jet INTL JOURNAL OF ELECTRONICS AND TELECOMMUNICATIONS, 2021, VOL. 67, NO. 3, PP. 531-536
Manuscript received May 13, 2021; revised July, 2021 DOI: 10.24425/ijet.2021.137843

A method for Soft Fault Diagnosis of Linear Analog Circuits Using the Laplace Transform Technique

Michał Tadeusiewicz, Marek Ossowski, and Marek Korzybski

Abstract—**This paper is focused on multiple soft fault diagnosis of linear time-invariant analog circuits and brings a method that achieves all objectives of the fault diagnosis: detection, location, and identification. The method is based on a diagnostic test arranged in the transient state, which requires one node accessible for excitation and two nodes accessible for measurement. The circuit is specified by two transmittances which express the Laplace transform of the output voltages in terms of the Laplace transform of the input voltage. Each of these relationships is used to create an overdetermined system of nonlinear algebraic equations with the circuit parameters as the unknown variables. An iterative method is developed to solve these equations. Some virtual solutions can be eliminated comparing the results obtained using both transmittances. Three examples are provided where laboratory or numerical experiments reveal effectiveness of the proposed method.**

Keywords—**analog linear circuits, fault diagnosis, multiple soft faults, the Laplace transform**

I. Introduction

FAULT diagnosis of analog circuits is great important problem playing a crucial role in design validation of electronic devices. Numerous methods in this field have been developed over the past decades. A lot of them are collected in the references [1-4]. However, despite these achievements, the problem is still open and there is a need for further works in this area. The fault diagnosis question is commonly considered as the solution of a system of nonlinear equations with circuit parameters as the unknown variables.

A fault is termed soft if the parameter value deviates from the tolerance range but does not produce catastrophic changes such as open circuit and short circuit. Otherwise, the fault is classified as hard. There are two categories of the fault diagnosis techniques: simulation before the test (SBT) and simulation after the test (SAT). Under the soft fault scenario, the SAT approach dominates, whereas the hard fault diagnosis usually is based on the SBT approach. Numerous works in the fault diagnosis field are focused on the circuits including just a single faulty element, e.g. [5-8]. Multiple fault diagnosis is more complex and insufficiently resolved.

Soft fault diagnosis has attracted great attention, leading to numerous results relating to linear and nonlinear circuits, e.g. [9-15]. Soft faults arise mainly in the fabrication process. They influence the circuit specifications but usually do not violate circuit functionality. Various aspects of soft fault diagnosis have been considered over the last decades and a large number of diagnostic methods has been developed based on: linear and nonlinear programming [16-17], matrix theory [8], optimization techniques [18], evolutionary algorithms [19-21], neural networks [22], support vector machine [23-24], fuzzy logic [25], statistical modeling [26], *z*-transform [27-28]. An important role in fault diagnosis plays the testability analysis and the test point selection [29-32].

This paper deals with multiple soft fault diagnosis of linear analog circuits. The goal of the work is to identify the faulty elements from among a set of the elements considered as potentially faulty and estimate their values. For this purpose, the SAT approach is used, based on the measurement test in a transient state and two transmittances in symbolic form, determined in a preliminary stage of the diagnostic process. Each of the transmittances leads to an equation that expresses the Laplace transform of an output voltage in terms of the Laplace transform of the preset input voltage. Based on this equation, a system of overdetermined algebraic type nonlinear equations is written with the circuit parameters as the unknown variables. To solve it, an iterative method is proposed whose core is the Newton-Raphson algorithm. Comparing the results obtained on the basis of the two transmittances some sets of virtually faulty parameters are eliminated. Unlike the papers reporting the verification methods, e.g. [27-28], based on the assumption that the set of the faulty elements is given and their values are to be calculated, this paper achieves all objectives of the fault diagnosis: detection, location, and identification.

The paper is organized as follows. The basic methodology of the proposed approach is described in Section II. Some procedures which are exploited by the diagnostic method are presented in Section III. Three examples are demonstrated in Section IV. Section V concludes the paper.

II. The Diagnostic Method

A. The main idea

Consider a linear time-invariant dynamic circuit including N parameters $x_1, \ldots, x_N$ considered as potentially faulty. To perform multiple fault (n-fault) diagnosis $(n < N)$ the circuit is described in the frequency domain by the equation

$$V_o(s) = H(s, \boldsymbol{x}) V_{in}(s) \tag{1}$$

where $V_{in}(s)$ and $V_o(s)$ are the Laplace transforms of the input and output voltages $v_{in}(t)$ and $v_o(t)$, $H(s, \boldsymbol{x})$ is the transfer function (transmittance) of the circuit given in symbolic form, where $\boldsymbol{x} = [x_1 \cdots x_n]^T$, and T denotes the transpose. Since the

Authors are with Lodz University of Technology, Department of Electrical, Electronic, Computer and Control Engineering, Lodz, Poland (e-mail: {michal.tadeusiewicz, marek.ossowski, marek.korzybski} @p.lodz.pl).

orders of the parameters differ significantly one from the other we scale them using the formula

$$x_k = p_k x_k^{\text{nom}} \qquad k = 1, \ldots, n \tag{2}$$

where x_k^{nom} is the nominal value of the parameter x_k and $p_k = x_k / x_k^{\text{nom}}$. Then for soft faults, all p_k have similar orders. After substitution (2) into $H(s, \boldsymbol{x})$ the transmittance labeled $T(s, \boldsymbol{p})$ is obtained, where $\boldsymbol{p} = [p_1 \cdots p_n]^{\text{T}}$, and the equation (1) becomes

$$V_{\text{o}}(s) - T(s, \boldsymbol{p}) V_{\text{in}}(s) = \boldsymbol{0} \ . \tag{3}$$

