GEODESY AND CARTOGRAPHY
Vol. 58, No 2, 2009, pp. 119-135

A kinematic model of a modular network as applied for the determination of displacements

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Received: 5 June 2009 /Accepted: 18 September 2009

Abstract: The paper presents two alternative proposals for processing kinematic modular networks. The first method employs the idea of multi-group transformation which may be reduced to setting up a system of conditional equations with unknowns. The kinematic parameters (point motion velocities) are in this case determined after the observations are adjusted, together with point coordinates. The other proposal is based on the classic idea of the parametric method. The theoretical relationships for functional models of the network adjustment for each of the methods have been provided. The practical conditions have been presented for the application of the proposed models (methods) in constructing detailed computational algorithms. The modular network technology may be an appropriate method of geodetic determination of displacements, especially in difficult terrain conditions (slopes, trees, unfavourable exposition to satellite signals).

Keywords: Modular network, kinematic model, determination of displacements

1. Introduction

One of the main tasks of geodetic engineering is to handle objects which can be dislocated or deformed, i.e. areas or structures included in a group of potentially moving objects. This is in particular related to mining areas, landslide sites, engineering structures which are parts of communication routes (bridges, overpasses, embankments). The basic assumption in the determination (monitoring) of displacements is a periodical character (repeatability) of the measurements of the geometric state of an object. A set of observations dispersed in time and space can be treated as the observational system (Kadaj, 1998). An important factor for the choice of the model (method) of processing observations is the relationship between the duration of an individual measurement cycle and the predicted velocity of dislocation of points which represent the object (Prószyński and Kwaśniak, 2006). If one common epoch is assigned to all observations, the classic *static model* can be applied for processing them. However, if the changes taking place during the measurement are regarded as important, the use of the *kinematic model* to the adjustment of observations should be considered.

The essence of the model lies in the possibility of reducing all observations to one epoch by introducing kinematic parameters to the functional model, e.g. point movement velocity in the adopted reference system. The literature of the subject mentions other, intermediate models of adjustment, e.g. a *quasi-static* and a *quasi-kinematic* model (Kadaj, 1998; Preweda, 2002). Detailed discussion of the issues related to the classification of models (methods) of the determination of displacements can be found in such studies as (Pelzer, 1987; Kadaj and Plewako, 1991; Kadaj, 1992; Heunecke et al., 1998). An important element of the dislocation model is the reference system and the issue related to its identification, which has been reflected in numerous publications (e.g. Prószyński, 1986; Beluch and Piwowarski, 1996).

Publications devoted to the issue emphasise application of new methods of measurement (GPS technique, hybrid systems) (Bałut and Gocał, 1997; Asteriadis and Schwan, 1998). An alternative may be provided by the *modular network* technology (see e.g. GUGiK, 1986; Gargula, 2004), applied as a separate method of classic measurements or in combination with the GPS technique (Gargula, 2009). Among the advantages of modular networks is the flexibility of their structure, which facilitates not only the stage of the measurement design, but also its execution. The following properties of modular networks are important from this point of view: 1) free choice of stations – without marking (no instrument centring and errors related to it); 2) tie points are assumed to be target points – usually wall points, e.g. ones linked with the object under study (advantages similar as for the stations). Geodetic networks of a special structure, similar to that of modular networks, are discussed in the study of Kadaj (1975).

The principles of application of modular networks are provided in the Technical Guidelines G-4.1 (GUGiK, 1986). The concept of the measurement technology has been developed and partly modified by the author of this paper. For example, the concept of so called modular traverses has been introduced for geodetic handling of linear objects (Gargula, 1995), a proposition has been put forward to apply certain elements of modular networks as realization networks and control measurements (Gargula, 2003a), and a method of rigorous adjustment for this type of structures has been developed (Gargula, 2003b). Currently, studies of the issue of integration of modular network with the GPS technique are being conducted (within the scope of measurement and numerical processing) (Gargula, 2009) and of the possibility of application of the method in geodetic handling of kinematic (moving) objects, i.e. areas or engineering structures for which a model of displacements or deformations has to be determined. These issues are a subject of the research project which is being carried out by the author of this paper. The project includes periodical measurements in the area of a vast landslide which poses a threat to people living nearby. It turns out that due to difficult terrain conditions (great height differences, numerous slopes and micro-landslides, slopes with trees, property fences, etc.), it is impossible to conduct traditional horizontal and vertical surveys (at centred measurement stations). A GPS measurement also proved insufficient due to unfavourable exposure to satellite signals. In such case the measurement could only be performed with the use of the modular network technique. The results obtained in the survey were used for the determination of horizontal displacements. The modular network method has also been tested in the determination of displacements in mining areas (Gargula and Kwinta, 2005, 2008).

The aim of this study is to determine the functional model which will provide grounds for algorithms of adjustment of kinematic modular networks. A possibility of applying two methods of numerical processing: 1) transformational method, which essence lies in setting up the system of conditional equations with unknowns (cf. Gargula, 2003b), 2) parametric method (the transformational method in this case is used for the determination of approximate coordinates) will be discussed.

2. Basic functional relationships

The position of points in any geodetic network is determined from angular and distance measurements and the measurements of height differences. Computation algorithms usually require two observation equation sets to be created: one for planar coordinates and the other for the heights. The following considerations will be limited to the horizontal network. The general functional model for a static network can be written in the following form:

$$F(\mathbf{X}) = \mathbf{W} \tag{1}$$

where **X** is the vector of parameters to be determined (unknowns), and **W** is the vector of constants.

Assuming that observations and coordinates of the network reference points are uncorrelated, the stochastic model can be expressed as (cf. Preweda, 2002)

$$Cov(\mathbf{X}_R, \mathbf{W}_L) = \begin{bmatrix} Cov(\mathbf{X}_R) & \mathbf{0} \\ \mathbf{0} & Cov(\mathbf{W}_L) \end{bmatrix} \tag{2}$$

where $Cov(\mathbf{X}_R)$, $Cov(\mathbf{W}_L)$ are variance-covariance matrices for the vector of reference point coordinates $\mathbf{X}_R$ and observational vector $\mathbf{W}_L$, respectively.

