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Analytical description of Gauss-Krüger projection in spheroidal and spheroidal-and-spherical versions

The paper discusses Gauss-Krüger projection as a projection consisting of spheroidal and spherical parts. For the spheroidal part distribution of projection deformations has been determined and besides, its impact on the metric structure of the initial Gauss-Krüger projection, as an entirety, has been estimated.

1. *Analytical description of Gauss-Krüger projection in the fundamental version*

Gauss-Krüger projection [1] is a conformal projection of the entire surface of an oblate ellipsoid of revolution

$$\vec{r} = \vec{r}\,(B, L) = \left[\frac{a\cos B\cos l}{\sqrt{1-k^2\sin^2 B}}, \frac{a\cos B\sin l}{\sqrt{1-k^2\sin^2 B}}, \frac{a(1-k^2)\sin B}{\sqrt{1-k^2\sin^2 B}}\right], \tag{1}$$

$$(B, L) \in \omega = \{(B, L) : B \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2}\right\rangle, L \in \langle -\pi, \pi)\},$$

$$l = L - L_0,\ L_0 = \text{const},\ k^2 = \frac{a^2 - b^2}{a^2},$$

a and $b < a$ semiaxes of the ellipsoid surface (1),

into a plane

$$\left.\begin{aligned} F = x_e + iy_e &= \int_0^{\vartheta} M\,(t)\,dt = \omega\,(\vartheta) \\ z_e = q_e + il_e &= \int_0^{\vartheta} \frac{M\,(t)}{N\,(t)\cos t}\,dt = \psi(\vartheta) \end{aligned}\right., \quad \vartheta = \vartheta_1 + i\vartheta_2, \tag{2}$$

where:

$$M = M\,(B) = \frac{a\,(1-k^2)}{(\sqrt{1-k^2\sin^2 B})^3}, \qquad N = N\,(B) = \frac{a}{\sqrt{1-k^2\sin^2 B}},$$

B – geodetic ellipsoidal latitude

q_e – geodetic isometric ellipsoidal latitude determined by the formula

$$q_e = \ln\left[\left(\frac{1-k\sin B}{1+k\sin B}\right)^{\frac{k}{2}} \tan\left(\frac{\pi}{4}+\frac{B}{2}\right)\right], \tag{3}$$

(q_e, l_e) – isometric parameters of the point (B, L) of the ellipsoid surface (1).

Transition from (q_e, l_e) co-ordinates of the point (B, L) of the ellipsoidal surface (1) to x_e, y_e co-ordinates of a plane (2) is performed based on the distribution of a complex function (2) into the real part Re $\omega\,(\psi^{-1}(z))$ and the imaginary part Im $\omega\,(\psi^{-1}(z))$.

First, we transform the relation (3) to the form:

$$q_e = \frac{1}{2}\ln\left(\frac{1+\sin B}{1-\sin B}\right) - \frac{k}{2}\ln\left(\frac{1+k\sin B}{1-k\sin B}\right) = \tanh^{-1}[\mathrm{sn}\,(u,k)] - k\tanh^{-1}[k\,\mathrm{sn}\,(u,k)], \tag{4}$$

remembering that:

$$\tanh^{-1}(\sin(B)) = \frac{1}{2}\ln\left(\frac{1+\sin B}{1-\sin B}\right), \tag{5}$$

where sn (u, k) means [2] a certain function of an argument u and a parameter k, which is called the Jacobi elliptical sine.

Then, in the relation (4) we transfer from a real argument u to the complex argument $z = u + i\upsilon$. This leads to the distribution

$$q_e + il_e = \tanh^{-1}[\mathrm{sn}\,(u+i\upsilon)] - k\tanh^{-1}[k\,\mathrm{sn}\,(u+i\upsilon)] \tag{6}$$

of the function (6) into the real part

$$q_e = \tanh^{-1}[\mathrm{sn}\,u\,\mathrm{dn}'\upsilon] - k\tanh^{-1}[\mathrm{sn}\,u\,\mathrm{dn}'\,(K-u)\,\mathrm{tn}'\upsilon] \tag{7}$$

and the imaginary part

$$l_e = \tan^{-1}\left[\frac{\mathrm{tn}'\upsilon}{\mathrm{sn}\,(K-u)}\right] - k\tan^{-1}[k\,\mathrm{sn}\,(K-u)\,\mathrm{tn}'\,\upsilon]. \tag{8}$$

