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The problems of mathematical modelling of rolling bearing vibrations

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Abstract. The problems of mathematical modelling of vibration signal for bearings with specific geometrical structure or defect is important insofar as there are no model bearings (to facilitate carrying out a calibration procedure for industrial measurement systems). It is even more so that there are no precise reference systems to which we would compare the results. This article presents a general outline of the most important studies on modelling of vibrations in rolling bearings. Papers constituting the basis for the most recent studies and a review of articles from the past few years have been considered here. Five different models have been analyzed in detail in order to show the directions of the latest studies. Completed analysis presents different viewpoints on the issue of modelling a rolling bearing operation. This overview article makes it possible to derive the final conclusion that in order to include all factors affecting bearing vibrations, even those ignored in the most recent models, it is necessary to carry out practical statistical research including the principles of multicriteria statistics. This approach will facilitate developing a versatile model, also applicable to predicting vibrations of a new bearing just manufactured in a factory.

Key words: bearing vibration control, mathematical modelling, industrial measurement.

1. Introduction

Vibrations in rolling bearings have three sources: structural, production and operating. Structural sources of vibration occurrence result from the very nature of a rolling bearing operation. The number of rolling elements transmitting the load recurrently changes while the bearing works. As a result of this, variable elastic deformation develops in ball-raceway contact zones. Considerable self-excited vibrations may occur as a consequence of recurrent changes in the position of an imposed load. The causes of the development of vibrations in rolling bearings are related to the shape and size deviations, appearing at the stage of production or assembly. These include the following: excessive shape inaccuracies, roughness, or waviness of working surfaces of individual bearing components, and after assembly - bearing and basket slackness, as well. Moreover, the causes include point defects of a rolling element and raceway and bearing, and grease impurities formed during the production process. Operating causes for the generation of vibrations are related to the use of bearings and abrasive and fatigue wear processes. Existing deviations and the condition of surfaces of mating components in a rolling bearing change during operations. Also, new point and surface defects (e.g. pitting) appear [1, 2].

In general, the latest research on the measurement of rolling bearing vibrations can be divided into two groups: practical issues (publications on rolling bearing diagnostics) and theoretical issues (regarding mathematical modelling of bearing vibrations).

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A significant part of scientific articles on practical issues is devoted to the assessment of the technical condition of a bearing operating in a specific mechanical structure. The high popularity of this issue resulted in numerous papers represented by such articles as [3–8]. Articles of this type discuss diagnostic methods based on the analysis of real vibrations so as to precisely determine the specific reasons of a defect. The papers related to theoretical issues (described in Sections 2 and 3) are mostly of mathematical equations. However, some of these models are validated and were omitted in this article. In most cases, articles on modelling provide a historical overview of modelling, and then they discuss just one new model. This article has been written to demonstrate different viewpoints, and to show which factors are omitted in the latest studies.

During its service life, a rolling bearing is exposed to numerous and very complex physical phenomena, which make its mathematical model much more complicated if considered thoroughly. If it is necessary to model simultaneous deformations within a ball-raceway contact zone, lubrication, friction processes or geometrical structure of working surfaces, it becomes apparent that the use of analytical methods is very difficult or just impossible. It may even happen that the analytical solution of the equation is possible only when using approximations that make this solution practically useless. Numerical methods are applied in other cases [9]. Numerical methods used to solve differential equations are based on a geometrical interpretation of a differential equation. The essence of numerical methods is to replace usually labour-intensive calculations with other methods. Indeed, the majority of currently introduced mathematical models of a rolling bearing include models developed as a result of using numerical methods.

At least a dozen or so factors affect the dynamic condition of any manufactured bearing free from operating and/or assembly defects. First of all, they concern the manufacturing accuracy and obtained geometrical structure of surfaces of major bearing components, basic parameters of any completed bearing (radial and axial clearance, frictional moment and rotation accuracy), and factors related to auxiliary bearing components (lubricant, cage and seals). The authors search for a model, which would facilitate predicting vibrations in all manufactured bearings subject to quality control.

Considering that to some extent even the most accurate model (taking into account many factors) will always depart from the real conditions, and taking into account the specific character of industrial vibration measurements in rolling bearings (brief measurement of a new bearing without operating defects, examination of an unmounted bearing, sometimes also without grease), the authors intend to develop a bearing model based on the principle of multicriteria statistics. Numerous studies on theoretical modelling of rolling bearing vibrations, including [10-16] have been reviewed for this purpose. There are models used for experimental studies; however, they do not often include all the factors affecting vibrations and typically do not refer to quality control in industrial plants. Moreover, the available literature occasionally discusses empirical models [17, 18], although those are incomplete. One can also find comparative studies showing differences in simulations between two models only [19].

