\, (\boldsymbol{w} \times \boldsymbol{B})\right] \times \boldsymbol{B} \tag{6}$$

Then, Eq. (3) may be rewritten in the following form

$$\frac{\partial \boldsymbol{w}}{\partial \tau} + (\boldsymbol{w} \cdot \nabla)\, \boldsymbol{w} = \nu \Delta \boldsymbol{w} + \frac{\left[\sigma_e\, (\boldsymbol{w} \times \boldsymbol{B})\right] \times \boldsymbol{B}}{\rho} \tag{7}$$

Eq. (7) may be expressed in the symbolic form as follows

$$\left(\frac{w_0}{\tau_0}\right)\left[\frac{\partial \boldsymbol{w}^*}{\partial \tau^*}\right] + \left(\frac{w_0^2}{l_0}\right)\left[(\boldsymbol{w}^* \cdot \nabla^*)\, \boldsymbol{w}^*\right] = \left(\frac{\nu_0 w_0}{l_0^2}\right)\left[\nu^* \Delta^* \boldsymbol{w}^*\right] + \left(\frac{\sigma_{e_0} w_0 B_0^2}{\rho_0}\right)\left[\frac{\left[\sigma_e^*\, (\boldsymbol{w}^* \times \boldsymbol{B}^*)\right] \times \boldsymbol{B}^*}{\rho^*}\right] \tag{8}$$

Eq. (8) may be scaled against the term $\left(\frac{v_0 w_0}{l_0^2}\right)$. This mathematical operation leads to the following dimensionless governing equation

$$\left(\frac{l_0^2}{\tau_0 v_0}\right)\left[\frac{\partial w^*}{\partial \tau^*}\right] + \left(\frac{w_0 l_0}{v_0}\right)\left[\left(\boldsymbol{w}^* \cdot \nabla^*\right)\boldsymbol{w}^*\right] = \left[\nu^* \Delta^* \boldsymbol{w}^*\right] + \left(\frac{\sigma_{e_0} B_0^2 l_0^2}{\rho_0 v_0}\right)\left[\frac{\left[\sigma_e^*\left(\boldsymbol{w}^* \times \boldsymbol{B}^*\right)\right] \times \boldsymbol{B}^*}{\rho^*}\right] \tag{9}$$

It should be noticed that Eq. (9) may be rewritten with the application of non-dimensional numbers (Rakoczy & Masiuk, 2011)

$$\Theta^{-1}\left[\frac{\partial \boldsymbol{w}^*}{\partial \tau^*}\right] + \mathrm{Re}\left[\left(\boldsymbol{w}^* \cdot \nabla^*\right)\boldsymbol{w}^*\right] = \left[\nu^* \Delta^* \boldsymbol{w}^*\right] + Q\left[\frac{\left[\sigma_e^*\left(\boldsymbol{w}^* \times \boldsymbol{B}^*\right)\right] \times \boldsymbol{B}^*}{\rho^*}\right] \tag{10}$$

The application of magnetic particles is associated with the use of different mathematical descriptions. In this case, the following equation can be applied

$$\frac{\partial \boldsymbol{w}}{\partial \tau} + \left(\boldsymbol{w} \cdot \nabla\right)\boldsymbol{w} = \nu \Delta \boldsymbol{w} + \frac{\boldsymbol{f}_m}{\rho} \tag{11}$$

The Kelvin force density, $\boldsymbol{f}_m$, may be defined in the form Boulware et al., 2010; Lange, 2002)

$$\boldsymbol{f}_m = \mu_0 \left(\boldsymbol{M} \cdot \nabla\right)\boldsymbol{H} \tag{12}$$

In the linear portion of the Langevin function, volumetric magnetic susceptibility, χ, is the ratio of the magnetization vector, $\boldsymbol{M}$, to the applied field vector, $\boldsymbol{H}$

$$\chi = \frac{\boldsymbol{M}}{\boldsymbol{H}} \tag{13}$$

By substituting for the magnetization vector, $\boldsymbol{M}$, Eq. (12) can be rewritten as follows

$$\boldsymbol{f}_m = \mu_0 \chi \left(\boldsymbol{H} \cdot \nabla\right)\boldsymbol{H} \tag{14}$$

Using the vector identity

$$\left(\boldsymbol{H} \cdot \nabla\right)\boldsymbol{H} = \frac{1}{2}\nabla\left(\boldsymbol{H} \cdot \boldsymbol{H}\right) - \boldsymbol{H} \times \left(\nabla \times \boldsymbol{H}\right) \tag{15}$$

with the current-free magnetostatic Ampére's law

$$\nabla \times \boldsymbol{H} = 0 \tag{16}$$

allows one to express Eq. (14) as follows

$$\boldsymbol{f}_m = \frac{1}{2}\mu_0 \chi \nabla \boldsymbol{H}^2 \tag{17}$$

With a constant temperature, the relative permeability, μ_T, is constant and may be expressed as

$$\mu_r = \mu_0 \left(1 + \chi\right) \tag{18}$$

This relative permeability is the ratio between the magnetic flux density, $\boldsymbol{B}$, and applied magnetic field, $\boldsymbol{H}$

$$\mu_r = \frac{\boldsymbol{B}}{\boldsymbol{H}} \tag{19}$$

Taking into consideration Eq. (18) and Eq. (19), the Kelvin force density may be expressed as follows

$$\boldsymbol{f_m} = \frac{1}{2\mu_0}\frac{\chi}{\left(1 + \chi\right)^2}\nabla \boldsymbol{B}^2 \tag{20}$$

Then Eq. (11) is given in the following form

$$\frac{\partial w}{\partial \tau} + (w \cdot \nabla)\, w = \nu \Delta w + \frac{1}{\rho}\frac{1}{2\mu_0}\frac{\chi}{(1+\chi)^2}\nabla \boldsymbol{B}^2 \tag{21}$$