We use the substitution

$$\sin B = \mathrm{sn}\,(u,k), \tag{9}$$

and introduce the following notation:

$\mathrm{dn}'\upsilon$ – a certain function, called "a delta of Jacobi amplitude", taken from an auxiliary module $k' = \sqrt{1 - k^2}$,

$\mathrm{tn}'\upsilon = \dfrac{\mathrm{sn}'\upsilon}{\mathrm{cn}'\upsilon}$ – the Jacobi elliptical tangent, depending on the argument υ and the auxiliary module k',

$\mathrm{sn}'\upsilon, \mathrm{cn}'\upsilon$ – the Jacobi elliptical sine and cosine, depending on the argument υ and the auxiliary module k',

K – a quarter-period of the Jacobi elliptical functions, depending on the module k of the ellipsoid surface,

K' – a quarter-period of the Jacobi elliptical functions, depending on the auxiliary module k'.

We apply a similar procedure in the case of distribution of the second function $F = w(\vartheta)$, which appears in (2).

We transform an element of the meridian arc

$$ds_e = MdB = \frac{a(1 - k^2)}{(\sqrt{1 - k^2 \sin^2 B})^3}\, dB \tag{10}$$

of the ellipsoid surface (1) to the form

$$ds_e = \frac{ak'\, du}{\mathrm{dn}^2\, u} = a\, \mathrm{dn}^2 (K + u)\, du, \tag{11}$$

where

$$\cos B = \mathrm{cn}\, u, \quad \mathrm{sn}\, (K' - u) = \frac{\mathrm{cn}\, u}{\mathrm{dn}\, u}, \quad dB = (\mathrm{dn}\, u)\, du. \tag{12}$$

Integrating both sides of (11) we obtain the formula

$$s_e = a \int_0^u \mathrm{dn}^2 (K + u)\, d(K + u) = aE(K + u) - aE. \tag{13}$$

In this expression $E(K + u)$ and $E(K)$ mean [2] the so-called complete Jacobi integrals of the second type, taken from their appropriate arguments $K + u$ and K.

Expanding the argument u in (13) with an imaginary part, i.e. assuming a complex variable $(u + i\upsilon)$ instead of u, we obtain

$$F = x_e + iy_e = a\,[E(K + u + i\upsilon) - E(K)]. \tag{14}$$

Distribution of (14) into the real part x_e and the imaginary part y_e leads to the distribution

$$\frac{x_e}{a} = E(u) - \frac{k^2 \operatorname{sn} u \operatorname{cn} u \operatorname{dn} u}{\operatorname{dn}^2 u + (\operatorname{dn}' \upsilon)^2 - 1}, \qquad (15)$$

$$y_e = \upsilon - E'(\upsilon) + \frac{(k')^2 \operatorname{sn}' \upsilon \operatorname{cn}' \upsilon \operatorname{dn}' \upsilon}{\operatorname{dn}^2 u + (\operatorname{dn}' \upsilon)^2 - 1},$$

where $E'(\upsilon)$ means the $E(\upsilon)$ function taken from the auxiliary module k'.

Variables x_e and y_e in (15) mean co-ordinates in the Gauss-Krüger projection in the ortho-Cartesian plane system.

2. *Analytical description of the spheroidal part in Gauss-Krüger projection*

A real function which creates the speroidal part in the Gauss-Krüger projection is a certain function

$$q_e = f(q_k) \qquad (16)$$

which relates the ellipsoidal isometric geodetic latitude q_e with the isometric spherical latitude q_k of the sphere surface. For the axial meridian $l_e = 0$ of the ellipsoid surface (1) this function is generated by the equation

$$s_e(B) = s_k(\varphi). \qquad (17)$$

In this equation:

- $s_e(B)$ – means length of an arc of the meridian of the ellipsoid surface, calculated from the Equator $B = 0$,
- $s_k(\varphi)$ – means length of the axial meridian of a sphere of radius R, calculated from the Equator $\varphi = 0$.

