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Local modelling of the disturbing potential with the use of quasigeoid[1] heights, gravimetric data and digital terrain model

In the paper the model of the disturbing potential based on the lower-degree harmonic spherical polynomials and the local density model of topographic masses has been proposed. Topographic masses are represented by DTM. The model parameters are fixed by the use of quasigeoid heights as well as a dense network of gravity points. Preliminary analyses of the model's robustness of gravimetric data errors have also been included.

INTRODUCTION

Interpolation of quasigeoid ζ heights based on a network of points with known normal and geometric (GPS – ellipsoidal) heights, is probably the simplest method of satellite levelling. Because the density of points with known ζ is not sufficient enough the method is not useful (especially in the mountains). Connecting the quasigeoid height and the gravity disturbances δg with disturbing potential *T*, we can replace quasigeoid height interpolation problem onto constructing a proper model of the disturbing potential problem [3, 6]:

$$\begin{aligned} &\zeta + \frac{(W_o - W_o)}{\gamma} + \frac{\bar{\gamma}_H - \bar{\gamma}_h}{\gamma} H = \frac{T}{\gamma} \\ &\delta g = -T_z \end{aligned} \tag{1}$$

where: H is normal height, $\bar{\gamma}_H$ is mean normal gravity value between ellipsoid and telluroide, $\bar{\gamma}_h$ is mean normal gravity value between ellipsoid and terrain surface, γ is normal gravity value on ellipsoid, W_o is value of the gravity potential on geoid, U_o is value of the normal potential on ellipsoid, T_z is derivative of the disturbing potential in the *Z*-direction, in local horizontal co-ordinate system, with the *Z*-axis directed towards Zenith.

[1] Quasigeoid of Molodensky.

Quantity of data, which can be taken to build this model is much higher because, except of points with known ζ, we can include points with measured gravity.

There are doubts if the gravity data (gravity anomaly, gravity disturbances) is precise enough for such kind of models. With geometrical interpretation of gravity shifts using vertical gradient (~0.3 mGal/m) (1 mGal = 10^{-5} ms^{-2}) we can obtain:

- accuracy of the gravity measuring (~0.02 mGal) ~6.6 cm,
- real accuracy of the gravity anomaly (~1 mGal) ~3.3 m.

In case when the disturbing potential is being modelled, this interpretation seems to be too simple because both the gravity and the gravity potential depend on more factors than the position of point. I can illustrate this by a simple example. On Fig.1 it is showed a homogeneous sphere and the point *P* on the sphere surface.

If we want to change gravity by 0.3 mGal, we must move the point *P* along the radius by 1 m. It will make change of the gravity potential value by 9.82 m^2 s^{-2}.

The same change of the gravity (0.3 mGal) at the point *P* we can obtain by putting mass point *mp* under this point in the *r* distance. However the change of the gravity potential, produced by the mass *mp* is just 3.2×10^{-4} m^2 s^{-2}. In the height it is concerned as a few hundreds of a part of millimetre.

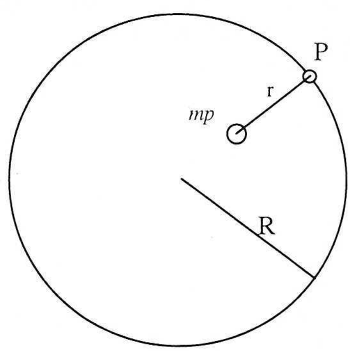

Fig 1. Homogeneous sphere with the radius $R = 6371$ km and the constant $GM = 3986005 \times 10^8$ m^3 s^{-2}, point with mass $mp = 5 \times 10^8$ kg in the distance $r = 105$ m from point *P*

This example shows that it is not easy to define how the errors of the gravity anomaly have influence on the disturbing potential. Everything depends on the used gravity potential model. In models which use the law of universal gravitation (i.e. mass points model of Bjerhammar-Aronow or density of topographical masses model proposed in the paper) these errors may only slightly disturb the calculated potential.

The model of disturbing potential

The proposed model consists of two components:

- harmonic spherical polynomials of lower degree representing main component of the influences of topographical masses and irregular mass distribution within the Earth.