Eq. (21) may be rewritten in the symbolic form as follows

$$\left(\frac{w_0}{\tau_0}\right)\left[\frac{\partial \boldsymbol{w}^*}{\partial \tau^*}\right] + \left(\frac{w_0^2}{l_0}\right)\left[(\boldsymbol{w}^* \cdot \nabla^*)\, \boldsymbol{w}^*\right] = \\ = \left(\frac{\nu_0 w_0}{l_0^2}\right)\left[\nu^* \Delta^* \boldsymbol{w}^*\right] + \left(\frac{B_0^2}{2\rho_0 \mu_0 l_0}\frac{\chi_0}{(1+\chi_0)^2}\right)\left[\frac{1}{\rho^*}\frac{1}{\mu_0^*}\frac{\chi^*}{(1+\chi^*)^2}\nabla^* \boldsymbol{B}^* \boldsymbol{B}^*\right] \tag{22}$$

When Eq. (22) is scaled against the term $\left(\frac{\nu_0 w_0}{l_0^2}\right)$, we obtain the following non-dimensional relationship

$$\left(\frac{l_0^2}{\tau_0 \nu_0}\right)\left[\frac{\partial \boldsymbol{w}^*}{\partial \tau^*}\right] + \left(\frac{w_0 l_0}{\nu_0}\right)\left[w^* \mathrm{grad}^* \boldsymbol{w}^*\right] = \\ = \left[\nu^* \Delta^* w^*\right] + \left(\frac{B_0^2 l_0}{2\rho_0 \mu_0 \nu_0 w_0}\frac{\chi_0}{(1+\chi_0)^2}\right)\left[\frac{1}{\rho^*}\frac{1}{2\mu_0^*}\frac{\chi^*}{(1+\chi^*)^2}\nabla \boldsymbol{B}^* \boldsymbol{B}^*\right] \tag{23}$$

Notice the presence of a core region of the fluid flow, rotating as a rigid body with angular velocity (Moffatt, 1965)

$$\Omega_l = \frac{B_0^2}{2\rho_0 \mu_0 \nu_0} \tag{24}$$

Under the action of the RMF, the particle experiences the torque, and it rotates with angular velocity (Moffatt, 1990)

$$\Omega_p = \frac{B_0^2 \alpha^{(i)}}{2\rho_0 \mu_0 \nu_0} \tag{25}$$

The parameter $\alpha^{(i)}$ is dependent on the following term

$$\lambda = \left(\frac{1}{2} a^2 \mu_0 \sigma_p \omega_{\mathrm{RMF}}\right)^{\frac{1}{2}} \tag{26}$$

It should be noticed that when $\lambda \ll 1$ the parameter $\alpha^{(i)}$ is expressed as $\alpha^{(i)} \approx 0.0667\lambda^2$.

The last term in Eq. (23) may be defined as follows

$$\left(\frac{B_0^2 l_0}{2\rho_0 \mu_0 \nu_0 w_0}\frac{\chi_0}{(1+\chi_0)^2}\right) \Rightarrow \left(\frac{B_0^2 l_0}{2\rho_0 \mu_0 \nu_0 w_0}\frac{\chi_0}{(1+\chi_0)^2}\right)\left(\frac{\alpha^{(i)}}{\alpha^{(i)}}\right) \Rightarrow \\ \Rightarrow \left(\frac{B_0^2 \alpha^{(i)}}{2\rho_0 \mu_0 \nu_0}\frac{l_0}{w_0}\frac{1}{\alpha^{(i)}}\frac{\chi_0}{(1+\chi_0)^2}\right) \Rightarrow \left(\Omega_p \frac{l_0}{w_0}\frac{1}{\alpha^{(i)}}\frac{\chi_0}{(1+\chi_0)^2}\right) \tag{27}$$

Introducing to Eq. (27) the term $\left(\frac{l_0^2}{l_0^2}\right)\left(\frac{\nu_0}{\nu_0}\right)$, we obtain the following expressions

$$\Omega_p \frac{l_0}{w_0}\frac{1}{\alpha^{(i)}}\frac{\chi^*}{(1+\chi^*)^2}\left(\frac{l_0^2}{l_0^2}\right)\left(\frac{\nu_0}{\nu_0}\right) \Rightarrow \left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\Omega_p l_0^2}{\nu_0}\right)\left(\frac{\nu_0}{w_0 l_0}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right) \tag{28}$$

It follows that the relation (28) may be defined as follows

$$\left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\Omega_p l_0^2}{\nu_0}\right)\left(\frac{\nu_0}{w_0 l_0}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right) \Rightarrow \left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right)\mathrm{Re}_p \mathrm{Re}^{-1} \tag{29}$$

Taking into account the definition of the non-dimensional numbers and the relation (29), Eq. (23) may be rewritten in the form

$$\Theta^{-1}\left[\frac{\partial \boldsymbol{w}^*}{\partial \tau^*}\right] + \mathrm{Re}\left[\boldsymbol{w}^* \mathrm{grad}^* \boldsymbol{w}^*\right] = \\ = \left[\nu^* \Delta^* \boldsymbol{w}^*\right] + \left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right)\mathrm{Re}_p \mathrm{Re}^{-1}\left[\frac{1}{\rho^*}\frac{1}{2\mu_0^*}\frac{\chi^*}{(1+\chi^*)^2}\nabla \boldsymbol{B}^* \boldsymbol{B}^*\right] \tag{30}$$

Based on Eq. (10), we obtain the following general relationship describing the influence of the RMF on the mixing process (Sedov, 1959)

$$f(\Theta, \mathrm{Re}, Q) = 0 \tag{31a}$$

Based on the above relationship, the useful correlation for the mixing process under the action of RMF may be introduced. In this experimental work, the dimensionless mixing time number was chosen as an independent number. The dimensionless Reynolds and Chandrasekhar numbers were selected as dependent numbers. This approach allowed to define the following correlation (Ruzicka, 2008)

$$\Theta = f(\mathrm{Re}, Q) \quad \Rightarrow \quad \Theta = p_{01}\mathrm{Re}^{p_{02}} Q^{p_{03}} \tag{31b}$$

The influence of RMF on the mixing process with the application of magnetic particles is defined by the following relation (this relation is based on Eq. (30))

$$f\left(\Theta, \mathrm{Re}, \left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right)\mathrm{Re}_p \mathrm{Re}^{-1}\right) = 0 \tag{32a}$$