The value of the radius R in (17) determines the equation

$$R\frac{\pi}{2} = s_e\left(\frac{\pi}{2}\right). \qquad (18)$$

But φ in (17) depends on the isometric spherical latitude q_k and the parameter B depends on the geodetic isometric ellipsoidal latitude q_e. Therefore the following system of relations must occur for the axial meridian

$$w(\psi^{-1}(q_k)) = s_k, \quad w(\psi^{-1}(q_e)) = s_e \qquad (19)$$

which meet the equation

$$w(\psi^{-1}(q_k)) = w(\psi^{-1}(q_e)). \qquad (20)$$

After substitution of the isometric latitude q_e by the complex parameter $z_e = q_e + il_e$, the right side of (20) leads to the Gauss-Krüger projection. The left side of the equation (20), after substitution of the parameter q_k by the complex parameter $z_k = q_k + il_k$, also describes a conformal projection. In fact, it is the transverse Mercator projection. In this projection, the equatorial semimajor axis a is substituted by the sphere parameter R, with simultaneous setting to zero of the eccentricity value k. Thus, reversing the first functions of (19) and under the assumption that $s_k = s_e$ allows to find the argument q_k from the assumed value s_e.

After expanding the parameter q_k with the imaginary part l_k and after reversing s_k we will find, basing on (2)

$$z_k = q_k + il_k = \ln \tan\left(\frac{\pi}{4} + \frac{F}{2R}\right). \tag{21}$$

In (21) the variable F means the point (x_e, y_e), i.e. the point in the plane of the complex variable $z_e = x_e + iy_e$.

3. *Transition from the point co-ordinates in the Gauss-Krüger projection to the point co-ordinates on the sphere surface*

Determination of q_k, l_k co-ordinates, which occur in z_k, is reduced to the decomposition of the function (21) into the real and imaginary parts.

For that purpose, we will present the relation (21) in the following form

$$\left[e^{z_k} = \tan\left(\frac{\pi}{4} + \frac{F}{2R}\right) = \frac{\sin\left(\dfrac{\frac{\pi}{2} + \frac{F}{R}}{2}\right)}{\cos\left(\dfrac{\frac{\pi}{2} + \frac{F}{R}}{2}\right)} = \sqrt{\frac{1 - \cos\left(\frac{\pi}{2} + \frac{F}{R}\right)}{1 + \cos\left(\frac{\pi}{2} + \frac{F}{R}\right)}} = \right.$$

$$\left. = \sqrt{\frac{1 + \sin\left(\frac{F}{R}\right)}{1 - \sin\left(\frac{F}{R}\right)}}\right] \Leftrightarrow \left[(e^{z_k})^2 = \frac{1 + \sin\left(\frac{F}{R}\right)}{1 - \sin\left(\frac{F}{R}\right)}\right], \tag{22}$$

and then in the form

$$\left[(e^{z_k})^2 - (e^{z_k})^2 \sin\left(\frac{F}{R}\right) = 1 + \sin\left(\frac{F}{R}\right)\right] \equiv \left[\sin\left(\frac{F}{R}\right) = \frac{e^{2z_k} - 1}{e^{2z_k} + 1} = \tanh z_k\right]. \tag{23}$$

Then, based on [3], in (23) we will utilise the equivalence

$$\arcsin w = \frac{1}{i} \ln (iw + \sqrt{1 + w^2}) \tag{24}$$

and, basing on (24) we find

$$\begin{aligned}
&\left\{\frac{F}{R} = \frac{1}{i} \ln \left[i \tanh z_k + \sqrt{1 - (\tanh z_k)^2} \right] = \frac{1}{i} \ln \left[\frac{i \sinh z_k + 1}{\cosh z_k} \right] = \right. \\
&= \frac{1}{i} \ln \left[\frac{i (\sin (iz_k) + 1)}{\cos (iz_k)} \right] = \frac{1}{i} \ln \left[\frac{1 + \sin (iz_k)}{\sqrt{1 - \sin^2 (iz_k)}} \right] = \frac{1}{i} \ln \sqrt{\frac{1 + \sin (iz_k)}{1 - \sin (iz_k)}} = \\
&\left. = \frac{1}{i} \ln \left(\frac{\pi}{4} + \frac{iz_k}{2} \right) \right\} \equiv \left\{ \frac{F}{R} = \frac{1}{i} \ln \tan \left(\frac{\pi}{4} + \frac{iz_k}{2} \right) \right\}.
\end{aligned} \tag{25}$$