Models (1) and (2) provide grounds for the least squares algorithm, based on the condition

$$F = \mathbf{V}^T \cdot \mathbf{P} \cdot \mathbf{V} = (\mathbf{W} - \mathbf{A} \cdot \mathbf{X})^T \cdot Cov(\mathbf{X}_R, \mathbf{W}_L)^{-1} \cdot (\mathbf{W} - \mathbf{A} \cdot \mathbf{X}) = \min \tag{3}$$

where **V** is the vector of corrections to both observations and reference coordinates, **A** is the design matrix, and **P** is the weight matrix.

When kinematic models are considered, a function of time t is introduced to static models (1), (2)

$$F[\mathbf{X}(t)] = \mathbf{W}(t) \tag{4}$$

$$Cov\,(\mathbf{X}_R\,(t)\,,\mathbf{W}_L\,(t)) = \begin{bmatrix} Cov\,(\mathbf{X}_R\,(t)) & \mathbf{0} \\ \mathbf{0} & Cov\,(\mathbf{W}_L\,(t)) \end{bmatrix} \tag{5}$$

The kinematic model (4), (5) is used for the approximation of the function of displacements (as a function of time) of individual points of the object and the vector field of displacements (see: Kadaj, 1998; Preweda, 2002).

A kinematic model for any individual point P (moving at a uniform motion) is based on the assumption

$$\begin{cases} X_P^{[i]} = X_P^{[0]} + A_P \cdot t_{[i]} \\ Y_P^{[i]} = Y_P^{[0]} + B_P \cdot t_{[i]} \end{cases} \tag{6}$$

where $[i]$ represents the number of the observation cycle, $t_{[i]}$ is time when the observation cycle was performed, measured from the initial observation cycle, and A_P, B_P are kinematic parameters of the moving point (velocity of movement, in the directions of x and y axis, respectively).

Algorithms of modular network development employ the principle of multi-group transformation. Transformation of planar Cartesian coordinates is based on the following formulae:

$$\begin{cases} X_P = X_0 + x_P \cdot C + y_P \cdot S \\ Y_P = Y_0 + y_P \cdot C - x_P \cdot S \end{cases} \tag{7}$$

with

$$C = f \cdot \cos\alpha; \qquad S = f \cdot \sin\alpha \tag{8}$$

where (x_P, y_P), (X_P, Y_P) are Cartesian coordinates of point P in the primary and secondary coordinate system, respectively; (X_0, Y_0) is the displacement vector (coordinates of the origin of the primary coordinate system in the secondary coordinate system); C, S are transformation coefficients; f is a scale coefficient; α is the angle of rotation.

Introducing equations (7) to the model (6) provides kinematic transformation equations for moving points

$$\begin{cases} X_P^{[i]} = X_P^{[0]} + A_P \cdot t_{[i]} = X_0 + x_P^{[i]} \cdot C + y_P^{[i]} \cdot S \\ Y_P^{[i]} = Y_P^{[0]} + B_P \cdot t_{[i]} = Y_0 + y_P^{[i]} \cdot C - x_P^{[i]} \cdot S \end{cases} \tag{9}$$

The equations (9) provide the basis for the functional model (system of conditional equations with unknowns) in the transformational method of modular network adjustment.

3. Numerical processing of a kinematic modular network by the transformation method (*TRANS*)

The task may be divided into several conventional stages (cf. Gargula, 2009):

- calculation of approximate transformation parameters $\tilde{X}_0$, $\tilde{Y}_0$, $\tilde{C}$, $\tilde{S}$ for each module;
- transformational adjustment by the conditional method with unknowns (adjustment of observed polar coordinates d, β, transformation parameters X_0, Y_0, C, S and local Cartesian coordinates x_P, y_P);
- determination of adjusted global Cartesian coordinates X_P, Y_P (in the reference system) and kinematic parameters A_P, B_P.

3.1. Calculation of approximate transformation parameters

With the use of equations (7), a conditional equation, linking coordinates of the points expressed in various systems (modules), is set up for each tie point (in particular for the reference point).

Conditions for the reference points r, i.e. the equations linking the local system for the given module j with the global reference system (Fig. 1) are

$$\begin{cases} X_r = X_0^{(j)} + x_r^{(j)} \cdot C^{(j)} + y_r^{(j)} \cdot S^{(j)} \\ Y_r = Y_0^{(j)} + y_r^{(j)} \cdot C^{(j)} - x_r^{(j)} \cdot S^{(j)} \end{cases} \tag{10}$$

while conditions for the tie points k (between adjacent modules j and j+1)

$$\begin{cases} X_k = X_0^{(j)} + x_k^{(j)} \cdot C^{(j)} + y_k^{(j)} \cdot S^{(j)} = X_0^{(j+1)} + x_k^{(j+1)} \cdot C^{(j+1)} + y_k^{(j+1)} \cdot S^{(j+1)} \\ Y_k = Y_0^{(j)} + y_k^{(j)} \cdot C^{(j)} - x_k^{(j)} \cdot S^{(j)} = Y_0^{(j+1)} + y_k^{(j+1)} \cdot C^{(j+1)} - x_k^{(j+1)} \cdot S^{(j+1)} \end{cases} \tag{11}$$

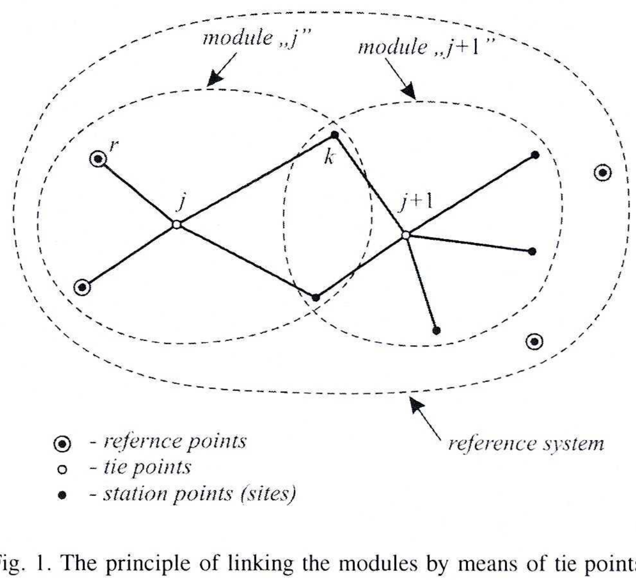

Fig. 1. The principle of linking the modules by means of tie points

It is noteworthy that the equations do not contain any information on $[i]$ in which the coordinates are determined. This is because the conditions relate to the coordinates expressed in the systems of local modules, and these in turn depend on the choice of a measuring station during a periodical measurement.