The following correlation for the mixing process with the synergistic effect of magnetic particles and the RMF may be proposed

$$\Theta = f\left(\mathrm{Re}, \left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right)\mathrm{Re}_p \mathrm{Re}^{-1}\right) \quad \Rightarrow \\ \Rightarrow \quad \Theta = p_{11}\mathrm{Re}^{p_{12}}\left[\left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right)\mathrm{Re}_p \mathrm{Re}^{-1}\right]^{p_{13}} \tag{32b}$$

It should be noticed that the influence of RMF on the mixing process may be defined with the following dimensionless numbers

$$\Theta = \frac{\tau_0 \nu_0}{l_0^2} \quad \Rightarrow \quad \Theta = \frac{t_{\mathrm{mix}}\nu}{D^2} \tag{33}$$

$$Q = \left(\frac{\sigma_{e_0} B_0^2 l_0^2}{\rho_0 \nu_0}\right) \quad \Rightarrow \quad Q = \frac{\sigma_e B^2 D^2}{\rho \nu} \tag{34}$$

$$\mathrm{Re} = \left(\frac{w_0 l_0}{\nu_0}\right) \quad \Rightarrow \quad \mathrm{Re} = \frac{w_{\mathrm{RMF}} D}{\nu} \tag{35}$$

$$\mathrm{Re}_{\mathrm{mod}} = \left(\left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right)\mathrm{Re}_p\right) \tag{36}$$

The velocity of fluid under the action of RMF, w_{RMF}, may be defined as (Spitzer, 1999)

$$w_{\mathrm{RMF}} = BD\sqrt{\frac{\sigma_e \omega_{\mathrm{RMF}}}{\rho}} \tag{37}$$

3. EXPERIMENTAL

3.1. Experimental set-up

All the experimental investigations were carried out using the set-up shown in Fig. 1.

The apparatus consisted of the housing (1) and the cylindrical conduit (2) placed inside the RMF generator (3). In the case of these experimental investigations, the RMF was generated using the three-phase stator of the squirrel cage induction motor. The inverter filter (4) and inverter (5) were used to adjust the power supply frequency and the RMF frequency in the range between 5 and 50 Hz. Container (6) was placed inside the RMF generator (3). The experimental set-up was also equipped with pH electrodes (7 and 8), feeder (9), and multifunctional computer meters (10 and 11). These meters were connected with the computer (12) equipped with the software that was used to record pH changes. To receive the excess heat generated during the process, the RMF generator (3) was placed in the cooling jacket connected with the outer circulation loop. The main elements of this loop were the three-way valve (13) and the heat exchanger (14). The silicone oil was used to cool the RMF generator windings.

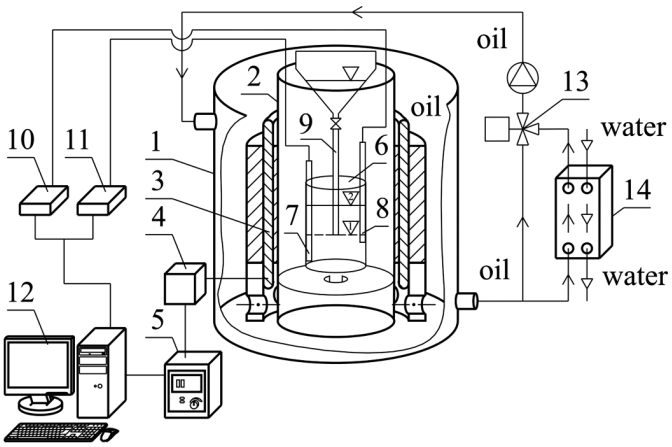

Fig. 1. Sketch of experimental set-up: 1 – housing; 2 – cylindrical conduit; 3 – RMF generator; 4 – inverter filter; 5 – inverter; 6 – container; 7, 8 – pH electrodes; 9 – feeder; 10, 11 – multifunction computer meters; 12 – computer; 13 – three-way valve; 14 – heat exchanger

3.2. Mixing time

The mixing time was applied for the description of the mixing effect of the alternating magnetic field. The mixing time is the time measured from the instant of addition of tracer until the vessel contents have reached a specified degree of uniformity when the system is said to be mixed (Harnby et al., 1985). It should be noticed that pH measurements may be applied for the characterization of the mixing process in the magnetically applied mixer (Poulsen and Iversen, 1997). Tan et al. showed that the dynamic responses of the pH value of the applied indicator might be used to define the mixing time (Tan et al., 2011).

To identify the mixing time, the neutralization reaction (titration of a weak acid with a strong base) was used (Sołoducha et al., 2020). The defined volume of 0.1 M CH_3COOH (100 mL) was placed in the glass container (6). The volume (100 mL) of base (0.1 M NaOH) was introduced into the analyte. The time of the base injection into the acid was equal to 10 s. The pH changes were measured as a function of time using the pH electrodes (7 and 8 in Fig. 1) and the multifunctional computer meters (10 and 11 in Fig. 1). The pH signals were recorded digitally every 1 s. The pH electrodes were temperature compensated. It was assumed that the mixing process was regarded as complete when the equivalence point of the neutralization

process was reached (Sołoducha et al., 2020). For the analyzed neutralization process of the weak acid with the strong base, the mixing process was regarded as complete when the pH solution was equal to 8.7.

The obtained pH curves are described by means of the following relationship

$$\mathrm{pH}(t) = p_1 + \frac{p_2 - p_1}{1 + \left(\frac{t}{p_3}\right)^{p_4}} \tag{38}$$

The above Eq. (38) may be rewritten in the following form

$$t = \sqrt[p_4]{\frac{(p_2 - p_1) - (\mathrm{pH} - p_1)}{(\mathrm{pH} - p_1)\left(\frac{1}{p_3}\right)^{p_4}}} \quad \Rightarrow \quad t = \sqrt[p_4]{\frac{(p_2 - p_1) - (8.7 - p_1)}{(8.7 - p_1)\left(\frac{1}{p_3}\right)^{p_4}}} \tag{39}$$

Eq. (39) allows to obtain the time at which the equivalence point of the solution (pH = 8.7) is reached. This time may be used to calculate the dimensionless mixing time number.