This article is intended to compare several different models and to show factors which are included in the most recent theoretical models. Descriptions provided by the authors facilitate finding those factors, which are often omitted. For this purpose, besides a general description of some models, five concepts representing different approaches to the problem have been demonstrated here in detail. The discussed models are also selected to demonstrate different numbers of degrees of freedom.

2. Industrial measurement of rolling bearing vibrations in quality control

Industrial measuring systems for rolling bearing vibrations belong to the most critical equipment in production plants. Larger companies have even tens of devices of this type. Most often, three devices measuring vibrations are located at the end of each production line. Two of them are operated continuously to measure vibrations in all finished products. Any bearing that leaves production line is automatically placed in one device, and after completing the measurement of one bearing side, it is put in the second one, the other way round. The measurement automation is less complicated if two bearing sides are measured at two different stations. Also, time required for a check becomes shorter. The third device at the production line end is used for a recheck (manual) of products possibly rejected by automatic control. Moreover, the laboratories of industrial companies are equipped with manually operated extra equipment used for thorough bearing checks, testing new solutions and performing statistical analyses. Figure 1 shows one of these devices.

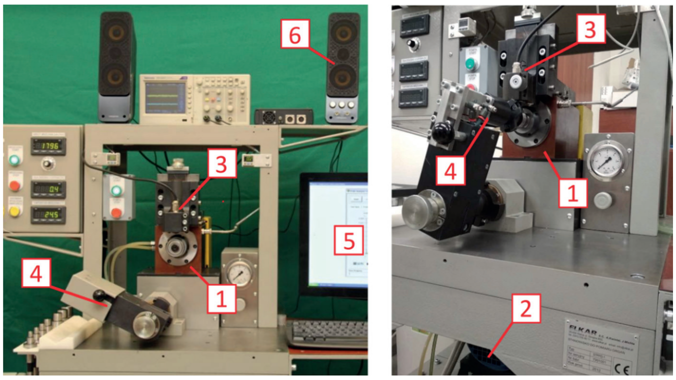

Fig. 1. The industrial testing stand used to measure rolling bearing vibrations: 1 – spindle, 2 – electric motor, 3 – positioning set including vibration sensor, 4 – pusher, 5 – monitor, 6 – loudspeaker

A spindle (1) driven by an electric motor (2) is the central element of the testing equipment. The positioning set including a vibration sensor (3) is located above the spindle. Axial load is exerted on the bearing through the pusher (4). Apart from a display panel or monitor (5) allowing us to read out the result, the system is also provided with a loudspeaker (6) for audio monitoring of the received measuring signal. Often, very experienced factory workers are able to diagnose a bearing defect on the basis of the signal. The methods used to measure rolling bearing vibrations are based on the internal procedures, which must comply with the official standards [20–22]. According to the measuring principle, the tested rolling bearing is set on a shaft spinning inside the spindle. The spindle has a multi-purpose seat for shafts adapted to different bearing types. The rotational speed of the shaft, and thus the inner ring of the bearing, is precisely defined – 1800 rpm. If agreed by the manufacturer and the buyer, in justified cases the rate of rotation can be altered to 3600 rpm, 900 rpm, or 700 rpm. The tested bearing requires axial load to ensure the correct measurement of vibrations. The load is applied by way of pushing the outer ring by a force dependent on a bearing type. Usually, pushers have adequate adjustment, and are replaceable or to some extent versatile so as to match the tested bearing size. Radial vibrations of working and loaded bearing are registered by an electrodynamic vibration velocity sensor, in direct contact with fixed outer ring. The sensor is mounted in a clamp, which can move it along two axes. Figure 2 shows the discussed principle of operation. Minimum measurement duration should be 0.5 s (for 1800 rpm). The measurement should be quick enough not to reduce the production output (each manufactured bearing undergoes a vibration measurement at the production plant, without any exceptions), and slow enough for readings to stabilize. Changes in the results may arise due to random factors only. The measurement procedure should also consider minimizing the effect of the unstable dynamic condition of a bearing, connected with the beginning of the interaction between the bearing balls and raceway, or still insufficiently spread lubricant. In most cases, production plants check bearing vibrations with applied target grease. The measurement of lubricated bearings represents actual behavior of the manufactured bearing during regular service. However, some companies test bearing vibrations before applying grease. In this case, interacting surfaces are covered with a thin film of oil. Then, it is assumed that if the vibration level in unlubricated bearings does not exceed a critical value, all the more it will not exceed it after applying the lubricant, since grease dampens vibrations.