The relation (25) may have various forms. If, for example, we assume that

$$\xi = \tan \left(\frac{\pi}{4} + \frac{iz_k}{2} \right) = \frac{1 + \tan (iz_k)}{1 - \tan (iz_k)} = e^{iz_k}, \tag{26}$$

then we will obtain

$$\tan \left(\frac{i \frac{F}{R}}{2} \right) = \frac{\xi - 1}{\xi + 1} = \frac{e^{iz_k} - 1}{e^{iz_k} + 1} = i \tan \left(\frac{z_k}{2} \right) = \tanh \left(\frac{iz_k}{2} \right). \tag{27}$$

This means that the following relation occurs

$$\tan \left(\frac{z_k}{2} \right) = \tanh \left(\frac{F}{2R} \right). \tag{28}$$

But the following relation also occurs

$$\sinh z_k = \tan \left(\frac{F}{R} \right) \tag{29}$$

since

$$\sin (iz_k) = i \sinh z_k = \frac{2 \tan \left(\frac{iz_k}{2} \right)}{1 + \tanh^2 \left(\frac{iF}{R} \right)} = \tanh \left(i \frac{F}{R} \right) = i \tan \left(\frac{F}{R} \right), \tag{30}$$

as well as the relation

$$\frac{1}{\cosh z_k} = \cos\left(\frac{F}{R}\right) \tag{31}$$

since

$$\frac{1}{\cos(iz_k)} = \frac{1}{\cosh z_k} = \frac{1+\tan^2\left(\frac{iz_k}{2}\right)}{1-\tan^2\left(\frac{iz_k}{2}\right)} = \frac{1+\tanh^2\left(\frac{i\frac{F}{R}}{2}\right)}{1-\tanh^2\left(\frac{i\frac{F}{R}}{2}\right)} = \cosh\left(i\frac{F}{R}\right) \tag{32}$$

and the relation

$$\tanh z_k = \sin\left(\frac{F}{R}\right), \tag{33}$$

due to the relation

$$\tan(iz_k) = i\tanh z_k = i\sin\left(\frac{F}{R}\right) = \sinh\left(i\frac{F}{R}\right). \tag{34}$$

Determination of the argument $z_k = q_k + il_k$ from the relation (21), expressed by variables $x_k = x_e$, $y_k = y_e$ requires [3] that the following equivalence is considered

$$\ln\tan\hat{z} = -\operatorname{arctanh}\left(\frac{\cos 2\hat{x}}{\cosh 2\hat{y}}\right) + i\arctan\left(\frac{\sinh 2\hat{y}}{\sin 2\hat{x}}\right) \tag{35}$$

taken for

$$\left(\hat{z} = \hat{x} + i\hat{y} = \frac{\pi}{4} + \frac{F}{2R} = \frac{\pi}{4} + \frac{x_e}{2R} + i\frac{y_e}{2R}\right) \Rightarrow \left(2\hat{x} = \frac{\pi}{2} + \frac{x_e}{R},\quad 2\hat{y} = \frac{y_e}{R}\right). \tag{36}$$

Considering (35) and (36) we will find

$$\begin{gathered}\hat{z} = q_k + il_k = \operatorname{arctanh}\left(\frac{\sin\left(\frac{x_e}{R}\right)}{\cosh\left(\frac{y_e}{R}\right)}\right) + i\arctan\left(\frac{\sinh\left(\frac{y_e}{R}\right)}{\cos\left(\frac{x_e}{R}\right)}\right) \Leftrightarrow \\ \Leftrightarrow \left[\tanh q_k = \frac{\sin\left(\frac{x_e}{R}\right)}{\cosh\left(\frac{y_e}{R}\right)},\quad \tan l_k = \frac{\sinh\left(\frac{y_e}{R}\right)}{\cos\left(\frac{x_e}{R}\right)}\right].\end{gathered} \tag{37}$$

If, however, we assume that

$$\left[\hat{z} = \frac{\pi}{4} + i\frac{z_k}{2} = \left(\frac{\pi}{4} - \frac{l_k}{2}\right) + i\frac{q_k}{2}\right] \Rightarrow \left[2\hat{x} = \frac{\pi}{2} - l_k,\ 2\hat{y} = q_k\right], \tag{38}$$

then

$$\frac{F}{R} = \frac{x_e}{R} + i\frac{y_e}{R} = \arctan\left(\frac{\sinh q_k}{\cos l_k}\right) + i \operatorname{arctanh}\left(\frac{\sin l_k}{\cosh q_k}\right). \tag{39}$$