Approximate parameters $\tilde{X}_0$, $\tilde{Y}_0$, $\tilde{C}$, $\tilde{S}$ of transformation are calculated for each module j by solving the system of equations consisting of equations (10) and (11):

$$\begin{cases} X_r = \tilde{X}_0^{(j)} + \tilde{x}_r^{(j)} \cdot \tilde{C}^{(j)} + \tilde{y}_r^{(j)} \cdot \tilde{S}^{(j)} \\ Y_r = \tilde{Y}_0^{(j)} + \tilde{y}_r^{(j)} \cdot \tilde{C}^{(j)} - \tilde{x}_r^{(j)} \cdot \tilde{S}^{(j)} \\ \vdots \\ \tilde{X}_0^{(j)} + \tilde{x}_k^{(j)} \cdot \tilde{C}^{(j)} + \tilde{y}_k^{(j)} \cdot \tilde{S}^{(j)} = \tilde{X}_0^{(j+1)} + \tilde{x}_k^{(j+1)} \cdot \tilde{C}^{(j+1)} + \tilde{y}_k^{(j+1)} \cdot \tilde{S}^{(j+1)} \\ \tilde{Y}_0^{(j)} + \tilde{y}_k^{(j)} \cdot \tilde{C}^{(j)} - \tilde{x}_k^{(j)} \cdot \tilde{S}^{(j)} = \tilde{Y}_0^{(j+1)} + \tilde{y}_k^{(j+1)} \cdot \tilde{C}^{(j+1)} - \tilde{x}_k^{(j+1)} \cdot \tilde{S}^{(j+1)} \\ \vdots \end{cases} \tag{12}$$

Approximate coordinates $\tilde{x}$, $\tilde{y}$ are referred to the local polar coordinate system (Fig. 2) of the origin at the surveying site in a specific module$j$ (for the reference points r and the tie points k, respectively)

$$\begin{cases} \tilde{x}_r^{(j)} = d_{jr} \cdot \cos\beta_{jr} \\ \tilde{y}_r^{(j)} = d_{jr} \cdot \sin\beta_{jr} \end{cases} \tag{13}$$

$$\begin{cases} \tilde{x}_k^{(j)} = d_{jk} \cdot \cos\beta_{jk} \\ \tilde{y}_k^{(j)} = d_{jk} \cdot \sin\beta_{jk} \end{cases} \tag{14}$$

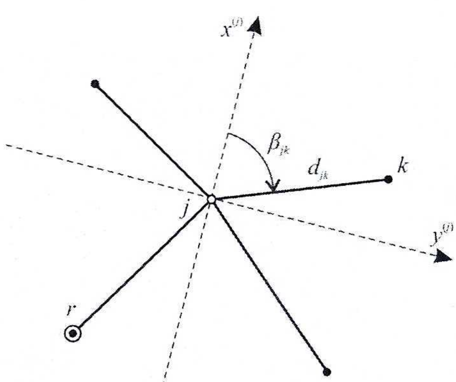

Fig. 2. Observations and coordinates in the local polar coordinate system of the elementary module j

The total number l_R of independent equations (12) should be equal to the number of unknowns. Since there are four transformation parameters for each module, consequently

$$l_R = 4 \cdot m \tag{15}$$

where m is a number of elementary modules.

The condition (15) is, however, not sufficient, as each module has to have at least 2 points from the group of tie points k or reference points r (condition of module determinability – see Gargula, 2004).

The system (12) can be expressed in the matrix form

$$\tilde{\mathbf{A}} \cdot \tilde{\mathbf{X}} = \mathbf{L} \tag{16}$$

or, when expanded

$$\left[\begin{array}{cccc|cccc|c}
1 & 0 & \tilde{x}_r^{(j)} & \tilde{y}_r^{(j)} & 0 & 0 & 0 & 0 & \cdots \\
0 & 1 & \tilde{y}_r^{(j)} & -\tilde{x}_r^{(j)} & 0 & 0 & 0 & 0 & \cdots \\
\hline
0 & 0 & 0 & 0 & 1 & 0 & \tilde{x}_r^{(j+1)} & \tilde{y}_r^{(j+1)} & \cdots \\
0 & 0 & 0 & 0 & 0 & 1 & \tilde{y}_r^{(j+1)} & -\tilde{x}_r^{(j+1)} & \cdots \\
\hline
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\
\hline
1 & 0 & \tilde{x}_k^{(j)} & \tilde{y}_k^{(j)} & 0 & 0 & 0 & 0 & \cdots \\
0 & 1 & \tilde{y}_k^{(j)} & -\tilde{x}_k^{(j)} & 0 & 0 & 0 & 0 & \cdots \\
\hline
0 & 0 & 0 & 0 & 1 & 0 & \tilde{x}_k^{(j+1)} & \tilde{y}_k^{(j+1)} & \cdots \\
0 & 0 & 0 & 0 & 0 & 1 & \tilde{y}_k^{(j+1)} & -\tilde{x}_k^{(j+1)} & \cdots \\
\hline
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right]
\times
\left[\begin{array}{c}
\tilde{X}_0^{(j)} \\ \tilde{Y}_0^{(j)} \\ \tilde{C}^{(j)} \\ \tilde{S}^{(j)} \\ \hline
\tilde{X}_0^{(j+1)} \\ \tilde{Y}_0^{(j+1)} \\ \tilde{C}^{(j+1)} \\ \tilde{S}^{(j+1)} \\ \hline
\vdots
\end{array}\right]
=
\left[\begin{array}{c}
X_r \\ Y_r \\ \hline X_r \\ Y_r \\ \hline \vdots \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline \vdots
\end{array}\right]$$