- based on the digital terrain model featured as parallelepiped blocks (DTM), the density model which represents remainder influences.

The DTM grid contains area, where results of measurements have been given.

Generally, this model has the following form:

$$T(X_P, Y_P, Z_P) = W(X_P, Y_P, Z_P) + G\sum_{j=1}^{n}\sum_{i=1}^{m_j}\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{\varrho_j(x_i, y_i, z_i)}{\sqrt{(x_i-X_P)^2+(y_i-Y_P)^2+(z_i-Z_P)^2}}dxdydz \tag{2}$$

$$T_Z(X_P, Y_P, Z_P) = W_Z(X_P, Y_P, Z_P) + G\sum_{j=1}^{n}\sum_{i=1}^{m_j}\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{\varrho_j(x_i, y_i, z_i)(z_i-Z_P)}{\left(\sqrt{(x_i-X_P)^2+(y_i-Y_P)^2+(z_i-Z_P)^2}\right)^3}dxdydz \tag{3}$$

where: n – quantity of DTM zones (there will be composed different linear density model for each zone $\varrho_j(x_i, y_i, z_i)$, m – quantity of parallelepiped blocks (DTM) in zone j, $x_{i1}, x_{i2}, y_{i1}, y_{i2}, z_{i2}, z_{i2}$ – coordinates defining singular, parallelepiped DTM block, $P(X_P, Y_P, Z_P)$ – point where the potential is calculated, $T_Z(X_P, Y_P, Z_P)$ – derivative $T(X_P, Y_P, Z_P)$ in the Z direction, $W(x_P, y_P, z_P) = \sum_{k=1}^{l} f_k(x_p, y_p, z_p)b_k$ – harmonic spherical polynomials of lower degree with unknown coefficients b_k [1], $\varrho_j(x_i, y_i, z_i) = a_{0j}+a_{1j}x_i+a_{2j}y_i+a_{3j}z_i$ – linear density model with unknown coefficients $a_{0j}, a_{1j}, a_{2j}, a_{3j}$.

Application of the method demands solving of integrals presented in the equations (2) and (3). The solutions are presented in the appendix.

The model described by equations (2) and (3) allows us to write observation equations for disturbing potential and its derivative in a form:

$$\begin{aligned} T+v &= T(X_P, Y_P, Z_P) \\ \delta g+v &= -T_Z(X_P, Y_P, Z_P) \end{aligned} \tag{4}$$

where: v is residuum, T and δg are observations (gravity disturbance can be obtained from the fundamental equation of physical geodesy [3], using known, approximate quasigeoid model), $T(X_P, Y_P, Z_P)$ and $T_Z((X_P, Y_P, Z_P)$ are given by equations (2) and (3) models with unknown coefficients b_k and $a_{0j}, a_{1j}, a_{2j}, a_{3j}$.

Unknown coefficients have been determined by least squares method using covariance matrix.

Numerical example

The data prepared for the example was generated on the base of DTM grid covering the square of 100 × 100 km with constant density of topographical masses (2750 kg/m^3). Test network was located at central part of the square. Within this region (test network) in different depth there were put 7 mascons. The DTM represents the area of the Sudety Mountains. On the basis of this data model the observable values were calculated: T at 9 points, δg at 121 points as well as test values of the disturbing potential T_{test}, which will be calculated from the proposed model which bases on 121 points (Fig. 2).

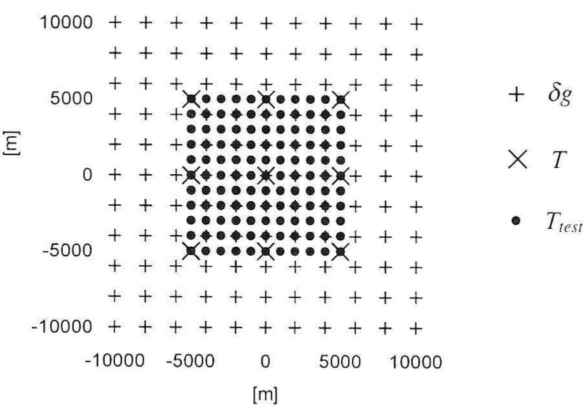

Fig. 2. Location of points with „observed" disturbing potential values T, „observed" the gravity disturbances δg and calculated, testing, disturbing potential values T_{test}

The following maps present gravity anomaly and disturbing potential within the area of the test network.