3.3. Characteristics of applied RMF

The applied magnetic field was characterized by the measurements of magnetic induction. The experiments were performed according to the procedure of Rakoczy and Masiuk (2011). The dependence of the obtained values of the averaged values of the magnetic induction on the frequency of RMF is shown in Fig. 2. It was found that the scatter of averaged values of magnetic induction as a function of RMF frequency can be defined as follows

$$B_{\mathrm{avg}} = 19.25 - 13.77 \exp\left(-\frac{f}{6.49}\right) \tag{40}$$

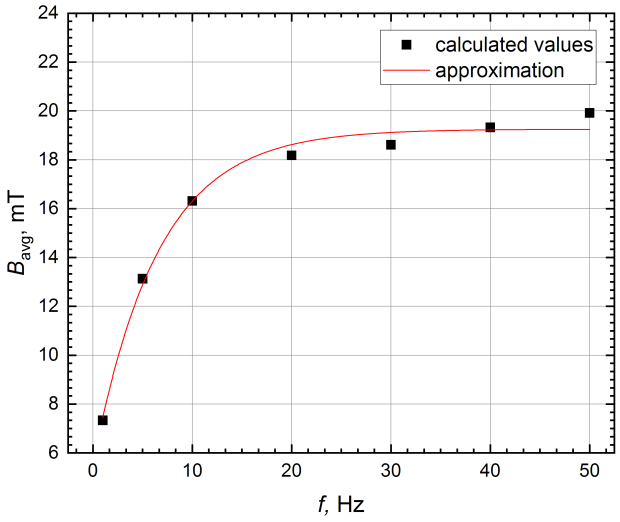

Fig. 2. Relation between the averaged values of magnetic induction (calculated values) and the RMF frequency

3.4. Magnetic particles

The mixing process was also carried out with the application of magnetic particles. The particles of Fe_3O_4 were also introduced into the solutions and suspended using RMF. The mass of magnetic particles used in the experimental procedure was equal to 2.5 g. The linear dimension of these particles was equal to 282 μm (in the experimental work, the sieve diameter was treated as the linear dimension of magnetic particles).

4. RESULTS AND DISCUSSION

A typical change in pH during the experiment without the application of RMF is shown in Fig. 3.

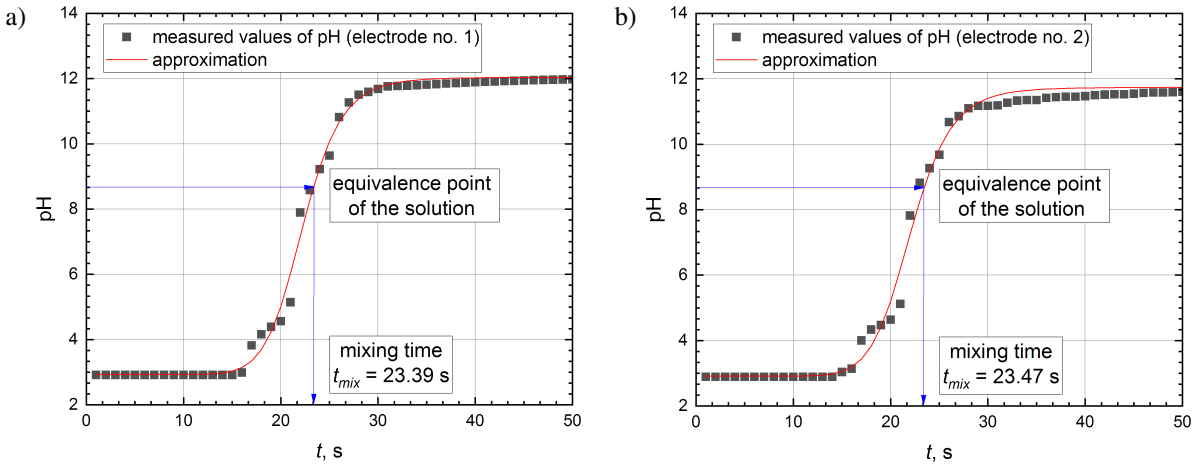

Fig. 3. Variation of pH values versus time for the experiment without the application of RMF ($f = 0$ Hz) for pH electrode no. 1 (a) and pH electrode no. 2 (b). The measured values of pH are marked as points. The approximation of pH values using Eq. (38) is represented as the solid line

The obtained change of pH in time for the experiments without the application of RMF is described by the following relation (solid line in Fig. 3a)

$$\mathrm{pH}(t) = 12.03 + \frac{2.93 - 12.03}{1 + \left(\dfrac{t}{22.28}\right)^{11.23}} \tag{41a}$$

and (solid line in Fig. 3b)

$$\mathrm{pH}(t) = 11.74 + \frac{2.91 - 11.74}{1 + \left(\dfrac{t}{22.08}\right)^{10.56}} \tag{41b}$$

Based on Eq. (39), the equivalence time for the neutralization process without the application of RMF and magnetic particles was calculated. This calculation was done for the measurements obtained from two pH electrodes. Then, the obtained values of the equivalence time were averaged and the results of this calculation may be treated as the mixing time (in this case, this time was equal to 23.43 s).

The mixing process under the action of RMF can be described using the following relation (see Eq. (31b))

$$\Theta = f(\mathrm{Re}, Q) \quad \Rightarrow \quad \Theta = p_{01}\mathrm{Re}^{p_{02}}Q^{p_{03}} \quad \Rightarrow \quad \Theta = p_1(\mathrm{Re}\,Q)^{p_2} \tag{42}$$

In Eq. (42) it is assumed that the influence of the dimensionless Re and Q numbers on the mixing process under the action of RMF is the same. Therefore, it is assumed that $p_{02} = p_{03} = p_2$.