The solution of an unambiguous system (16) is obtained after a simple matrix operation is performed

$$\tilde{\mathbf{X}} = \tilde{\mathbf{A}}^{-1} \cdot \mathbf{L} \tag{17}$$

3.2. Transformational adjustment by the conditional method with unknowns

After observations d, β have been inserted in the conditional equations (10), (11)

$$\begin{cases} x_k^{(j)} = \left(d_{jk} + v_{jk}^{(d)}\right) \cdot \cos\left(\beta_{jk} + v_{jk}^{(\beta)}\right) \\ y_k^{(j)} = \left(d_{jk} + v_{jk}^{(d)}\right) \cdot \sin\left(\beta_{jk} + v_{jk}^{(\beta)}\right) \end{cases} \tag{18}$$

and the same has been done with approximate parameters $\tilde{X}_0$, $\tilde{Y}_0$, $\tilde{\alpha}$ (for simplicity let us assume that the scale is invariable, i.e. $f = 1$)

$$\begin{cases} X_0^{(j)} = \tilde{X}_0^{(j)} + \delta X_0^{(j)} \\ Y_0^{(j)} = \tilde{Y}_0^{(j)} + \delta Y_0^{(j)} \end{cases} \tag{19}$$

$$\begin{cases} C^{(j)} = \cos\left(\tilde{\alpha}^{(j)} + \delta\alpha^{(j)}\right) \\ S^{(j)} = \sin\left(\tilde{\alpha}^{(j)} + \delta\alpha^{(j)}\right) \end{cases} \tag{20}$$

one obtains the following non-linear equations (for an example coordinate X of point k, linking two neighbouring modules j and $j+1$):

$$\begin{aligned} &\left(\tilde{X}_0^{(j)} + \delta X_0^{(j)}\right) + \left(d_{jk} + v_{jk}^{(d)}\right) \cdot \cos\left(\beta_{jk} + v_{jk}^{(\beta)}\right) \cdot \cos\left(\tilde{\alpha}^{(j)} + \delta\alpha^{(j)}\right) + \\ &+ \left(d_{jk} + v_{jk}^{(d)}\right) \cdot \sin\left(\beta_{jk} + v_{jk}^{(\beta)}\right) \cdot \sin\left(\tilde{\alpha}^{(j)} + \delta\alpha^{(j)}\right) = \\ &= \left(\tilde{X}_0^{(j+1)} + \delta X_0^{(j+1)}\right) + \left(d_{j+1,k} + v_{j+1,k}^{(d)}\right) \cdot \cos\left(\beta_{j+1,k} + v_{j+1,k}^{(\beta)}\right) \cdot \cos\left(\tilde{\alpha}^{(j+1)} + \delta\alpha^{(j+1)}\right) + \\ &+ \left(d_{j+1,k} + v_{j+1,k}^{(d)}\right) \cdot \sin\left(\beta_{j+1,k} + v_{j+1,k}^{(\beta)}\right) \cdot \sin\left(\tilde{\alpha}^{(j+1)} + \delta\alpha^{(j+1)}\right) \end{aligned} \tag{21}$$

where $v^{(d)}, v^{(\beta)}$ are observational residuals, and (j) is the index of module linked with the measuring station j.

After being expanded in the Taylor's series, equations (21) become (example):

$$\begin{cases} \delta X_0^{(j)} - \delta X_0^{(j+1)} + a_1^{(j)} \cdot \delta\alpha^{(j)} - a_1^{(j+1)} \cdot \delta\alpha^{(j+1)} = \\ \qquad = a_2^{(j)} \cdot v_{jk}^{(d)} - a_3^{(j)} \cdot v_{jk}^{(\beta)} + a_2^{(j+1)} \cdot v_{j+1,k}^{(d)} - a_3^{(j+1)} \cdot v_{j+1,k}^{(\beta)} + \omega_x^{(j,j+1)} \\ \delta Y_0^{(j)} - \delta Y_0^{(j+1)} + b_1^{(j)} \cdot \delta\alpha^{(j)} - b_1^{(j+1)} \cdot \delta\alpha^{(j+1)} = \\ \qquad = b_2^{(j)} \cdot v_{jk}^{(d)} - b_3^{(j)} \cdot v_{jk}^{(\beta)} + b_2^{(j+1)} \cdot v_{j+1,k}^{(d)} - b_3^{(j)} \cdot v_{j+1,k}^{(\beta)} + \omega_y^{(j,j+1)} \end{cases} \tag{22}$$

Coefficients a, b corresponding to partial derivatives are determined by linearization of (21) based on approximate transformation parameters and observations (d, β) (Gargula, 2003b). The terms ω_x, ω_y (deviations) are calculated with the use of the approximate parameters (13), (14), (17), which have been determined earlier

$$\begin{cases} \omega_x^{(j,j+1)} = -\tilde{C}^{(j)} \cdot \tilde{x}_k^{(j)} + \tilde{S}^{(j)} \cdot \tilde{y}_k^{(j)} + \tilde{C}^{(j+1)} \cdot \tilde{x}_k^{(j+1)} - \tilde{S}^{(j+1)} \cdot \tilde{y}_k^{(j+1)} - \tilde{X}_0^{(j)} + \tilde{X}_0^{(j+1)} \\ \omega_y^{(j,j+1)} = -\tilde{S}^{(j)} \cdot \tilde{x}_k^{(j)} - \tilde{C}^{(j)} \cdot \tilde{y}_k^{(j)} + \tilde{S}^{(j+1)} \cdot \tilde{x}_k^{(j+1)} + \tilde{C}^{(j+1)} \cdot \tilde{y}_k^{(j+1)} - \tilde{Y}_0^{(j)} + \tilde{Y}_0^{(j+1)} \end{cases} \tag{23}$$

Similarly to (22), linear equations for the reference point r are as follows

$$\begin{cases} \delta X_0^{(j)} + a_1^{(j)} \cdot \delta\alpha^{(j)} = a_2^{(j)} \cdot v_{jr}^{(d)} + a_3^{(j)} \cdot v_{jr}^{(\beta)} + \omega_x^{(j)} \\ \delta Y_0^{(j)} + b_1^{(j)} \cdot \delta\alpha^{(j)} = b_2^{(j)} \cdot v_{jr}^{(d)} + b_3^{(j)} \cdot v_{jr}^{(\beta)} + \omega_y^{(j)} \end{cases} \tag{24}$$