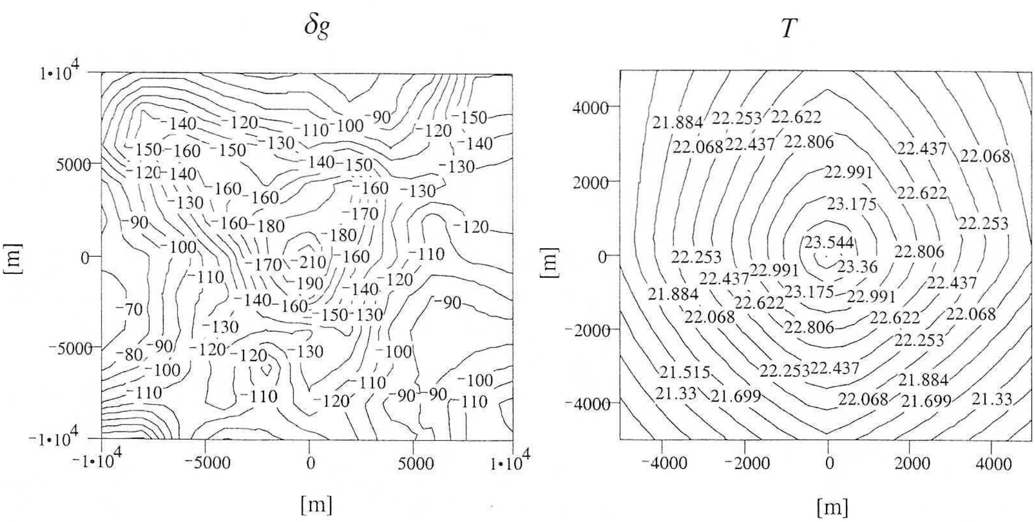

Fig. 3. Maps of gravity disturbances δg (mGal) and disturbing potential T (based on 9 points) (m^2s^{-2}), taken as observed values

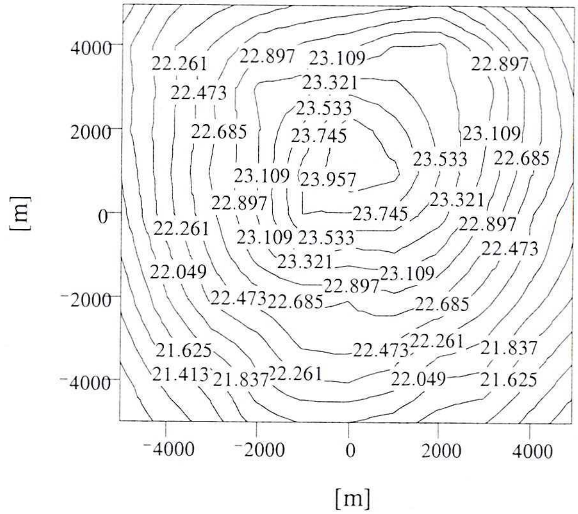

Fig. 4. Map of calculated (theoretical) disturbing potential values (T_{test}) on the base of 121 testing points (m^2s^{-2})

2025 DTM blocks with the site of 500 m, used in disturbing potential model have been divided into 25 zones (Fig. 5).

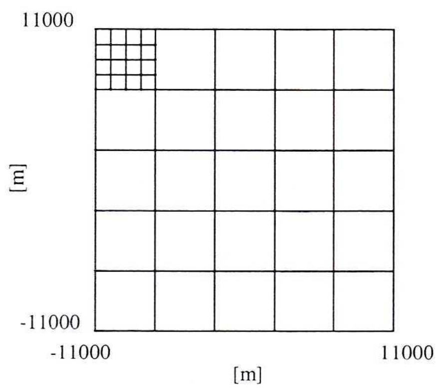

Fig. 5. The DTM division into zones and schematic marked blocs at the one zone

Based on this data the model (2, 3) was formed.