Equation (3) describes the circuit with unknown relative parameters $p_1, \ldots, p_n$, whereas the other circuit parameters have nominal values. Let us apply a trapezoidal voltage $v_{\text{in}}(t)$ to the circuit having actual parameters $p_1, \ldots, p_n$ and measure the output voltage $v_{\text{o}}(t)$ in the course of the diagnostic test. Next, we find the Laplace transforms $V_{\text{in}}(s)$ and $V_{\text{o}}(s)$ as described in Section III. To determine $p_1, \ldots, p_n$ which meet the diagnostic test, we substitute into (3) $s = s_j$, $j = 1, \ldots, 2n$ where s_j are real positive numbers, selected in the way presented in Section III

$$\begin{gathered} V_{\text{o}}(s_1) - T(s_1, \boldsymbol{p}) V_{\text{in}}(s_1) = 0 \\ \vdots \\ V_{\text{o}}(s_{2n}) - T(s_{2n}, \boldsymbol{p}) V_{\text{in}}(s_{2n}) = 0 . \end{gathered} \tag{4}$$

This system consisting of 2*n* nonlinear algebraic equations with *n* unknown variables cannot be solved directly. To find the variables $p_1, \ldots, p_n$ the following numerical method will be used. We divide (4) into two systems of *n* equations

$$\begin{gathered} V_{\text{o}}(s_1) - T(s_1, \boldsymbol{p}) V_{\text{in}}(s_1) = 0 \\ \vdots \\ V_{\text{o}}(s_n) - T(s_n, \boldsymbol{p}) V_{\text{in}}(s_n) = 0 \end{gathered} \tag{5}$$

and

$$\begin{gathered} V_{\text{o}}(s_{n+1}) - T(s_{n+1}, \boldsymbol{p}) V_{\text{in}}(s_{n+1}) = 0 \\ \vdots \\ V_{\text{o}}(s_{2n}) - T(s_{2n}, \boldsymbol{p}) V_{\text{in}}(s_{2n}) = 0 \ . \end{gathered} \tag{6}$$

and apply the Newton-Raphson method separately to (5) and (6) using the same initial guess $\boldsymbol{p}^{(0)} = [1 \cdots 1]^{\text{T}}$, and in each case, perform just one iteration. The Newton-Raphson iteration formula has the form

$$\begin{bmatrix} -\dfrac{\partial T}{\partial p_1}\left(s_j, \boldsymbol{p}^{(k-1)}\right) V_{\text{in}}(s_j) & \cdots & -\dfrac{\partial T}{\partial p_n}\left(s_j, \boldsymbol{p}^{(k-1)}\right) V_{\text{in}}(s_j) \\ -\dfrac{\partial T}{\partial p_1}\left(s_{j+1}, \boldsymbol{p}^{(k-1)}\right) V_{\text{in}}(s_{j+1}) & \cdots & -\dfrac{\partial T}{\partial p_n}\left(s_{j+1}, \boldsymbol{p}^{(k-1)}\right) V_{\text{in}}(s_{j+1}) \\ \cdots & \cdots & \cdots \\ -\dfrac{\partial T}{\partial p_1}\left(s_{j+n-1}, \boldsymbol{p}^{(k-1)}\right) V_{\text{in}}(s_{j+n-1}) & \cdots & -\dfrac{\partial T}{\partial p_n}\left(s_{j+n-1}, \boldsymbol{p}^{(k-1)}\right) V_{\text{in}}(s_{j+n-1}) \end{bmatrix} \begin{bmatrix} p_1^{(k)} - p_1^{(k-1)} \\ p_2^{(k)} - p_2^{(k-1)} \\ \vdots \\ p_n^{(k)} - p_n^{(k-1)} \end{bmatrix} = - \begin{bmatrix} V_{\text{o}}(s_j) - T\left(s_j, \boldsymbol{p}^{(k-1)}\right) V_{\text{in}}(s_j) \\ V_{\text{o}}(s_{j+1}) - T\left(s_{j+1}, \boldsymbol{p}^{(k-1)}\right) V_{\text{in}}(s_{j+1}) \\ \vdots \\ V_{\text{o}}(s_{j+n-1}) - T\left(s_{j+n-1}, \boldsymbol{p}^{(k-1)}\right) V_{\text{in}}(s_{j+n-1}) \end{bmatrix} \tag{7}$$

where $j = 1$ in the case of the system (5) and $j = n+1$ in the case of the system (6). Since $T(s, \boldsymbol{p})$ is given in symbolic form, the formulas for the elements of the matrix and the vector in equation (7) can be derived before the test. Thus, at first, we set $k = 1$ and solve (7), adapted to the systems (5) and (6), in parallel. The obtained vectors $\left[p_1^{(1)} \cdots p_n^{(1)}\right]^{\text{T}}$ for (5) and (6) are denoted by $\boldsymbol{p}_{\text{A}}^{(1)}$ and $\boldsymbol{p}_{\text{B}}^{(1)}$, respectively. Next, we create

$$\boldsymbol{p}^{(1)} = \frac{1}{2}\left(\boldsymbol{p}_{\text{A}}^{(1)} + \boldsymbol{p}_{\text{B}}^{(1)}\right) , \tag{8}$$