A system of conditional equations in the matrix form with unknowns, containing equations (22) and (24) is as follows

$$\mathbf{A} \cdot \hat{\mathbf{X}} = \mathbf{C} \cdot \mathbf{V} + \mathbf{W} \tag{25}$$

that is

$$\left[\begin{array}{ccc|ccc|c}
1 & 0 & a_1^{(j)} & 0 & 0 & 0 & \cdots \\
0 & 1 & b_1^{(j)} & 0 & 0 & 0 & \cdots \\
\hline
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\
\hline
1 & 0 & a_1^{(j)} & -1 & 0 & a_1^{(j+1)} & \cdots \\
0 & 1 & b_1^{(j)} & 0 & -1 & b_1^{(j+1)} & \cdots \\
\hline
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right] \times \left[\begin{array}{c}
\delta X_0^{(j)} \\ \delta Y_0^{(j)} \\ \delta \alpha_0^{(j)} \\ \hline \delta X_0^{(j+1)} \\ \delta Y_0^{(j+1)} \\ \delta \alpha_0^{(j+1)} \\ \hline \vdots
\end{array}\right] =$$

$$= \left[\begin{array}{cc|c|cc|cc|c}
a_2^{(j)} & a_3^{(j)} & \cdots & 0 & 0 & 0 & 0 & \cdots \\
b_2^{(j)} & b_3^{(j)} & \cdots & 0 & 0 & 0 & 0 & \cdots \\
\hline
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots \\
\hline
0 & 0 & \cdots & a_2^{(j)} & -a_3^{(j)} & a_2^{(j+1)} & -a_3^{(j+1)} & \cdots \\
0 & 0 & \cdots & b_2^{(j)} & -b_3^{(j)} & b_2^{(j+1)} & -b_3^{(j+1)} & \cdots \\
\hline
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right] \times \left[\begin{array}{c}
v_{jr}^{(d)} \\ v_{jr}^{(\beta)} \\ \hline \vdots \\ \hline v_{jk}^{(d)} \\ v_{jk}^{(\beta)} \\ \hline v_{j+1,k}^{(d)} \\ v_{j+1,k}^{(\beta)} \\ \hline \vdots
\end{array}\right] + \left[\begin{array}{c}
\omega_x^{(j)} \\ \omega_y^{(j)} \\ \hline \vdots \\ \hline \omega_x^{(j,j+1)} \\ \omega_y^{(j,j+1)} \\ \hline \vdots
\end{array}\right]$$

Solving the system of equations (25) by the least squares method (Gargula, 2009) yields increments of transformation parameters δX_0, δY_0, $\delta\alpha$ (for each module) and corrections $v^{(d)}$, $v^{(\beta)}$ (for all the observations in the entire network). In the subsequent step, adjusted parameters of transformation X_0, Y_0, α (19) and (20) are determined together with coordinates of the tie points x_k, y_k in local module polar coordinate systems (18) (see Fig. 2).

3.3. Determination of global coordinates and kinematic parameters

When a kinematic modular network is considered, parameters are determined (coordinates, kinematic parameters) only with reference to the tie points k. The stations j are not monumented and each time when periodical measurements are performed they are chosen at different places. Kinematic equations for the tie points in the i-th observation cycle are noted with the use of a simplified model (6)

$$\begin{cases} X_k^{[i]} = X_k^{[0]} + A_k \cdot t_{[i]} = X_0^{(j)} + x_k^{(j)} \cdot C^{(j)} + y_k^{(j)} \cdot S^{(j)} \\ Y_k^{[i]} = Y_k^{[0]} + B_k \cdot t_{[i]} = Y_0^{(j)} + y_k^{(j)} \cdot C^{(j)} - x_k^{(j)} \cdot S^{(j)} \end{cases} \tag{26}$$

Parameters A and B can easily be obtained from (26)

$$\begin{cases} A_k = \dfrac{X_0^{(j)} + x_k^{(j)} \cdot C^{(j)} + y_k^{(j)} \cdot S^{(j)} - X_k^{[0]}}{t_{[i]}} = \dfrac{X_k^{[i]} - X_k^{[0]}}{t_{[i]}} \\ B_k = \dfrac{Y_0^{(j)} + y_k^{(j)} \cdot C^{(j)} - x_k^{(j)} \cdot S^{(j)} - Y_k^{[0]}}{t_{[i]}} = \dfrac{Y_k^{[i]} - Y_k^{[0]}}{t_{[i]}} \end{cases} \tag{27}$$

If a polynomial model of n-th degree is applied in equations (26), one obtains a system of equations of the following type

$$\begin{cases} X_k^{[i]} = X_k^{[0]} + A_{k1} \cdot t_{[i]} + A_{k2} \cdot t_{[i]}^2 + \ldots + A_{kn} \cdot t_{[i]}^n \\ Y_k^{[i]} = Y_k^{[0]} + B_{k1} \cdot t_{[i]} + B_{k2} \cdot t_{[i]}^2 + \ldots + B_{kn} \cdot t_{[i]}^n \end{cases} \tag{28}$$

In order to determine kinematic parameters A_k, B_k, an appropriate number of measurement cycles $(1, 2, \ldots, n)$ is required. The types of models applied in description of kinematic networks and methods of identification of kinematic parameters (direct, indirect) have been widely discussed in the literature (e.g. Kadaj, 1998; Preweda, 2002).

4. A kinematic model of a modular network adjustment by the parametric method (*PARAM*)

Numerical processing of a modular network by the parametric method is possible only after approximate coordinates of network points are obtained, obviously expressed in the reference system (common for both the tie points and the observation stations). The task can be performed by multi–group transformation, like at the first stage of previously presented *TRANS* method – equations (10) to (18). Using approximate parameters of transformation (17) and approximate local coordinates (18), global approximate Cartesian coordinates can be calculated:
– for tie points k

$$\begin{cases} \tilde{X}_k = \tilde{X}_0^{(j)} + \tilde{C}^{(j)} \cdot \tilde{x}_k^{(j)} + \tilde{S}^{(j)} \cdot \tilde{y}_k^{(j)} \\ \tilde{Y}_k = \tilde{Y}_0^{(j)} + \tilde{C}^{(j)} \cdot \tilde{y}_k^{(j)} - \tilde{S}^{(j)} \cdot \tilde{x}_k^{(j)} \end{cases} \tag{29}$$

– for the points of the observation stations j

$$\begin{cases} \tilde{X}_j = \tilde{X}_0^{(j)} \\ \tilde{Y}_j = \tilde{Y}_0^{(j)} \end{cases} \tag{30}$$

(with the assumption that $x_0^{(j)} = y_0^{(j)} = 0$).