Thus, the following relation occurs

$$\tan\left(\frac{x_e}{R}\right) = \frac{\sinh q_k}{\cos l_k} = \frac{\tan\varphi}{\cos l_k},\ \tanh\left(\frac{y_e}{R}\right) = \frac{\sin l_k}{\cosh q_k} = \sin l_k \cos\varphi. \tag{40}$$

Therefore, on the axial meridian we have

$$\tanh\left(\frac{q_k}{2}\right) = \tan\left(\frac{\varphi}{2}\right),\ \tanh q_k = \sin\varphi \tag{41}$$

and

$$\sinh q_k = \tan\varphi,\ \frac{1}{\cosh q_k} = \cos\varphi. \tag{42}$$

Relations (37) and (40) determine the transverse Mercator projection.

4. *Distribution of projection deformations in the spheroidal part of the Gauss-Krüger projection*

In a cartographic, conformal projection of an ellipsoid surface (1) on a sphere surface

$$\begin{aligned} &\vec{\tau} = \vec{\tau}(\varphi, l_k) = [R\cos\varphi\cos l_k,\ R\cos\varphi\sin l_k,\ R\sin\varphi],\\ &l_k = \lambda - \lambda_0\ \lambda_0 = \text{const},\\ &(\varphi,\lambda) \in \omega_k = \{(\varphi,\lambda) : \varphi \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2}\right\rangle, \lambda \in \langle -\pi, \pi)\}, \end{aligned} \tag{43}$$

of the radius R, which meets the condition (18), for which

$$q_e = \int_0^B \frac{M(t)}{N(t)\cos t}\,dt = \psi_e(B) \tag{44}$$

means the geodetic isometric ellipsoidal latitude of an ellipsoid surface, and

$$q_k = \int_0^{\varphi} \frac{dt}{\cos t} = \psi(\varphi), \tag{45}$$

means the isometric spherical latitude, projection functions have the form (21) and (7), (8) and (15).

Therefore, the formula, which describes a local scale of lengths $\vec{\mu} = \frac{d\vec{\tau}}{|d\vec{r}|}$ in the plane of the complex variable z_e may be expressed by means of the relation

$$\mu = |\vec{\mu}| = \frac{R \cos \varphi}{N \cos B} \left|\frac{dz_k}{dz_e}\right| = \frac{R \cos \varphi}{N \cos B} \left|\frac{dz_k}{dF} \frac{dF}{dz_e}\right|. \tag{46}$$

The right side of (46) may be presented in the form of a quotient

$$\mu = \frac{1}{\frac{1}{R \cos \varphi} \left|\frac{dF}{dz_k}\right|} \frac{1}{N \cos B} \left|\frac{dF}{dz_e}\right| = \frac{\mu_{GK}}{\mu_{PM}}, \tag{47}$$

where

$$\mu_{GK} = \frac{1}{N \cos B} \left|\frac{dF}{dz_e}\right| \tag{48}$$

describes a local scale of length in the Gauss-Krüger projection, and

$$\mu_{PM} = \frac{1}{\cos \varphi} \left|\frac{dF}{dz_k}\right| \tag{49}$$

describes a local scale of length in the transverse Mercator projection.

The variable F, which occurs here, determines the complex length of the meridian arc, common for the ellipsoid surface and for the sphere surface, $s_e = s_k$, which is described by the relation (20).

We calculate the derivative $\frac{dF}{dz_e}$ from a system of relations (2)

$$\frac{dF}{dz_e} = \frac{dF}{dt} \frac{dt}{dz_e} = M(\hat{B}) \frac{N(\hat{B}) \cos \hat{B}}{M(\hat{B})} = N(\hat{B}) \cos(\hat{B}) \text{ where } \hat{B} = B_1 + iB_2. \tag{50}$$

Similarly to this, we can calculate the derivative $\frac{dF}{dz_k}$ basing on the formula (50), following the assumption the value R, calculated from the relation (14) or (42), instead of a, and $(\varphi + l_k)$ instead of $\hat{B}$ for the self-zeroing parameter k.