substitute $k = 2$ and perform the second iteration using the Newton-Raphson iteration formula (7) for (5) and (6). The results, labeled $\boldsymbol{p}_{\text{A}}^{(2)}$ and $\boldsymbol{p}_{\text{B}}^{(2)}$, are used to find $\boldsymbol{p}^{(2)} = \frac{1}{2}\left(\boldsymbol{p}_{\text{A}}^{(2)} + \boldsymbol{p}_{\text{B}}^{(2)}\right)$. This process is continued leading to the sequence $\boldsymbol{p}^{(0)}, \boldsymbol{p}^{(1)}, \boldsymbol{p}^{(2)}, \ldots$ until

$$\left\| \boldsymbol{p}^{(k)} - \boldsymbol{p}^{(k-1)} \right\| = \sqrt{\sum_{i=1}^{n} \left(p_i^{(k)} - p_i^{(k-1)}\right)^2} < \tilde{\varepsilon} \tag{9}$$

and

$$\sqrt{\sum_{j=1}^{2n} \left(V_{\text{o}}(s_j) - T\left(s_j, \boldsymbol{p}^{(k)}\right) V_{\text{in}}(s_j)\right)^2} < \tilde{\tilde{\varepsilon}} \tag{10}$$

where $\tilde{\varepsilon}$ and $\tilde{\tilde{\varepsilon}}$ are the convergence tolerances and $\boldsymbol{p}^{(k)}$ is considered as an approximate solution.

B. *Sketch of the n-fault diagnostic method*

Step 1

Create a set $\mathcal{A}$ of vectors consisting of *n* parameters considered as potentially faulty.

Step 2

In the circuit under test choose an input node and two output nodes. Denote the input voltage by $v_{\text{in}}(t)$ and the output voltages by $\tilde{v}_{\text{o}}(t)$ and $\tilde{\tilde{v}}_{\text{o}}(t)$ and their Lapalce transforms by $V_{\text{in}}(s)$, $\tilde{V}_{\text{o}}(s)$, and $\tilde{\tilde{V}}_{\text{o}}(s)$, respectively. Create two transmittances $\tilde{T}(s, p_1, \ldots, p_N) = \tilde{V}_{\text{o}}(s) / V_{\text{in}}(s)$ and $\tilde{\tilde{T}}(s, p_1, \ldots, p_N) = \tilde{\tilde{V}}_{\text{o}}(s) / V_{\text{in}}(s)$, where $p_1, \ldots, p_N$ are the parameters considered as potentially faulty, in symbolic form.

Step 3

Arrange the diagnostic test as follows. Apply a trapezoidal voltage $v_{\text{in}}(t)$ and measure the output voltages $\tilde{v}_{\text{o}}(t)$ and $\tilde{\tilde{v}}_{\text{o}}(t)$.

Find their Laplace transforms $V_{\mathrm{in}}(s)$, $\widetilde{V}_{\mathrm{o}}(s)$, and $\widetilde{\widetilde{V}}_{\mathrm{o}}(s)$ using the procedure described in Section III.

Step 4

Chose one of the elements (vectors $n\times 1$) of the set $\mathcal{A}$, label it $\boldsymbol{p}$, and consider the transmittance $\widetilde{T}(s,\,\boldsymbol{p})$. Write the equation

$$\widetilde{V}_{\mathrm{o}}(s)-\widetilde{T}(s,\,\boldsymbol{p})V_{\mathrm{in}}(s)=0 \qquad (11)$$

and reproduce it to $2n$ equations

$$\widetilde{V}_{\mathrm{o}}(s_j)-\widetilde{T}(s_j,\,\boldsymbol{p})V_{\mathrm{in}}(s_j)=0 \qquad j=1,\ldots,2n\ . \qquad (12)$$

Divide (12) into two systems of n equations as described at the beginning of this Section and solve (12) using the proposed iterative method. If the solution does not exist because the method does not converge or the obtained solution is not accepted (e.g., some parameters are negative), the considered vector of the parameters is discarded.

Step 5

Repeat step 4 for all vectors of the set $\mathcal{A}$ in succession. As a result, one or more vectors of the parameters are obtained.

Step 6

In the case of one solution, we are given the set of faulty parameters and their values. Otherwise, repeat steps 4 and 5 for the transmittance $\widetilde{\widetilde{T}}(s,\,\boldsymbol{p})$ and proceed to step 7.

Step 7

From among the solution vectors corresponding to both transmittances $\widetilde{T}(s,\,\boldsymbol{p})$ and $\widetilde{\widetilde{T}}(s,\,\boldsymbol{p})$ select all the pairs of the vectors consisting of the same parameters and determine the distance between them. The Euclidean norm of the difference of the vectors is taken as the measure of the distance. It is labeled DIS and assigned to each of the pair. Choose the pairs having the smallest values of DIS and find the average values of the corresponding elements of the vectors creating the pair. They form the sets of the parameters which meet the diagnostic test.

III. PROCEDURES USED BY THE METHOD

A. *Finding the Laplace transforms*

1) Input voltage

The input voltage is chosen as a trapezoidal function as shown in Fig. 1.