Unknown global Cartesian coordinates X, Y and velocities A, B of displacements will be the parameters of observation equations for the distances and angles measured in the modular network.

4.1. Kinematic equation for the horizontal distance

The elementary distance measured at station j to the tie point k (Fig. 3), taking into account the kinematic parameters of the simplified model (3), can be expressed by the following equation

$$d_{jk}+v_{jk}^{(d)} = \sqrt{\left[\left(X_k^{[0]} + A_k \cdot t_{[i]}\right) - \left(X_j^{[0]} + A_j \cdot t_{[i]}\right)\right]^2 + \left[\left(Y_k^{[0]} + B_k \cdot t_{[i]}\right) - \left(Y_j^{[0]} + B_j \cdot t_{[i]}\right)\right]^2} \tag{31}$$

When expanded into the Taylor's series against the coordinates X, Y and their kinematic parameters A, B, this yields

$$\begin{aligned} v_{jk}^{(d)} = &- \frac{\tilde{X}_k^{[i]} - \tilde{X}_j^{[i]}}{\tilde{d}_{jk}} \cdot \delta X_j - \frac{\tilde{Y}_k^{[i]} - \tilde{Y}_j^{[i]}}{\tilde{d}_{jk}} \cdot \delta Y_j + \frac{\tilde{X}_k^{[i]} - \tilde{X}_j^{[i]}}{\tilde{d}_{jk}} \cdot \delta X_k \\ &+ \frac{\tilde{Y}_k^{[i]} - \tilde{Y}_j^{[i]}}{\tilde{d}_{jk}} \cdot \delta Y_k - t_{[i]} \cdot \frac{\tilde{X}_k^{[i]} - \tilde{X}_j^{[i]}}{\tilde{d}_{jk}} \cdot \delta A_j - t_{[i]} \cdot \frac{\tilde{Y}_k^{[i]} - \tilde{Y}_j^{[i]}}{\tilde{d}_{jk}} \cdot \delta B_j \\ &+ t_{[i]} \cdot \frac{\tilde{X}_k^{[i]} - \tilde{X}_j^{[i]}}{\tilde{d}_{jk}} \cdot \delta A_k + t_{[i]} \cdot \frac{\tilde{Y}_k^{[i]} - \tilde{Y}_j^{[i]}}{\tilde{d}_{jk}} \cdot \delta B_k + \tilde{d}_{jk} - d_{jk} \end{aligned} \tag{32}$$

where $(\tilde{\cdot})$ denotes the value calculated from approximate parameters, and δ is the residual of the determined parameter.

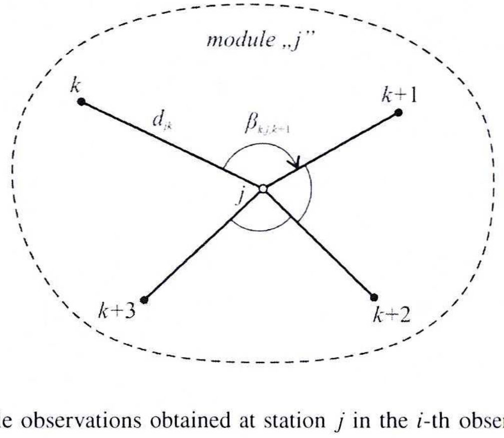

Fig. 3. Sample observations obtained at station j in the i-th observation cycle

If a polynomial model (28) is adopted, the equation (32) will contain n "kinematic" elements of the following type

$$-t_{[i]}^{q}\cdot\frac{\tilde{X}_k^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{jk}}\cdot\left(\delta A_j\right)_q-t_{[i]}^{q}\cdot\frac{\tilde{Y}_k^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{jk}}\cdot\left(\delta B_j\right)_q+t_{[i]}^{q}\cdot\frac{\tilde{X}_k^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{jk}}\cdot(\delta A_k)_q+t_{[i]}^{q}\cdot\frac{\tilde{Y}_k^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{jk}}\cdot(\delta B_k)_q \tag{33}$$

where $q = 1, 2, ..., n$.

In matrix notation (32) will thus be as follows

$$\left[v_{jk}^{(d)}\right]=\begin{bmatrix}-\dfrac{\tilde{X}_k^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{jk}}\\ -\dfrac{\tilde{Y}_k^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{jk}}\\ \dfrac{\tilde{X}_k^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{jk}}\\ \dfrac{\tilde{Y}_k^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{jk}}\end{bmatrix}^{\mathrm{T}}\times\left[\begin{array}{c|cccc}\delta X_j & t_{[i]}\cdot\left(\delta A_j\right)_1 & t_{[i]}^2\cdot\left(\delta A_j\right)_2 & \cdots & t_{[i]}^n\cdot\left(\delta A_j\right)_n\\ \delta Y_j & t_{[i]}\cdot\left(\delta B_j\right)_1 & t_{[i]}^2\cdot\left(\delta B_j\right)_2 & \cdots & t_{[i]}^n\cdot\left(\delta B_j\right)_n\\ \delta X_k & t_{[i]}\cdot(\delta A_k)_1 & t_{[i]}^2\cdot(\delta A_k)_2 & \cdots & t_{[i]}^n\cdot(\delta A_k)_n\\ \delta Y_k & t_{[i]}\cdot(\delta B_k)_1 & t_{[i]}^2\cdot(\delta B_k)_2 & \cdots & t_{[i]}^n\cdot(\delta B_k)_n\end{array}\right]+\left[\tilde{d}_{jk}-d_{jk}\right] \tag{34}$$