CONCLUSIONS

As it turns out from performed experiments, for limited projection zones, the Gauss-Krüger projection of an ellipsoid and the transverse Mercator projection of the respective sphere, are mutually equivalent. Differences of linear deformations on the axial meridian, within the distant of 5° (10-degree projection zone) from the axial meridian, do not exceed 10 mm/km for medium geographic latitudes. For narrow projection zones (3° and 6°) they are very small. The smallest differences of deformations occur in latitudes close to 30°. Inversion of signs may be also observed for those cases. For the meridian, located 10° apart from the axial meridian, those differences reach 50-40 mm/km. Their highest values occur in areas located close to the Equator.

B°	l_e°	φ°	l_k°	μ_{GK}	μ_{PM}	$(\mu_{GK} - \mu_{PM})$ mm/km
0.0000	5.0000	0.0000	5.0084	1.0038457	1.0038327	**13.0**
0.0000	10.0000	0.0000	10.0170	1.0155330	1.0154797	53.3
10.0000	5.0000	9.9509	5.0082	1.0037283	1.0037173	**11.0**
10.0000	10.0000	9.9512	10.0164	1.0150507	1.0150053	45.4
20.0000	5.0000	19.9076	5.0074	1.0033909	1.0033848	**6.1**
20.0000	10.0000	19.9082	10.0149	1.0136670	1.0136423	24.7
30.0000	5.0000	29.8754	5.0063	1.0028752	1.0028752	**0.0**
30.0000	10.0000	29.8761	10.0126	1.0115614	1.0115617	0.3
40.0000	5.0000	39.8581	5.0049	1.0022451	1.0022500	**−4.9**
40.0000	10.0000	39.8588	10.0098	1.0090017	1.0090220	−20.3
50.0000	5.0000	49.8580	5.0035	1.0015773	1.0015845	**−7.2**
50.0000	10.0000	49.8584	10.0069	1.0063052	1.0063343	−29.1
60.0000	5.0000	59.8750	5.0021	1.0009525	1.0009589	**−6.4**
60.0000	10.0000	59.8752	10.0042	1.0037968	1.0038226	−25.8
70.0000	5.0000	69.9071	5.0010	1.0004449	1.0004487	**−3.8**
70.0000	10.0000	69.9072	10.0019	1.0017697	1.0017848	−15.1
80.0000	5.0000	79.9505	5.0003	1.0001146	1.0001157	**−1.1**
80.0000	10.0000	79.9506	10.0005	1.0004550	1.0004594	−4.4
85.0000	5.0000	84.9749	5.0001	1.0000289	1.0000291	**−0.2**
89.0000	5.0000	88.9950	5.0000	1.0000012	1.0000012	**0,0**

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Opis analityczny odwzorowania Gaussa-Krügera w wersji sferoidalnej i sferoidalno-sferycznej

S t r e s z c z e n i e

W pracy wykazano możliwość ujęcia odwzorowania Gaussa-Krügera powierzchni sferoidy w postaci odwzorowania Mercatora stosownej sfery w położeniu poprzecznym. Podane zostały zależności występujące pomiędzy parametrami położenia punktu na sferoidzie i sferze. Wyznaczony został stan rozkładu zniekształceń liniowych odwzorowania Gaussa-Krügera jako odwzorowania poprzecznego Mercatora sfery. Wykazano współzależność zniekształceń liniowych od składowej sferoidalnej i sferycznej. Przeprowadzono obliczenia testowe, które wykazały, że wyznaczanie wartości zniekształceń liniowych w wąskich strefach odwzorowawczych może być ograniczone tylko do ich sferycznego czynnika.

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Аналитическое описание проекции Гаусса-Крюгера в сфероидальной и сфероидально-сферической версии

Р е з ю м е

В работе указана возможность трактовки поверхности сфероида в проекции Гаусса-Крюгера в виде проекции Меркатора соотвествующей сферы в поперечном положении. Представлены зависмости, выступающие между параметрами положения пункта на сфероиде и сфере. Определено состояние хода линейных деформаций проекции Гаусса-Крюгера как поперечной проекции Меркатора сферы. Проведены тестовые вычисления, которые показали, что определение величин линейных деформаций в узких проекционных зонах может быть ограничено только к их сферическому фактору (элементу).