It should be noticed that the product of the dimensionless Reynolds and Chandrasekhar numbers is the dimensionless Taylor number (Molokov et al., 2007). This number may be expressed as follows

$$\mathrm{Ta} = \frac{\textit{centrifugal force}}{\textit{viscous force}} \quad \Rightarrow \quad \mathrm{Ta} = Q\mathrm{Re} \quad \Rightarrow \quad \mathrm{Ta} = \left(\frac{\sigma_{e_0} B_0^2 l_0^2}{\rho_0 \nu_0}\right)\left(\frac{w_0 l_0}{\nu_0}\right) \quad \Rightarrow$$

$$\Rightarrow \quad \mathrm{Ta} = \left(\frac{\sigma_e B^2 D^2}{\rho \nu}\right)\left(\frac{w_{\mathrm{RMF}} D}{\nu}\right) \tag{43}$$

The diameter of container is equal to 0.15 m. The density and kinematic viscosity of the mixed liquid (solution) is equal to 1004 $kg \cdot m^{-3}$ and $1 \cdot 10^{-6}$ $m^2 \cdot s^{-1}$. The electrical conductivity of this liquid is equal to 0.433 $A^2 \cdot s^3 \cdot kg^{-1} \cdot m^{-3}$.

Therefore, the relation (42) may be rewritten in the following form

$$\Theta = p_1 (\mathrm{Ta})^{p_2} \tag{44}$$

The effect of the mixing process under the action of RMF without the application of magnetic particles can be defined by the dimensionless mixing time number, Θ, proportional to the mixing time, t_{mix}. The experimental results obtained in this work are graphically illustrated in the log–log scale in Fig. 4.

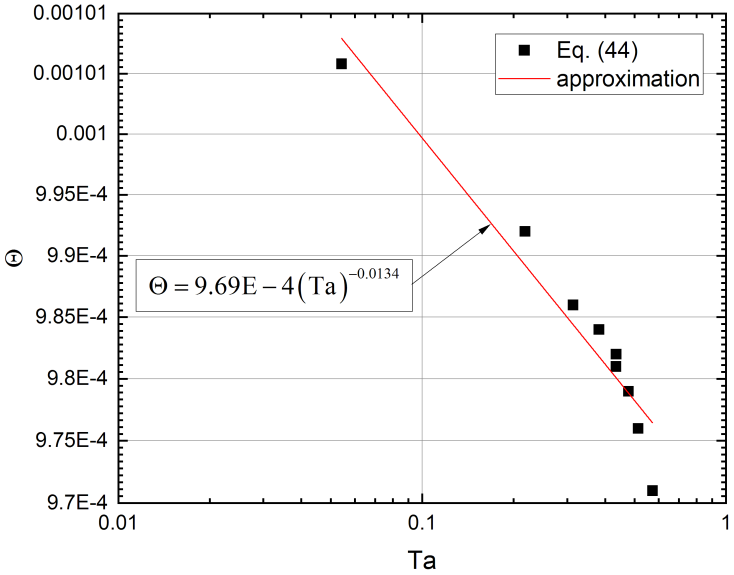

Fig. 4. Dimensionless mixing characteristics for the action of RMF without magnetic particles

The influence of magnetic particles on the mixing process with the RMF application may be defined using the following relationship (based on Eq. (32b))

$$\begin{aligned}
&\Theta = f\left(\mathrm{Re}, \left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right)\mathrm{Re}_p\mathrm{Re}^{-1}\right) \quad \Rightarrow \\
&\Theta = p_{11}\mathrm{Re}^{p_{12}}\left[\left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right)\mathrm{Re}_p\mathrm{Re}^{-1}\right]^{p_{13}} \quad \Rightarrow \\
&\Theta = p_3\mathrm{Re}^{p_4}\left[\left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right)\mathrm{Re}_p\mathrm{Re}^{-1}\right]^{p_4} \quad \Rightarrow \\
&\Theta = p_3\left[\left(\frac{1}{\alpha^{(i)}}\right)\left(\frac{\chi_0}{(1+\chi_0)^2}\right)\mathrm{Re}_p\right]^{p_4} \quad \Rightarrow \quad \Theta = p_3\,(\mathrm{Re}_{\mathrm{mod}})^{p_4}
\end{aligned} \tag{45}$$

It is assumed that the influence of dimensionless Re and the term in the square bracket in Eq. (45) on the mixing process with the application of magnetic particles under the action of RMF is the same. This simplifying assumption allows the following notation $p_{12} = p_{13} = p_4$.

To describe the mixing process with magnetic particles, the modified Reynolds number was introduced. The dimensionless parameter $\alpha^{(i)}$ is dependent on the parameter λ that was calculated as follows (see Eq. (26))

$$\lambda = \left(\frac{1}{2}a^2\mu_0\sigma_p\omega_{\mathrm{RMF}}\right)^{\frac{1}{2}} \quad \Rightarrow \quad \lambda = \left(\frac{1}{2}d_p^2\mu_m\sigma_p\omega_{\mathrm{RMF}}\right)^{\frac{1}{2}} \tag{46}$$

In the case of these experimental results, the parameter λ is much smaller than 1, therefore the parameter $\alpha^{(i)}$ is equal to $0.0667\lambda^2$. The electrical conductivity and volumetric magnetic susceptibility of Fe_3O_4 are about $\sigma_p = 2.5 \cdot 10^{-4}$ A^2·s^3· kg^{-1}· m^{-3} and $\chi_0 = 6$, respectively (Sołoducha et al., 2020).

To establish the effect of magnetic particles under the RMF action on the mixing process, the experimental data obtained in this work are presented in the log–log system in Fig. 5.