The following maps show the values calculated from the model of the disturbing potential T_m as well as differences $dT = T_{test} - T_m$ between theoretical and calculated values:

- standard deviation stdev(dT) = 0.07 m^2s^{-2}
- maximum value max(dT) = 0.222 m^2s^{-2}
- minimum value min(dT) = –0.139 m^2s^{-2}

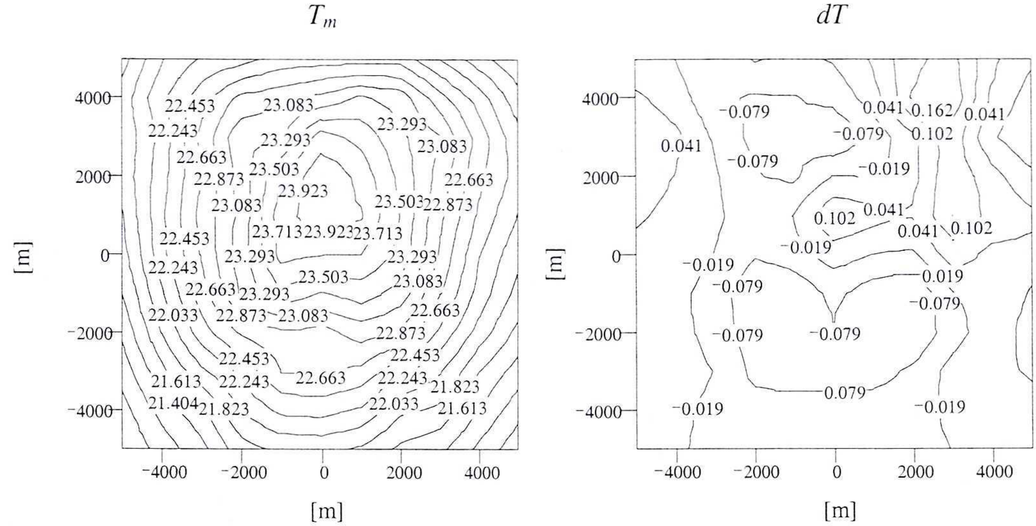

Fig. 6. Maps of the values calculated from the model of the disturbing potential T_m and the differences $dT = T_{test} - T_m$ (m^2s^{-2})

CONCLUSIONS

In my opinion – very high accuracy of the disturbing potential calculated from the model may appear because of simplicity of the data model. Final verification of the proposed model should be done using real data.

Looking back at the required accuracy of the gravity anomaly I would like to add that random disturbing of its values by ± (0÷1) mGal does not make any significant change of calculated disturbing potential at testing points.

Appendix

Integrals solutions:
Auxiliary marks:

$$x_i - X_P = \Delta x$$
$$y_i - Y_P = \Delta y$$
$$z_i - Z_P = \Delta z$$
$$\sqrt{(x_i - X_p)^2 + (y_i - Y_P)^2 + (z_i - Z_P)^2} = r$$

a) Disturbing potential

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{0j}+a_{1j}x_i+a_{2j}y_i+a_{3j}z_i}{r}\,dxdydz =$$

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{0j}}{r}\,dxdydz + \qquad (1)$$

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{1j}x_i}{r}\,dxdydz + \qquad (2)$$

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{1j}y_i}{r}\,dxdydz + \qquad (3)$$

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{1j}z_i}{r}\,dxdydz + \qquad (4)$$

(1) see Forsberg, Tscherning C. (1997)
(2) the author's solution

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{1j}x_i}{r}\,dxdydz = a_{1j}\Big[\Big\|\Big|\frac{1}{3}\Big(\Delta z\frac{\Delta z^2+\Delta x^2}{2}\ln|\Delta y+r| + \Delta y\frac{\Delta y^2+\Delta x^2}{2}\ln|\Delta z+r| + \Delta z\Delta y r\Big)\Big|_{x_{i1}}^{x_{i2}}\Big|_{y_{i1}}^{y_{i2}}\Big|_{z_{i1}}^{z_{i2}} + \left|\frac{\Delta x^2}{3}\int_{y_{i1}}^{y_{i2}}\int_{z_{i1}}^{z_{i2}}\frac{1}{r}\,dzdy\right|_{x_{i1}}^{x_{i2}} + X_P\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}}\frac{1}{r}\,dxdydz\Big] \qquad (5)$$