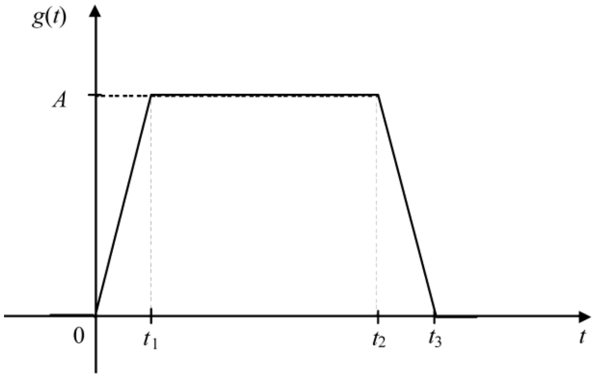

Fig. 1. Trapezoidal function $g(t)$

It is described in the time domain as

$$g(t)=\frac{A}{t_1}tu(t)-\frac{A}{t_1}(t-t_1)u(t-t_1)-\frac{A}{t_3-t_2}(t-t_2)u(t-t_2)+$$
$$+\frac{A}{t_3-t_2}(t-t_3)u(t-t_3)$$

where $u(t)$ is the unit step function. To find its Laplace transform we apply the equations: $\mathcal{L}(tu(t))=\dfrac{1}{s^2}$ and $\mathcal{L}((t-b)u(t-b))=\dfrac{1}{s^2}\mathrm{e}^{-bs}$ $(b>0)$ finding

$$G(s)=\frac{A}{s^2}\left[\frac{1-\mathrm{e}^{-st_1}}{t_1}+\frac{\mathrm{e}^{-st_3}-\mathrm{e}^{-st_2}}{t_3-t_2}\right]\ . \qquad (13)$$

2) Output voltage

The output voltage $v_{\mathrm{o}}(t)$ measured in the course of the diagnostic test does not have a standard function description in the time domain. To find its Laplace transform, we use the following procedure. Let us consider a continuous function $f(t)$ represented by a smooth line in Fig. 2. This function can be approximated by a staircase function $f_a(t)$ consisting of rectangles with hights $f(h), f(2h),\ldots, f(Nh)$ and identical width $h=t_{j+1}-t_j$ where $f(t)\cong 0$ for $t>Nh$.

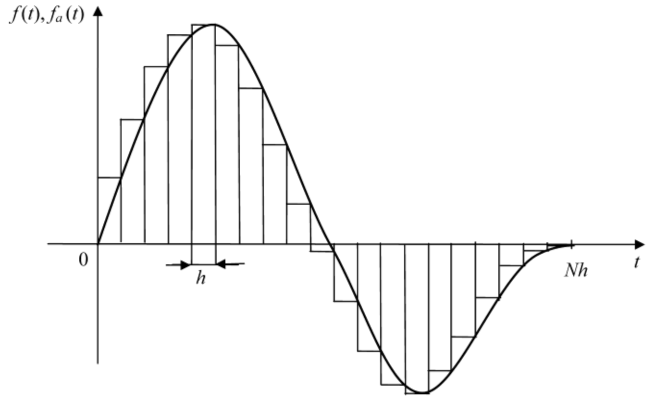

Fig. 2. A smooth line $f(t)$ approximated by a staircase function $f_a(t)$

The staircase function has the following representation

$$f_a(t)=f(h)\big(u(t)-u(t-h)\big)+f(2h)\big(u(t-h)-u(t-2h)\big)+$$
$$+\cdots+f(Nh)\big(u(t-(N-1)h)-u(t-Nh)\big).$$

Since $\mathcal{L}(u(t))=\dfrac{1}{s}$ and $\mathcal{L}(u(t-b))=\dfrac{1}{s}\mathrm{e}^{-sb}$ $(b>0)$, the Laplace transform $F_a(s)$ of $f_a(t)$ is as follows

$$F_a(s)=\frac{1}{s}\left\{f(h)+\sum_{i=1}^{N-1}\big[f((i+1)h)-f(ih)\big]\mathrm{e}^{-sih}-f(Nh)\mathrm{e}^{-sNh}\right\} \qquad (14)$$

B. *Selection of the values of s*

To write the system of equations (4), $2n$ real positive values of s are needed. They are chosen in the preliminary stage before the test as follows.

Consider the transmittance $T(s,\,\boldsymbol{p})$ at nominal values of the parameters. Take the trapezoidal input voltage $v_{\mathrm{in}}(t)$ and find its Laplace transform $V_{\mathrm{in}}(s)$ (see Section III.A). Calculate the Laplace transform of the output voltage $V_{\mathrm{o}}(s)=T(s,\,\boldsymbol{p})V_{\mathrm{in}}(s)$.

Find the output voltage in the time domain and using the procedure described in Section III.A determine its Laplace transform labeled $\hat{V}_o(s)$. Consider an interval $[s^-, s^+]$ of real positive values of s and divide it into $L >> 2n$ subintervals by points s_i. For each s_i find $\eta(s_i) = |V_o(s_i) - \hat{V}_o(s_i)|$ and select a subinterval where $\eta(s_i) < 10^{-4}$. Divide it uniformly by $6n$ points s_k and choose 100 sets consisting of $2n$ points of s_k using random selection assuming uniform distribution. Decompose each of the $2n$ point sets into two n point sets ranked in ascending order of values. For any of them, calculate the determinants of the Jacoby matrices of (5) and (6) at the starting points and choose the ones which give their largest absolute values. Repeat this procedure for all the transmittances and the Jacoby matrices at the starting points.

IV. EXAMPLES

To illustrate the method described in Sections II and III we consider three examples. The diagnosed circuits are shown in Figures 3, 4, and 5, where nominal values of the parameters are indicated. All the operational amplifiers included in the circuits are characterized by the ideal model. The computations were executed on a PC with the processor Intel (R) Xeon (R) E3-1230 using MATLAB R2012a with Symbolic Toolbox.