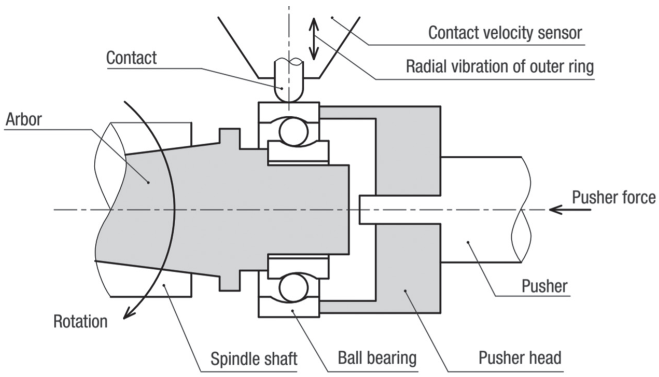

Fig. 2. Schematic presentation of industrial measurement of rolling bearing vibrations [23]

3. The development of mathematical models of rolling bearing vibrations and general review of the most recent publications

The first mathematical model of rolling bearing vibrations was presented by Sunnersjo in 1978 [24]. Published studies were focusing on vibrations of bearings with positive slackness, loaded radially. Self-excited vibrations (occurring independently of the bearing quality and precision) were analysed. The model had two degrees of freedom (displacement of inner ring in two perpendicular directions). Displacement of vibrations was modelled using contact theory according to Hertz equations, omitting mass and inertia of rolling elements. Generally, the foundations for the models discussed today were developed in the 1980s and 1990s. The essential studies on the discussed topic are described below.

In 1979, Gupta published a cycle of four articles describing the dynamics of a rolling bearing motion (determining the forces and moments generated at rolling contact). The first two studies [25, 26] concern a mathematical description of interactions between rolling components and raceways in roller bearings, whereas the next two [27, 28] contain a description of mechanical phenomena occurring in ball bearings. Moreover, these studies also include the effect of lubrication and interactions of rolling elements with the cage. In 1980, Meyer, Ahlgren and Weichbrodt presented in [29] the method used to predict vibrations in a ball bearing with imperfections including the misalignment of rings and wrong ball size. Lagrange equations were solved for bearing raceway moving in time under rotation forces of balls. Authors McFadden and Smith modelled vibrations in a ball bearing containing one- (1984) and multi-point (1985) defects, located on an inner raceway [30, 31]. The signal model is based on a generation of a series of pulses induced by ball surface impacts with a defect located on the raceway. These pulses appear recurrently, depending on the rotational speed of the inner ring. In 1985, Rahnejat and Gohar presented a theoretical analysis of vibrations in a setup consisting of two bearings supporting a rotating shaft [32]. In this study, operation of bearings with grease is modelled as a nonlinear system of springs and dampers, and the analysis includes the case of imbalance or changing structure of an inner raceway. In 1990, Aini presented the study on the analysis of motion for precise grinding spindle with rolling bearings. Ball contact with raceway was shown as nonlinear springs simulating elastic strain. Additionally, the model version was demonstrated, which included lubrication of bearings as well. The model was put to validation, proving good correlation of both theoretical and empirical results. In 1993, the same author (with Gohar) presented an extended analysis of this problem [33], and in 1995 (with Rahnejat and Gohar) demonstrated a wide range of experimental works used for comparison with the results of previous simulations [34]. Frequently referenced work [35] by Yhland from 1992 demonstrates the model of a rolling bearing motion considering the waviness of both raceways and the ball and uneven distribution of cage pockets. In 1997, Tandodn and Choudhury proposed an analytical model of a bearing with 3 degrees of freedom (expanded in 2006). The model was used inter alia to predict frequencies and amplitudes of vibration components in rolling bearings (for vibrations resulting from a defect located either on an outer raceway, inner raceway, or on one of the rolling elements [36, 37]. An ordinary ball bearing under radial and axial load was modelled. The demonstrated model simulates a spectrum of vibrations that contains peaks with characteristic frequencies of defects and their harmonics. In 2002, Liew, Feng and Hahn demonstrated four different models of bearing vibrations [38]. The most versatile model has 5 degrees of freedom and includes the following: loading induced by a centrifugal force of the rolling element, working angle, or radial play. A bearing model with five degrees of freedom not only contains radial displacement of inner raceway in two directions, but also axial displacement and rotation around radial axes. In 2003, Sopanen and Mikkola were the first to present the model of a bearing with six degrees of freedom [39, 40]. Additional sixth degree of freedom of a rolling bearing (rotation around a bearing axis) is generated by friction forces. However, an empirical approach has been applied due to the high complexity of this added equation. The discussed model includes a lot of factors, inter alia, basic rolling bearing kinematics, elastic deformation of bearing components, elastohydrodynamic lubrication, waviness and roughness of working surfaces, or single-point and scattered defects.