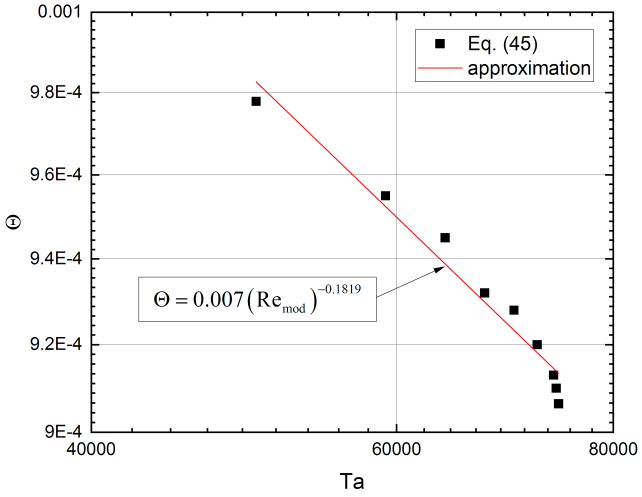

Fig. 5. Dimensionless mixing characteristics for the action of RMF with magnetic particles

To assess the efficiency of the mixing process with the RMF application, the following enhancement factor was used

$$E = \left| \frac{\Theta_{f=\text{var}} - \Theta_{f=0\,\text{Hz}}}{\Theta_{f=0\,\text{Hz}}} \right| 100\% \tag{47}$$

This factor can be expressed as a relative improvement of the dimensionless mixing time in the presence of RMF. Moreover, the E–factor may be also used to describe the effect of applying magnetic particles in the presence of RMF.

Figure 6 shows the E-factor values obtained for the analyzed cases (with and without magnetic particles under RMF) as a function of the frequency of the current flowing through the coil, f, and the averaged values of magnetic induction, B_{avg}.

As can be observed, the E-factor values in the presence of RMF with the application of magnetic particles are smaller compared to using only RMF. From the results presented in Figure 6, it is clear that the magnetic particles under the action of RMF are responsible for the reduction of mixing time. The effect of RMF with the application of the magnetic particles is more visible for $f > 30$ Hz ($B_{\text{avg}} > 19.11$ mT). For this case, the values of the E–factor are the smallest.

The current study found that the RMF may be used to intensify the mixing process instead of mechanical mixing (Rakoczy and Masiuk, 2010). This finding supports previous research into these non-instructive electromagnetic stirrers area which links the mixing process and the time varied fields, e.g., RMF (Molokov et al., 2007). The RMF induces eddy currents in the mixed liquids. These currents create their magnetic fields that in co-operation with the external field create small dynamos mixing the liquids. This effect is called the dynamo concept or the dynamo effect (Hristov, 2010). Some authors have speculated that these dynamos may be converted into micro-stirrers that mix the liquid at the micro-level (Olivier et al., 2014). It should also be emphasized that the mixing process-induced using RMF can be used to enhance liquid transport properties (Hristov, 2002).

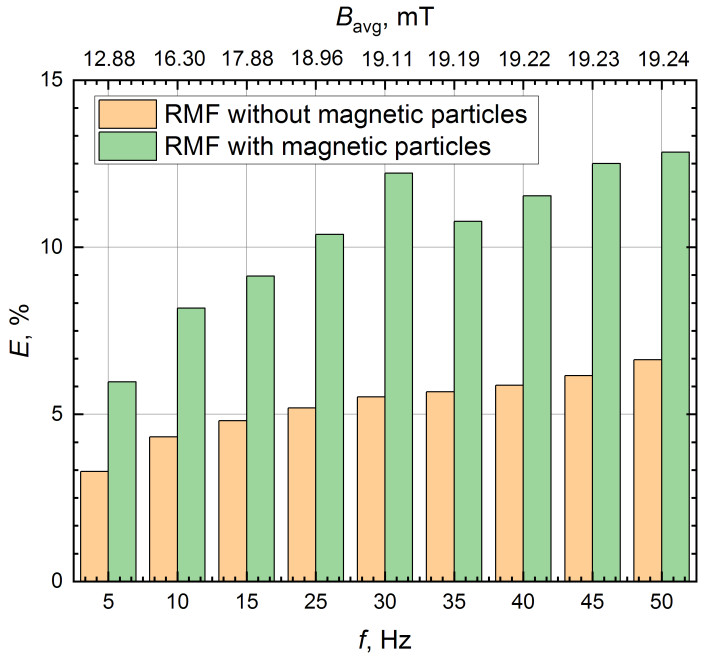

Fig. 6. The E–factor (see Eq. (47)) for the mixing process with and without magnetic particles under RMF

The results of this study indicate that the mixing process may be improved by considering the synergic effect of the applied RMF and magnetic particles (in this case Fe_3O_4). Magnetically assisted fluidization (MAF) is a technique combining fluidization with an externally applied magnetic field (Penchev and Hristov, 1990). The external magnetic field can be used to control the powder bed (Lu and Li, 2000). This approach causes magnetically driven fluidization (MDF) (Hristov, 2002). The particle motions inside the fluidization volume are caused by the magnetic field driving particles through a relatively stagnant fluid. The application of RMF improves the hydrodynamic conditions inside the fluidization volume (Hao et al., 2008). Moreover, the use of magnetic particles under the RMF action enhanced magnetic stirring and improved mixing process (Hausmann et al., 2004; Reichert et al., 2004). The magnetic particles could rotate with the external RMF in a liquid-solid particle system, thus they acted as small agitators (Hristov, 2007).

5. CONCLUSIONS

The present study makes several noteworthy contributions to the mixing process theory under the action of the alternating magnetic field. Based on the theoretical considerations, the mixing process with magnetic particles was expressed as the mixing characteristic with the modified dimensionless Reynolds number. It was also shown that the RMF with and without the application of magnetic particles could be used to enhance the mixing process. The obtained experimental results were mathematically described with the proposed relations based on equations of fluid mechanics. Our research has shown that the hydrodynamic state in the mixed liquid involving RMF can be described with equations containing dimensionless numbers considering magnetic field effects. It should also be emphasized that this research extended current knowledge on the application of synergic effect of the RMF and magnetic particles in the mixing process. This study has shown that the efficiency of the mixing process was enhanced by a maximum of 6.5% owing to the application of RMF. This process can be intensified by means of magnetic particles (in this case Fe_3O_4). The research results showed that the mixing process efficiency with the application of the synergistic effect of magnetic particles under the action of RMF can be improved by 12.8%.