4.2. *Kinematic equation for the horizontal angle*

An equation of residual for the horizontal angle β (Fig. 3) expressed in a linear form, similarly to (34), is

$$\begin{aligned}v_{k,j,k+1}^{(\beta)}=&\frac{\tilde{Y}_k^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{jk}^2}\cdot\delta X_k-\frac{\tilde{X}_k^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{jk}^2}\cdot\delta Y_k-\frac{\tilde{Y}_{k+1}^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{j,k+1}^2}\cdot\delta X_{k+1}+\frac{\tilde{X}_{k+1}^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{j,k+1}^2}\cdot\delta Y_{k+1}+\\&+\left(\frac{\tilde{Y}_{k+1}^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{j,k+1}^2}-\frac{\tilde{Y}_k^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{jk}^2}\right)\cdot\delta X_j+\left(\frac{\tilde{X}_{k+1}^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{j,k+1}^2}-\frac{\tilde{X}_k^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{jk}^2}\right)\cdot\delta Y_j+\\&+t_{[i]}\cdot\frac{\tilde{Y}_k^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{jk}^2}\cdot\delta A_k-t_{[i]}\cdot\frac{\tilde{X}_k^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{jk}^2}\cdot\delta B_k-t_{[i]}\cdot\frac{\tilde{Y}_{k+1}^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{j,k+1}^2}\cdot\delta A_{k+1}+\\&+t_{[i]}\cdot\frac{\tilde{X}_{k+1}^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{j,k+1}^2}\cdot\delta B_{k+1}+t_{[i]}\cdot\left(\frac{\tilde{Y}_{k+1}^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{j,k+1}^2}-\frac{\tilde{Y}_k^{[i]}-\tilde{Y}_j^{[i]}}{\tilde{d}_{jk}^2}\right)\cdot\delta A_j+\\&+t_{[i]}\cdot\left(\frac{\tilde{X}_{k+1}^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{j,k+1}^2}-\frac{\tilde{X}_k^{[i]}-\tilde{X}_j^{[i]}}{\tilde{d}_{jk}^2}\right)\cdot\delta B_j+\tilde{\beta}_{k,j,k+1}-\beta_{k,j,k+1}\end{aligned} \tag{35}$$

When the linear model (6) is replaced with the polynomial model (28), the equation (35) will have the general form (in the matrix notation)

$$\left[v^{(\beta)}_{k,j,k+1}\right] - \left[\tilde{\beta}_{k,j,k+1} - \beta_{k,j,k+1}\right] =$$

$$= \begin{bmatrix} \dfrac{\tilde{Y}^{[i]}_k - \tilde{Y}^{[i]}_j}{\tilde{d}^2_{jk}} \\ -\dfrac{\tilde{X}^{[i]}_k - \tilde{X}^{[i]}_j}{\tilde{d}^2_{jk}} \\ -\dfrac{\tilde{Y}^{[i]}_{k+1} - \tilde{Y}^{[i]}_j}{\tilde{d}^2_{j,k+1}} \\ \dfrac{\tilde{X}^{[i]}_{k+1} - \tilde{X}^{[i]}_j}{\tilde{d}^2_{j,k+1}} \\ \left(\dfrac{\tilde{Y}^{[i]}_{k+1} - \tilde{Y}^{[i]}_j}{\tilde{d}^2_{j,k+1}} - \dfrac{\tilde{Y}^{[i]}_k - \tilde{Y}^{[i]}_j}{\tilde{d}^2_{jk}}\right) \\ \left(\dfrac{\tilde{X}^{[i]}_{k+1} - \tilde{X}^{[i]}_j}{\tilde{d}^2_{j,k+1}} - \dfrac{\tilde{X}^{[i]}_k - \tilde{X}^{[i]}_j}{\tilde{d}^2_{jk}}\right) \end{bmatrix}^T \times \left[\begin{array}{c|cccc} \delta X_k & t_{[i]}\cdot(\delta A_k)_1 & t^2_{[i]}\cdot(\delta A_k)_2 & \cdots & t^n_{[i]}\cdot(\delta A_k)_n \\ \delta Y_k & t_{[i]}\cdot(\delta B_k)_1 & t^2_{[i]}\cdot(\delta B_k)_2 & \cdots & t^n_{[i]}\cdot(\delta B_k)_n \\ \delta X_{k+1} & t_{[i]}\cdot(\delta A_{k+1})_1 & t^2_{[i]}\cdot(\delta A_{k+1})_2 & \cdots & t^n_{[i]}\cdot(\delta A_{k+1})_n \\ \delta Y_{k+1} & t_{[i]}\cdot(\delta B_{k+1})_1 & t^2_{[i]}\cdot(\delta B_{k+1})_2 & \cdots & t^n_{[i]}\cdot(\delta B_{k+1})_n \\ \delta X_j & t_{[i]}\cdot(\delta A_j)_1 & t^2_{[i]}\cdot(\delta A_j)_2 & \cdots & t^n_{[i]}\cdot(\delta A_j)_n \\ \delta Y_j & t_{[i]}\cdot(\delta B_j)_1 & t^2_{[i]}\cdot(\delta B_j)_2 & \cdots & t^n_{[i]}\cdot(\delta B_j)_n \end{array}\right] \tag{36}$$

A set of equations (34) and (36) for distances and angles will form a linear system $\mathbf{V} = \mathbf{A}\cdot\mathbf{X} - \mathbf{L}$ in which the vector of residuals $\mathbf{V}$ will be estimated by the classic least squares parametric method. Unlike in the *TRANS* method, adjustment of observations will be preceded by the stage of the determination of unknowns, including the kinematic parameters A, B.

5. Practical notes

The complexity of the computational algorithm may affect the choice of the method of processing of modular kinematic networks (transformation method *TRANS* or parametric method *PARAM*). For obvious reasons, the parametric method has an advantage over the conditional one (to which the transformation method comes down). However, for a modular network there is a problem of obtaining approximate coordinates, which requires application of the transformation method, regardless of the approach applied in exact adjustment of observations.