The majority of present-day scientific articles on mathematical modelling of rolling bearing vibrations are the continuation and gradual improvement of the first models. Authors around the world are involved in the discussed issue, and below there is a review of the most recent scientific works from the last few years.

Authors of study [41] present a mathematical model of bearing vibrations, derived on the basis of a rotor system analysis. Various defects are simulated, including faults related to the loading of a bearing, and inner or outer ring defects. Then, a model verification is shown at the test stand containing a three-axial vibration acceleration sensor. On the other hand, article [42] presents an analytical model simulating an interaction between rolling elements and a raceway, based on a model from 1984. This model considers the bearing geometry, relative sliding forces, and resultant normal and tractive forces. Moreover, the model considers strictly defined geometrical defect located on the outer raceway. Verification tests were carried out at two test stands, containing the tested bearings with fault resembling the modelled defect, and vibrations were registered by an accelerometer installed in the test area. Then, the authors of the other study [43] modelled bearing vibrations using the HOSTSMO {*higher-order super-twisting sliding mode observation*} technique expected to improve prediction accuracy for the effects of the operation of a bearing with a specific defect. The model developed in this way was verified at the test stand containing a vibration acceleration sensor. The tests included bearings with an outer or inner raceway or ball defects, and bearings without any defects. Article [44] discusses a numerical model of rolling bearing vibrations embodying the loading distribution in a bearing, elasticity of individual bearing elements, oil film properties and, also, a signal transmission between the bearing and vibration sensor. The verification of the diagram with vibrations measured at the test stand is carried out using many parameters characteristic for rolling bearings, including arithmetic mean, effective value, peak factor, shape factor, kurtosis, or impulsivity factor. Spectra of vibrations are directly compared, as well. Study [45] proposes a scheme applicable primarily in the analysis of the impact of shell rigidity and defect size on vibrational characteristics of the bearings in a rotor system. The mathematical formula of the model contains defects on both the inner and outer ring. The obtained results are compared on the basis of an effective value derived from the vibration acceleration signal. A more complex problem is discussed in study [46]. Its authors do not model point defect anymore, but faults distributed along the perimeter of both the outer and inner ring. Most often, defects of this type appear as a result of electro-erosion or propagation of point defects. The discussed paper contains a comparison of vibration spectrum for a bearing with a natural raceway defect with the vibration spectrum simulated using a bearing model with defects distributed evenly at specific angles along the perimeter of rings, among other things. An interesting line for further research is outlined in study [47], where authors focus on the discrepancies between the vibration signal of a bearing damaged as a result of its prolonged operation (or time-consuming durability tests), and the signal from a deliberately damaged bearing [48] (e.g. using an electric engraver, by means of drilling, or electro-abrasive treatment). The discussed algorithm is expected to predict the signal of vibrations in a rolling bearing damaged naturally on the basis of the signal of vibrations obtained for the bearing with a deliberately made defect.

4. Specification of selected latest models of rolling bearing vibrations

The models presented below are understood as a system of equations, the number of which depends on the assumed degrees of freedom. Due to highly complicated formulas, this paper is limited only to the general formulation of the modelling of bearing subassemblies and phenomena occurring during its operation. The analysis shown below concerns five different models considering various factors affecting the vibration level generated by a rolling bearing. Symbols of unambiguous parameters, e.g. the mass of the outer ring or the angular position of the rolling element, have been unified for all models. In other cases, when the parameter symbol is specific for a given model only, original designation has been left in order to make it easy to find a certain parameter in the reference material.