SYMBOLS

a	linear dimension of particle, m
$\boldsymbol{B}$	magnetic field vector, kg/(A·s^2)
d	diameter, m
D	diameter of container, m
$\boldsymbol{E}$	electric field vector, V/m
E	enhancement factor, %
f	frequency of electrical current or rotating magnetic field, s^{-1}
$\boldsymbol{f}$	force, N
f_L	Lorentz force, N
f_m	Kelvin force, N
$\boldsymbol{H}$	magnetic field strength vector, A/m
l	characteristic measurement, m
$\boldsymbol{J}$	electrical current density vector, A/m^2
$\boldsymbol{M}$	magnetization vector, A/m
l	linear dimension, m
p	hydrodynamic pressure, N/m^2
p_{01-03}	parameters in Eq. (31b)
p_{11-13}	parameters in Eq. (32b)
p_{1-4}	constants in Eq. (38) and Eq. (39)
t_{mix}	mixing time, s
$\boldsymbol{w}$	velocity vector, m/s
w_{RMF}	velocity of fluid under action of RMF m·s^{-1}·

Greek symbols

α	dimensionless parameter
λ	dimensionless parameter
χ	volumetric magnetic susceptibility
μ	dynamic viscosity, kg·m^{-1}·s^{-1}
μ_0	magnetic permeability, kg·m·A^{-2}s^{-2}
ν	kinematic viscosity, m^{2}s^{-1}
ρ	density, kg m^{-3}
σ_e	electrical conductivity of liquid, A^2·s^3·kg^{-1}·m^{-3}
σ_p	electrical conductivity of particle, A^2·s^3·kg^{-1}·m^{-3}
τ	time, s
ω_{RMF}	angular velocity of rotating magnetic field, rad·s^{-1}
Ω	angular velocity, rad·s^{-1}

Dimensionless numbers

Re	dimensionless Reynolds number
Ta	dimensionless Taylor number
Re_{mod}	dimensionless modified Reynolds number
Q	dimensionless Chandrasekhar number
Θ	dimensionless mixing time number

Superscripts

$*$	dimensionless value

Subscripts

avg	averaged

l	liquid
mod	modified
mix	mixing
p	particle
0	reference value
RMF	value related to the rotating magnetic field

Mathematical operator

$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ Laplace operator or Laplacian (It is a differential operator given by the divergence of the gradient of a function on Euclidean space. In a Cartesian coordinate system, this operator is defined by the sum of second partial derivatives of the function with respect to each independent variable.)

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REFERENCES

Baldyga J., Bourne J.R., 1988. Calculation of micromixing in inhomogenous stirred tank reactors. *Chem. Eng. Res. Des.*, 66(1), 33–38.

Baldyga J., Bourne J.R., 1992. Interactions between mixing on various scales in stirred tank reactors. *Chem. Eng. Sci.*, 47, 1839–1848. DOI: 10.1016/0009-2509(92)80302-S.

Bałdyga J., Pohorecki R., 2013. Editorial. 14th European Conference on Mixing. *Chem. Eng. Res. Des.,* 91(11), 2071–2072). DOI: 10.1016/j.cherd.2013.10.021.

Bao S.R., Zhang R.P., Rong Y., Zhi X.Q., Qiu L.M., 2019. Interferometric study of the heat and mass transfer during the mixing and evaporation of liquid oxygen and nitrogen under non-uniform magnetic field. *Int. J. Heat Mass Transfer*, 136, 10–19. DOI: 10.1016/j.ijheatmasstransfer.2019.02.044.

Boroun S., Larachi F., 2016. Role of magnetic nanoparticles in mixing, transport phenomena and reaction engineering – challenges and opportunities. *Curr. Opin. Chem. Eng.*, 13, 91–99. DOI: 10.1016/j.coche.2016.08.011.

Boulware J.C., Ban H., Jensen, S., Wassom S., 2010. Influence of geometry on liquid oxygen magnetohydrodynamics. *Exp. Therm Fluid Sci.*, 34, 1182–1193. DOI: 10.1016/j.expthermflusci.2010.04.007.

Chen X., Zhang L., 2019. A review on micromicers acuated with magnetic nanomaterials. *Microchim Acta*, 184, 3639–3649. DOI: 10.1007/s00604-017-2462-2.

Davidson P.A., 1999. Magnetohydrodynamics in materials processing. *Annu. Rev. Fluid Mech.*, 31, 273–300. DOI: 10.1146/annurev.fluid.31.1.273.

Davidson P.A., 2001. An introduction to magnetohydrodynamics. Cambridge Uniwversity Press. DOI: 10.1017/CBO9780511626333.

Ergin F.G., Watz B.B., Erglis K., Cēbers A., 2015. Time-resolved velocity measurements in a magnetic micromixer. *Exp. Therm Fluid Sci.*, 67. DOI: 10.1016/j.expthermflusci.2015.02.019.

Gao Y., 2013. *Active mixing and catching using magnetic particles*. Phd Thesis. Technische Universiteit Eindhoven. DOI: 10.6100/IR759475.

Gopalakrishnan S., Thess A., 2010. Chaotic mixing in electromagnetically controlled thermal convection of glass melt. *Chem. Eng. Sci.*, 65, 5309–5319. DOI: 10.1016/j.ces.2010.07.008.

Hajiani P., Larachi F., 2014. Magnetic-field assisted mixing of liquids using magnetic nanoparticles. *Chem. Eng. Process.*, 84, 31–37. DOI: 10.1016/j.cep.2014.03.012.

Hajiani P, Larachi F., 2013. Remotely excited magnetic nanoparticles and gas–liquid mass transfer in Taylor flow regime. *Chem. Eng. Sci.*, 93, 257–265. DOI: 10.1016/j.ces.2013.01.052.

Hao Z., Zhu Q., Jiang Z., Li H., 2008. Fluidization characteristics of aerogel Co/Al_2O_3 catalyst in a magnetic fluidized bed and its application to CH_4-CO_2 reforming. *Powder Technol.*, 183, 46–52. DOI: 10.1016/j.powtec.2007.11.015.

Harnby N., Edwards M.F., Nienow A.W., 1985. *Mixing in the process industries*. Butterworth-Heinemann. DOI: 10.1016/b978-0-7506-3760-2.x5020-3.

Hausmann R., Reichert C., Franzreb M., Höll W.H., 2004. Liquid-phase mass transfer of magnetic ion exchangers in magnetically influenced fluidized beds: II. AC fields. *React. Funct. Polym.*, 60, 17–26. DOI: 10.1016/j.reactfunctpolym.2004.02.007.