Example 1

Let us consider the Sallen-Key filter shown in Fig. 3

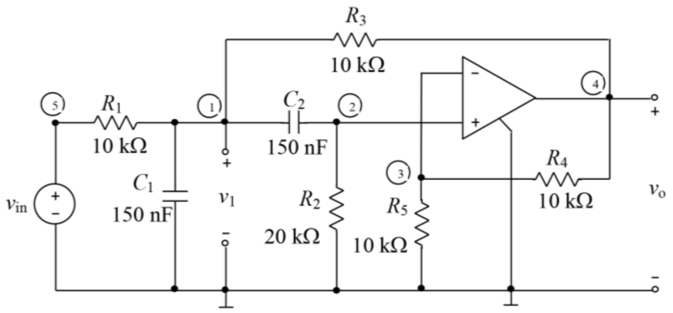

Fig. 3. The Sallen-Key bandpass filter

Double soft fault diagnosis is performed in this circuit. We consider the following sets of 12 potentially faulty pairs of the elements: R_1C_1, R_1C_2, R_1R_2, R_1R_3, C_1C_2, C_1R_2, C_1R_3, R_1R_5, C_1R_5, R_5C_2, R_5R_2, R_5R_3. The double fault diagnosis is carried out using the method described in Sections II and III with $n = 2$. Before the test we choose node 5 as the input node and nodes 1 and 4 as the output nodes and create two corresponding transmittances in symbolic form. The constants characterizing the trapezoidal input voltage (see Fig. 1) are picked out as: $A = 1\text{ V}$, $t_1 = 18\text{ ns}$, $t_2 - t_1 = 2\text{ ms}$, $t_3 - t_2 = 18\text{ ns}$.

In the course of the diagnostic test, the output voltages $v_o(t)$ and $v_1(t)$ are measured using the measurement system consisting of a two-channel digital phosphor oscilloscope with 2.5 GS/s real-time sample rate and 350 MHz bandwidth, and a programmable function generator with 250 MS/s sample rate.

The samples of the voltages at the instances mT_s, where $T_s = 4 \cdot 10^{-8}\text{s}$ is the sample spacing, are chosen and their Laplace transforms are calculated as described in Section III. Next, the diagnostic equations are written and solved, using the proposed iterative method, for each of the 12 pairs of the parameters. If only one pair of the acceptable parameters is determined, the double fault is located and identified. Otherwise, the procedure described in Section II, Step 7, is applied. While running this procedure, the smallest value of DIP, labeled MIN, is found and the pairs of the parameter vectors whose $\text{DIP} < 1.25\text{ MIN}$ are selected. The other pairs of the vectors, if any, are discarded. For each of the selected pairs, the parameter values are calculated as described in Section II, Step 7. As a result, one or more pairs of the parameters corresponding to the actual fault and the virtual ones can be found.

For illustration 12 double faults in the circuit were diagnosed. The outcomes are summarized in Table I where the relative error of the parameter x is defined as

$$\left|\frac{x^{\text{actual}} - x^{\text{determined}}}{x^{\text{actual}}}\right| 100\% .$$

In 11 cases (91.7%) the pairs of faulty parameters provided by the method include the actual one. In 9 cases this is the only one pair, whereas in 2 cases the actual pair is accompanied by one or two pairs of virtual faults. Every time the values of the actual faulty parameters are correctly estimated. In 81.8% of the cases the relative error does not exceed 4%. In 1 case (8.3%) the method fails.

TABLE I.
EXAMPLE 1. RESULTS OF DOUBLE SOFT FAULT DIAGNOSIS

Number of the case	Faulty parameters, resistances in Ω, capacitances in nF		Faulty parameters provided by the method		Relative errors in %	
1	$R_1 = 7471$	$C_1 = 105.6$	$R_1 = 7146$	$C_1 = 103.0$	4.35	2.46
2	$R_1 = 7471$	$C_2 = 224.3$	$R_1 = 7216$	$C_2 = 219.5$	3.41	2.14
3	$R_1 = 7471$	$R_2 = 22250$	$R_1 = 7199$	$R_2 = 22608$	3.64	1.61
4	$R_1 = 7471$	$R_3 = 12085$	$R_1 = 7473$ and the virtual faults: R_1, C_2 and R_1, R_2	$R_3 = 11949$	0.03	1.13
5	$C_1 = 105.6$	$C_2 = 224.3$	$C_1 = 101.5$	$C_2 = 207.6$	3.88	7.45
6	$C_1 = 105.6$	$R_2 = 22250$	$C_1 = 100.1$	$R_2 = 22420$	5.21	0.76
7	$C_1 = 105.6$	$R_3 = 12085$	Only the virtual faults: C_1, R_2	FAIL		
8	$R_1 = 7471$	$R_5 = 7471$	$R_1 = 7387$	$R_5 = 7346$	1.12	1.67
9	$C_1 = 105.6$	$R_5 = 7471$	$C_1 = 102.2$	$R_5 = 7424$	3.22	0.63
10	$R_5 = 7471$	$C_2 = 224.3$	$R_5 = 7580$	$C_2 = 215.7$	1.46	3.83
11	$R_5 = 7471$	$R_2 = 22250$	$R_5 = 7538$	$R_2 = 22170$	0.90	0.36
12	$R_5 = 7471$	$R_3 = 12085$	$R_5 = 7600$ and the virtual faults: C_1, R_5	$R_3 = 11200$	1.73	7.32

Example 2

In the circuit depicted in Fig. 4 we perform double fault diagnosis considering 9 pairs of the parameters as potentially faulty: C_1C_2, R_4R_6, R_4R_8, R_4C_1, R_4C_2, R_6R_8, R_8C_2, R_6C_1, R_8C_1. The diagnoses were carried out using the method described in Sections II and III. The diagnostic test was arranged using the input node 10 and the output nodes 6 and 9. The input voltage was the same as in Example 1. The test was simulated numerically. The results are presented in Table II.