$$\int_{y_{i1}}^{y_{i2}}\int_{z_{i1}}^{z_{i2}}\frac{1}{r}\,dzdy = \Big\|\Delta z\ln|\Delta y+r| + \Delta y\ln|\Delta z+r| - \Delta x\arctan\left[\frac{\Delta y\Delta z}{\Delta x r}\right]\Big|_{z_{i2}}^{z_{i2}}\Big|_{y_{i2}}^{y_{i2}} \qquad (6)$$

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}}\frac{1}{r}\,dxdydz \qquad \text{see Forsberg, Tscherning C. (1997)}$$

Solutions of the integrals (3) and (4) may be built on the basis of solutions of the integral (2), by exchanging proper variables.

b) Gravity anomaly

$$-\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{(a_{0j}+a_{1j}x_i+a_{2j}y_i+a_{3j}z_i)\Delta z}{r^3}\,dxdydz =$$

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{0j}\Delta z}{r^3}\, dxdydz + \quad (7)$$

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{1j}x\Delta z}{r^3}\, dxdydz + \quad (8)$$

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{2j}y\Delta z}{r^3}\, dxdydz + \quad (9)$$

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{3j}z\Delta z}{r^3}\, dxdydz + \quad (10)$$

(7) see Nagy (1996)
(8) the author's solution

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{1j}x\Delta z}{r^3}\, dxdydz =$$

$$-a_{1,j}\left[\frac{1}{2}\left|\left|\left| \Delta yr+(\Delta x^2+\Delta z^2)\ln|\Delta y+r| \right|_{x_{i1}}^{x_{i2}}\right|_{y_{i1}}^{y_{i2}}\right|_{z_{i1}}^{z_{i2}} + \left| X_P \int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{1}{r}\, dxdy \right|_{z_{i1}}^{z_{i2}}\right] \quad (11)$$

$\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{1}{r}\, dxdy$ exchange proper variable at solution (6)

Solutions of the integral (9) may be built on the basis of the solutions of integral (8), exchanging proper variables.

(10) the author's solution

$$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{a_{3j}z\Delta z}{r^3}\, dxdydz = a_{3,j}\left[\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{1}{r}\, dxdydz - \left| z\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{1}{r}\, dxdy \right|_{z_{i1}}^{z_{i2}}\right]$$

$\int_{z_{i1}}^{z_{i2}}\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{1}{r}\, dxdydz$ see Forsberg, Tscherning C. (1997)

$\int_{y_{i1}}^{y_{i2}}\int_{x_{i1}}^{x_{i2}} \frac{1}{r}\, dxdy$ exchange proper variable at solution (6)

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Lokalny model potencjału zakłócającego wyznaczony na podstawie odstępów quasigeoidy danych grawimetrycznych i cyfrowego modelu terenu

S t r e s z c z e n i e

W pracy zaproponowany jest model potencjału zakłócającego oparty na wielomianach harmonicznych niskiego stopnia oraz lokalnym modelu gęstości mas topograficznych reprezentowanych przez NMT. Do wyznaczenia parametrów modelu proponowane jest wykorzystanie danych wysokości quasigeoidy i gęstej sieci punktów z wyznaczonymi zakłóceniami grawimetrycznymi. Przeprowadzona jest także wstępna analiza odporności modelu na niedokładności przyjętych do obliczeń zakłóceń grawimetrycznych.

Марек Троянович

Местная модель потенциала возмущений определённого на основе расстояний квазигеода гравиметрических данных и цифровой модели местности

Р е з ю м е

В работе предложена модель потенциала возмущений, основана на гармонических полиномах низкой степени, а также на местной модели плотности топографическиц масс, представляемых цифровой моделью местности. Для определения параметров модели предлагается использовать высоты квазигеоида, и густой сети пунктов с определёнными гравиметрическими возмущениями. Проведен тоже предварительный анализ устойчивости модели к неточностям принятых для вычислений гравиметрических возмущений.