Hristov J., 2002. Magnetic field assisted fluidization – a unified aproach Part 1. Fundamentals and relevant hydrodynamics of gas-fluidized beds (batch solids mode). *Rev. Chem. Eng.*, 18, 295–512. DOI: 10.1515/REVCE.2002.18.4-5.295.

Hristov J., 2007. Magnetic field assisted fluidization-Dimensional analysis addressing the physical basis. *China Particuology*, 5, 103–110. DOI: 10.1016/j.cpart.2007.03.002.

Hristov J., 2010. Magnetic field assisted fluidization – A unified approach. Part 8. Mass transfer: Magnetically assisted bioprocesses. *Rev. Chem. Eng.*, 26, 55–128. DOI: 10.1515/REVCE.2010.006.

Hristov J.Y., 1998. Fluidization of ferromagnetic particles in a magnetic field Part 2: Field effects on preliminarily gas fluidized bed. *Powder Technol.*, 97, 35–44. DOI: 10.1016/S0032-5910(97)03392-5.

Krakov M.S., 2020. Mixing of miscible magnetic and non-magnetic fluids with a rotating magnetic field. *J. Magn. Magn. Mater.*, 498. DOI: 10.1016/j.jmmm.2019.166186.

Lange A., 2002. Kelvin force in a layer of magnetic fluid. *J. Magn. Magn. Mater.*, 241, 327–329. DOI: 10.1016/S0304-8853(01)01368-3.

Lu X., Li H., 2000. Fluidization of $CaCO_3$ and Fe_2O_3 particle mixtures in a transverse rotating magnetic field. *Powder Technol.*, 107, 66–78. DOI: 10.1016/S0032-5910(99)00092-3.

Moffatt H.K., 1965. On fluid flow induced by a rotating magnetic field. *J. Fluid Mech.*, 22, 521–528. DOI: 10.1017/S0022112065000940.

Moffatt H.K., 1990. On the behaviour of a suspension of conducting particles subjected to a time-periodic magnetic field. *J. Fluid Mech.*, 218, 509–529. DOI: 10.1017/S0022112090001094.

Moffatt H.K., 1991. Electromagnetic stirring. *Phys. Fluids A*, 3, 1336–1343. DOI: 10.1063/1.858062.

Molokov S., Moreau R., Moffat H.K., 2007. *Magnetohydrodynamics. Historical evolution and trends*. Springer Science+Business Media B.V. DOI: 10.1007/978-1-4020-4833-3.

Nouri D., Zabihi-Hesari A., Passandideh-Fard M., 2017. Rapid mixing in micromixers using magnetic field. *Sens. Actuators, A*, 255, 79–86. DOI: 10.1016/j.sna.2017.01.005.

Olivier G., Pouya H., Faďçal L., 2014. Magnetically induced agitation in liquid–liquid–magnetic nanoparticle emulsions: Potential for process intensification. *AIChE J.*, 60, 1176–1181. DOI: 10.1002/AIC.14331.

Penchev I.P., Hristov J.Y., 1990. Fluidization of beds of ferromagnetic particles in a transverse magnetic field. *Powder Technol.*, 62, 1–11. DOI: 10.1016/0032-5910(90)80016-R.

Poulsen B.R., Iversen J.J.L., 1997. Mixing determinations in reactor vessels using linear buffers. *Chem. Eng. Sci.*, 52, 979–984. DOI: 10.1016/S0009-2509(96)00466-6.

Rakoczy R., Masiuk S., 2010. Influence of transverse rotating magnetic field on enhancement of solid dissolution process. *AIChE J.*, 56, 1416–1433. DOI: 10.1002/aic.12097.

Rakoczy R., Konopacki M., Lechowska J., Bubnowska M., Hürter A., Kordas M., Fijałkowski K., 2018. Gas to liquid mass transfer in mixing system with application of rotating magnetic field. *Chem. Eng. Process.*, 130, 11–18. DOI: 10.1016/j.cep.2018.05.013.

Rakoczy R., Kordas M., Story G., Konopacki M., 2014. The characterization of the residence time distribution in a magnetic mixer by means of the information entropy. *Chem. Eng. Sci.*, 105, 191–197. DOI: 10.1016/j.ces.2013.10.014.

Rakoczy R., Masiuk S., 2011. Studies of a mixing process induced by a transverse rotating magnetic field. *Chem. Eng. Sci.*, 66, 2298–2308. DOI: 10.1016/j.ces.2011.02.021.

Reichert C., Hoell W.H., Franzreb M., 2004. Mass transfer enhancement in stirred suspensions of magnetic particles by the use of alternating magnetic fields. *Powder Technol.*, 145, 131–138. DOI: 10.1016/j.powtec.2004.06.010.

Ruzicka M.C., 2008. On dimensionless numbers. *Chem. Eng. Res. Des.*, 86, 835–868. DOI: 10.1016/j.cherd.2008.03.007.

Sedov L.I., 1959. *Similarity and dimensional methods in mechanics*. Academic Press Inc. DOI: 10.1016/C2013-0-08173-X.

Shanko E-S., van de Burgt Y., Anderosn P.D., den Toonder J.M.J., 2017. Microfluidic magnetic mixing at low Reynolds numbers and in stagnant fluids. *Micromachines*, 10, 731. DOI: 10.3390/mi10110731.

Sołoducha D., Borowski T., Kordas M., Konopacki M., Rakoczy R., 2020. Studies of neutralization reaction induced by rotating magnetic field. *Chem. Pap.*, 74, 3517–3526. DOI: 10.1007/s11696-020-01174-6.

Spitzer K.H., 1999. Application of rotating magnetic fields in Czochralski crystal growth. *Prog. Cryst. Growth Charact. Mater.*, 38, 59–71. DOI: 10.1016/S0960-8974(99)00008-X.

Tan R.K., Eberhard W., Büchs J., 2011. Measurement and characterization of mixing time in shake flasks. *Chem. Eng. Sci.*, 66, 440–447. DOI: 10.1016/j.ces.2010.11.001.

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