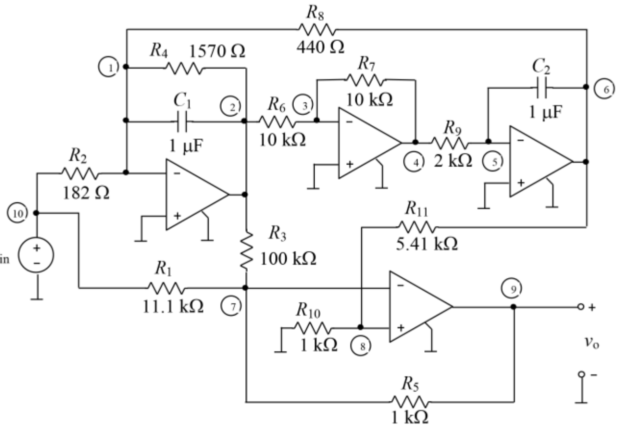

Fig.4. A circuit for Example 2

In 7 cases (77.8%) the method found the actual pair of the faulty parameters appeared as the only one in 5 cases and accompanied by one pair of virtual faults in 2 cases. In two cases (22.2%) the method failed.

TABLE II.
EXAMPLE 2. RESULTS OF DOUBLE SOFT FAULT DIAGNOSIS

Number of the case	Faulty parameters, resistances in Ω, capacitances in µF		Faulty parameters provided by the method		Relative errors in %	
1	$C_1=2.00$	$C_2=0.5$	$C_1=2.00$	$C_2=0.50$	0.00	0.00
2	$R_4=1000.0$	$R_6=12000.0$	$R_4=959.5$	$R_6=12035.5$	4.05	0.30
3	$R_4=1000.0$	$R_8=140.0$	$R_4=980.5$	$R_8=141.0$	2.00	0.71
4	$R_4=1000.0$	$C_1=2.00$	Only the virtual faults: R_4, R_8	FAIL		
5	$R_4=1000.0$	$C_2=0.50$	Only the virtual faults: C_1, C_2	FAIL		
6	$R_6=12000.0$	$R_8=140.0$	$R_6=12008.5$	$R_8=140.4$	0.07	0.28
7	$R_8=140.0$	$C_2=0.50$	$R_8=140.8$ and the virtual faults: R_8, R_6	$C_2=0.53$	0.57	0.60
8	$R_6=12000.0$	$C_1=2.00$	$R_6=11943.0$	$C_1=2.00$	0.47	0.00
9	$R_8=140.0$	$C_1=2.00$	$R_8=140.4$	$C_1=2.01$	0.29	0.50

Example 3

Figure 5 shows the Tow-Thomas filter containing three operational amplifiers. Double fault diagnoses are performed in this circuit choosing ten pairs of the parameters considered as potentially faulty: R_1R_2, R_1R_3, R_1R_4, R_2R_3, R_2R_4, C_2R_1, C_2R_2, C_2R_3, C_1R_4, C_1C_2. For this purpose, the method described in Sections II and III is used assuming $n=2$. Before the test, we choose the input node 1 and the output nodes 3 and 7, and create the corresponding transmittances in symbolic form. The input voltage has a trapezoidal shape (see Fig. 1) with $A = 10$ V, $t_1 = 18$ ns, $t_2 - t_1 = 0.12$ ms, $t_3 - t_2 = 18$ ns. The diagnostic test was simulated numerically. The results are presented in Table III. In 8 out of 10 cases, the method finds the actual pair of faulty parameters and correctly estimates their values, but in 6 of them it also provides one virtual faulty pair. Unlike Examples 1 and 2 there are many virtual faults in this circuit.