Characteristic features of the transformation method *TRANS* are as follows:

- a unified computational algorithm, based on multi-group transformation (a single type of conditional equations),
- it is easy to obtain approximate coordinates in local module polar coordinate systems (output data for transformations),
- it is not necessary to introduce information on the observation cycle $t_{[i]}$ when conditional equations are set up (cf. Fig. 4),
- a simple way of the determination of kinematic parameters based on adjusted observations,

- it is necessary to determine additional unknowns – parameters of transformations for local polar coordinate systems of all modules,
- it is difficult to define unambiguous, universal criteria for the determination of the number of independent conditional equations for tie points and for reference points.

Characteristic features of the parametric method *PARAM* are as follows:
- the problem of the determination of approximate coordinates (it is necessary to apply the approximate processing of network by the transformation method),
- it is easy to create observation equations,
- it is necessary to introduce information on the observation cycle $t_{[i]}$ to observation equations,
- a large number of estimated parameters of the system (coordinates and kinematic parameters).

A common feature of both methods is that it is not necessary to attribute to observations the time when such observations are made. This stems from the uniqueness of the geometric system of observations in particular cycles of a periodical measurement, which is in turn related to the basic property of modular networks – a freedom of choice of measuring stations (not marked in the ground) – cf. Fig. 4. The property makes it impossible to apply so called *intermediate* models of processing kinematic networks (see: Kadaj, 1998), which are based on observation equations F_j, created independently for each observation with the point of time ascribed to it

$$F_j\left[\mathbf{X}\left(t_j^{[i]}\right)\right] = l_j^{[i]} \tag{37}$$

where $\mathbf{X}(t)$ is the unknown vector of kinematic state of the network, and $l_j^{[i]}$ is the measure of a geometric element j at an observation cycle $t_{[i]}$.

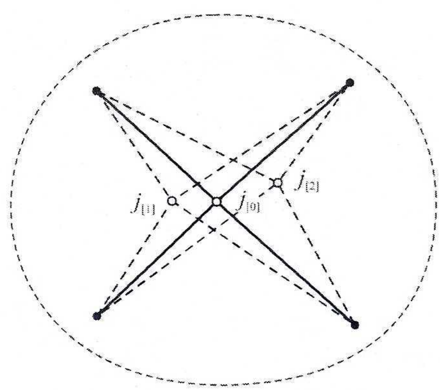

Fig. 4. Position of the surveying station $j_{[i]}$. in consecutive observation cycles (i = 0, 1, 2) of the periodical measurement

Equations (37) are typical for *kinematic* models, which allow for non-synchronous observations (freely dispersed in time). It is well known that the condition cannot be fulfilled in typical modular networks as particular modules are sets of synchronous observations. In consequence, the computational algorithm of a modular network will be based on *quasi-static* or *quasi-kinematic* models (Kadaj, 1998), which assume that an individual measurement campaign is a complete measurement process. Owing to it, it is possible to reduce the geometric state of an object to a certain time t.

Classification of the methods of displacement determination also includes *absolute* methods (positions of points) and *differential* methods (differences in observation measures) (Kadaj, 1998; Prószyński and Kwaśniak, 2006). It is a drawback of the differential methods that it is necessary to maintain a constant observation plan in different measurement cycles, which means invariability of a functional model (4), which defines the general geometric structure of the network. Due to that differential methods cannot be used for the determination of displacements with modular networks.

6. Summary and conclusions

The paper presents two proposals for numerical processing of kinematic modular networks. The theoretical relationships presented in the paper may provide grounds for creating a functional model of adjusting observations according to the chosen method. The first method (*TRANS*) employs the idea of multi-group transformation, which may be reduced to setting up a system of conditional equations with unknowns. The other, alternative method (*PARAM*) is a classic parametric procedure, in which kinematic parameters are estimated together with the coordinates of the points to be determined. This study also considers practical conditions to be fulfilled when the proposed methods are applied in identifying displacements.

The general conclusions which can be drawn from the paper are as follows:

- the technology of modular networks can be applied to determine horizontal displacements as an independent method of classic measurements or as a supplement for GPS measurements; the technology may prove indispensable in difficult terrain conditions;
- the process of network adjustment by the *TRANS* method does not involve the use of any kinematic parameters, as it consists in solving a system of conditional equations with unknown parameters of transformation; kinematic parameters are determined (together with coordinates of points) after adjusting observations;
- creating a functional model in the parametric method (*PARAM*) is a simpler task than in the *TRANS* method, but there appears the problem of the determination of approximate coordinates, which requires introducing an additional component in the computational algorithm – for approximate processing of modular network by the transformation method;
- due to the specificity of the modular network structure (measurement stations as temporary points), displacements can be identified only by absolute determinations; kinematic parameters can be referred only to a point (coordinates) because individual observations are not repeatable in subsequent measurement cycles.

The theoretical considerations contained in this study will be used in constructing computational algorithms of adjustment of kinematic modular networks, integrated with GPS measurements. These issues are the subject of the research project which is being carried out by the author of this paper.

Acknowledgments

This study has been conducted within the research project N N526 2094 33 financially supported by the Ministry of Science and Higher Education. The author expresses his special thanks to Prof. Jan Kryński, the Editor-in-Chief, for his valuable remarks when preparing the final version of the paper.

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Model kinematyczny sieci modularnej w zastosowaniu do wyznaczania przemieszczeń

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Streszczenie

W niniejszej pracy przedstawiono dwie alternatywne propozycje opracowania kinematycznych sieci modularnych. Pierwszy sposób polega na zastosowaniu idei transformacji wielogrupowej, co sprowadza się do zestawienia układu równań warunkowych z niewiadomymi. Parametry kinematyczne (prędkości ruchu punktów) wyznaczane są w tym przypadku po wyrównaniu obserwacji, łącznie ze współrzędnymi punktów. Druga pozycja opiera się na idei klasycznej metody parametrycznej. Podano zależności teoretyczne dla modeli funkcjonalnych wyrównania sieci według każdej z metod. Przedstawiono uwarunkowania praktyczne, dotyczące wykorzystania zaproponowanych modeli (metod) przy konstruowaniu szczegółowych algorytmów obliczeniowych. Technologia sieci modularnych może stanowić odpowiednią metodę geodezyjnego wyznaczania przemieszczeń, zwłaszcza w trudnych warunkach terenowych (skarpy, zadrzewienia, niekorzystna ekspozycja na sygnały satelitarne).