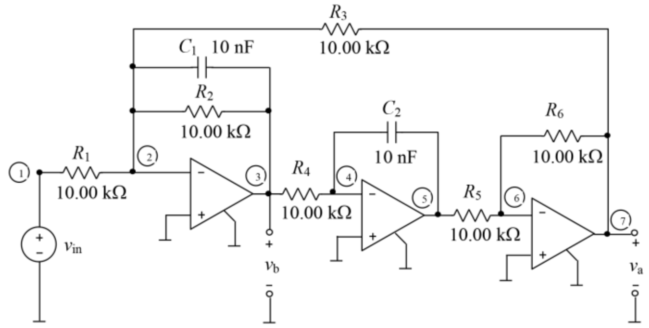

Fig.5. The Tow-Thomas filter for Example 3

TABLE III.
EXAMPLE 3. RESULTS OF DOUBLE SOFT FAULT DIAGNOSIS

Number of the case	Faulty parameters, resistances in kΩ, capacitances in nF		Faulty parameters provided by the method		Relative errors in %	
1	$R_1=15.00$	$R_2=5.00$	$R_1=14.99$	$R_2=5.00$	0.07	0.00
2	$R_1=15.00$	$R_3=4.40$	Only the virtual faults: R_2, R_3	FAIL		
3	$R_1=15.00$	$R_4=12.00$	$R_1=14.99$ and the virtual faults: C_2, R_1	$R_4=11.99$	0.07	0.08
4	$R_2=5.00$	$R_3=4.40$	$R_2=5.00$	$R_3=4.40$	0.00	0.00
5	$R_2=5.00$	$R_4=12.00$	$R_2=5.00$ and the virtual faults: C_2, R_2	$R_4=12.01$	0.00	0.08
6	$C_2=5.00$	$R_2=5.00$	$C_2=5.00$ and the virtual faults: R_2, R_4	$R_2=5.00$	0.00	0.00
7	$C_2=5.00$	$R_3=4.40$	Only the virtual faults: R_1, R_3	FAIL		
8	$C_2=5.00$	$R_1=15.00$	$C_2=5.02$ and the virtual faults: R_1, R_4	$R_1=15.04$	0.40	0.27
9	$C_1=12.0$	$R_4=12.00$	$C_1=12.01$ and the virtual faults: C_1, C_2	$R_4=12.00$	0.08	0.00
10	$C_1=12.0$	$C_2=5.00$	$C_1=12.03$ and the virtual faults: C_1, R_4	$C_2=5.00$	0.25	0.00

V. CONCLUSION

This paper offers a method for multiple soft fault diagnosis of analog linear circuits. Unlike the verification methods, e.g. [27-28], the proposed approach achieves all objectives of the fault diagnosis: detection, location, and identification. The diagnostic test, performed in the transient state, requires measurement devices including a two-channel digital oscilloscope, and a programmable function generator.

The equations which express the Laplace transform of the output signals in terms of the Laplace transform of the input signal are written on the basis of two transmittances. Each of the equations is reproduced to a system of overdetermined nonlinear algebraic equations. The procedure applied for this purpose is effective. The numerical method for solving the system of $2n$ equations in n unknown variables developed in this paper is simple, fast, and reliable. The transmittances in symbolic form, required by the diagnostic method, can be determined either by hand or using a dedicated computer program.

The diagnostic method can be applied to small and medium-size circuits. It concentrates on double fault diagnosis because triple, quadruple, and higher-order fault cases occur rarely.

ACKNOWLEDGEMENTS

The authors thank Professor S. Hałgas for the useful discussion they had with him.

REFERENCES

[1] D. Gizopoulos, *Advances in electronic testing. Challenges and methodologies*. (Springer, Dordrecht, 2006)

[2] P. Kabisatpathy, A. Barua and S. Sinha, *Fault diagnosis of analog integrated circuits*. (Springer, Dordrecht, 2005).

[3] Y. Sun (ed.), *Test and diagnosis of analog mixed-signal and RF integrated circuits: the system on chip approach*, (IET Digital Library, UK, 2008)

[4] D. Binu, B.S. Kariyappa, "A survey on fault diagnosis of analog circuits: Taxonomy and state of the art", *Int. J. Electron. Commun. (AEÜ)*, vol. 73, pp. 68-83, 2017. doi: 10.1016/j.aeue.2017.01.002.

[5] Z. Czaja, "Using a square-wave signal for fault diagnosis of analog parts of mixed-signal electronic embedded systems", *IEEE Trans. Instrum. Meas.*, vol. 57, pp. 1589-1595, 2008. doi: 10.1109/TIM.2008.925342

[6] H. Han, H. Wang, S. Tian, N. Zhang, "A new analog circuit fault diagnosis method based on improved Mahalanobis distance", *J. Electron. Test.*, vol. 29, pp. 95–102, 2013. https://doi.org/10.1007/s10836-012-5342-z.

[7] Ch. Yang, S. Tian, B. Long, F. Chen, "Methods of handling the tolerance and test-point selection problem for analog-circuit fault diagnosis", *IEEE Trans. Instrum. Means*., vol. 60, pp. 176-185, 2011. doi: 10.1109/TIM.2010.2050356

[8] Q.Z. Zhou, Y.L. Xie, X.F. Li, D.J. Bi, X. Xie, S.S. Xie, "Methodology and equipments for analog circuit parametric faults diagnosis based on matrix eigenvalues", *IEEE Trans. Appl. Superconductivity*, vol. 24, pp. 1–6, 2014. https://doi.org/10.1109/TASC.2014.2340447.

[9] Y. Deng, Y. N. Liu, "Soft fault diagnosis in analog circuits based on bispectral models", *J. Electron. Test.*, vol. 33, pp. 543-557, 2017. https://doi.org/10.1007/s10836-017-5686-5.

[10] S. Djordjevic, M.T. Pesic, "A fault verification method based on the substitution theorem and voltage-current phase relationship", *J. Electron. Test*., vol. 36, pp. 617-629, 2020. https://doi.org/10.1007/s10836-020-05901-5.

[11] T. Gao, J. Yang, S. Jiang, "A novel incipient fault diagnosis method for analog circuits based on GMKL-SVM and wavelet fusion feature". *IEEE Trans. Instrum. Meas*., vol. 70, 2021. https://doi.org/10.1109/TIM.2020.3024337.

[12] Y. Li, R. Zhang, Y. Guo, P. Huan, M. Zhang, "Nonlinear soft fault diagnosis of analog circuits based on RCCA-SVM", *IEEE Access*., vol. 8, pp. 60951-60963, 2020. doi.org/10.1109/ACCESS.2020.2982246.

[13] M. Tadeusiewicz, S. Hałgas, "A new approach to multiple soft fault diagnosis of analog BJT and CMOS circuits", *IEEE Trans. Instrum. Meas*., vol. 64, pp. 2688–2695, 2015. https://doi.org/10.1109/TIM.2015.2421712.

[14] M. Tadeusiewicz, S. Hałgas, "A method for local parametric fault diagnosis of a broad class of analog integrated circuits", *IEEE Trans. Instrum. Meas*., vol. 67, pp. 328–337, 2018. https://doi.org/10.1109/TIM.2017.2775438.

[15] Y. Xie, X. Li, S. Xie, X. Xie, Q. Zhou, "Soft fault diagnosis of analog circuits via frequency response function measurements", *J. Electron. Test*., vol. 30, pp. 243–249, 2014. https://doi.org/10.1007/s10836-014-5445-9.

[16] M. Tadeusiewicz, S. Hałgas, M. Korzybski, "An algorithm for soft-fault diagnosis of linear and nonlinear circuits", *IEEE Trans. Circ. Syst.-I.*, vol. 49, pp. 1648-1653, 2002. doi: 10.1109/TCSI.2002.804596.

[17] M. Tadeusiewicz, S. Hałgas, "Soft fault diagnosis of linear circuits with the special attention paid to the circuits containing current conveyors", *Int. J. Electron Commun. (AEÜ)*, vol. 115, 2020. https://doi.org/10.1016/j.aeue.2019.153036.

[18] M. Tadeusiewicz and S. Hałgas, "A method for multiple soft fault diagnosis of linear analog circuits", *Measurement*, vol. 131, pp. 714-722, 2019. doi: 10.1016/j.measurement.2018.09.001.

[19] M. Jahangiri, F. Razaghian, "Fault detection in analogue circuit using hybrid evolutionary algorithm and neural network", *Analog Int. Cir. Sig. Proc*., vol. 80, pp. 551-556, 2014. https://doi.org/10.1007/s10470-014-0352-7

[20] P. Jantos, D. Grzechca, J. Rutkowski, "Evolutionary algorithms for global parametric fault diagnosis in analogue integrated circuits*", Bull. Polish Acad. Scien*., vol. 60, pp. 133-142, 2012. doi: 10.2478/v10175-012-0019-4

[21] C. Yang, "Multiple soft fault diagnosis of analog filter circuit based on genetic algorithm", *IEEE Access*., vol. 8, pp. 8193-8201, 2020. https://doi.org/10.1109/ACCESS.2020.2964054.

[22] D. Grzechca, "Soft fault clustering in analog electronic circuits with the use of self organizing neural network", *Metrol Meas Syst.*, vol. 8, pp. 555–568, 2011. doi: 10.2478/v10178-011-0054-8

[23] B. Long, M. Li, H. Wang, S. Tian, "Diagnostics of analog circuits based on LS-SVM using time-domain features", *Circuits Syst. Signal. Process*., vol. 32, pp. 2683-2706, 2013. https://doi.org/10.1007/s00034-013-9614-3

[24] R. Sałat, S, Osowski, "Support Vector Machine for soft fault location in electrical circuits", *J. Intelligent Fuzzy Systems.*, vol. 22, pp. 21-31, 2011. doi: 10.3233/IFS-2010-0471.

[25] D. Grzechca, "Construction of an expert system based on fuzzy logic for diagnosis of analog electronic circuits", *Int. Journal of Electronic and Telecomunications,* vol. 61, pp. 77-82, 2015. doi: 10.1515/eletel-2015-0010

[26] P. Bilski, "Analysis of the ensemble of regression algorithms for the analog circuit parametric identification", *Measurement*, vol. 170, pp. 503–514, 2020. https://doi.org/10.1016/j.measurement.2020.107829.

[27] M. Tadeusiewicz, M. Ossowski, "A verification technique for multiple soft fault diagnosis of linear analog circuits", *Int. Journal of Electronic and Telecommunications*, vol. 64, pp. 83-89, 2018. doi: 10.24425/118150.

[28] M. Tadeusiewicz, M. Ossowski, "Modeling analysis and diagnosis of analog circuits in z-domain", *J. Circ. Syst. Comput*, vol. 29, no. 02, 2020. https://doi.org/10.1142/S0218126620500280

[29] G. Fedi, S. Manetti, M.C. Piccirilli, J. Starzyk, "Determination of an optimum set of testable components in the fault diagnosis of analog linear circuits", *IEEE Trans. Circ. Syst.-I*, vol. 46, pp. 779-787, 1999. doi: 10.1109/81.774222

[30] S. Manetti, M.C. Piccirilli, "A singular-value decomposition approach for ambiguity group determination in analog circuits", *IEEE Trans. Circ. Syst.-I*, vol. 50 pp. 477-487, 2003. doi: 10.1109/TCSI.2003.809811.

[31] S. Saeedi, S.H. Pishgar, M. Eslami, "Optimum test point selection method for analog fault dictionary techniques", *Analog Integr. Circuits Signal Processing.*, vol. 100, pp. 167-179, 2019. https://doi.org/10.1007/s10470-019-01453-7.

[32] X. Tang, A. Xu, R. Li, M. Zhu, J. Dai, "Simulation-based diagnostic model for automatic testability analysis of analog circuit", *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*., vol. 37, pp. 1483-1493, 2018. https://doi.org/10.1109/TCAD.2017.2762647.