4.1. Basic model of a ball bearing with four degrees of freedom . The first of the described models presents a standard dynamic model of a ball bearing with four degrees of freedom. Apart from self-excited vibrations derived on the basis of deformations generated as a result of applying Hertz contact theory, publication [49] from 2016, written by Shi, Su, and Han, includes in its equations local rectangle-shaped defects located on an inner or outer raceway, or a ball. In the demonstrated example, the contact between the ball and raceway is modelled as a simple system of spring and damper connected in parallel. This induces a nonlinear relation between the force and deformation. The outer raceway is installed on a rigid support, whereas the inner raceway is stiffly fixed to a rotating shaft. Permanent radial loading is applied to the shaft. The analyzed model is shown in Fig. 3.

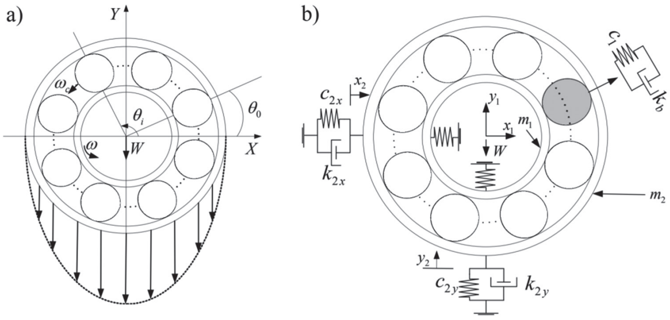

Fig. 3. Basic model of a rolling bearing with four degrees of freedom [49]

Four differential equations that form mathematical model of rolling bearing vibrations are shown by the formulas (1)–(4):

$$m_1\ddot{x}_1 + c_1\dot{x}_1 - c_1\dot{x}_2 + k_{1x}x_1 - k_{1x}x_2 = F_x\,, \tag{1}$$

$$m_1\ddot{y}_1 + c_1\dot{y}_1 - c_1\dot{y}_2 + k_{1y}y_1 - k_{1y}y_2 = F_y - W_s\,, \tag{2}$$

$$m_2\ddot{x}_2 + (c_1 + c_{2x})\dot{x}_2 - c_1\dot{x}_1 + k_{2x}x_2 = F_x\,, \tag{3}$$

$$m_2\ddot{y}_2 + (c_1 + c_{2y})\dot{y}_2 - c_1\dot{y}_1 + k_{2y}y_2 = F_y\,, \tag{4}$$

where: m_1, m_2 are the mass of the inner ring and the shaft $\{1\}$ and mass of the outer ring $\{2\}$; x_1, x_2, y_1, y_2 are the radial displacements in horizontal $\{x\}$ and vertical $\{y\}$ direction of the inner ring $\{1\}$ and the outer ring $\{2\}$; c_1, c_{2x}, c_{2y} are damping coefficients for the ball $\{1\}$ and bearing housing in a radial direction: horizontal $\{2x\}$ and vertical $\{2y\}$; k_{1x}, k_{1y}, k_{2x}, $k_{2y}$are the radial stiffness coefficient for the shaft $\{1\}$ in horizontal $\{x\}$ and vertical $\{y\}$ direction, for housing $\{2\}$ in horizontal $\{x\}$ and vertical $\{y\}$ direction; W_s denotes the radial load applied to the shaft; F_x, F_y are components of generated forces in the radial direction: horizontal $\{x\}$ and vertical $\{y\}$, given by the following formula:

$$F_x = \sum_{j=1}^{z} K\big[(x_1 - x_2)\cos\theta_j + + (y_1 - y_2)\sin\theta_j - C_r - \varsigma_j\big]^{3/2}\cos\theta_j\,, \tag{5}$$

$$F_y = \sum_{j=1}^{z} K\big[(x_1 - x_2)cos\theta_j + + (y_1 - y_2)\sin\theta_j - C_r - \varsigma_j\big]^{3/2}\sin\theta_j\,, \tag{6}$$

z denotes the number of balls; K is the deflection factor or constant for elastic deformation of Hertz contact; θ_j is the angular position of the ball relative to axis x; C_r denotes internal radial clearance; ς_j is the deformation due to a faulty location in an angular position of the j-th rolling element.

As mentioned above, this model also considers the defect in a form of a simple rectangle with sharp edges, visible in Fig. 4.

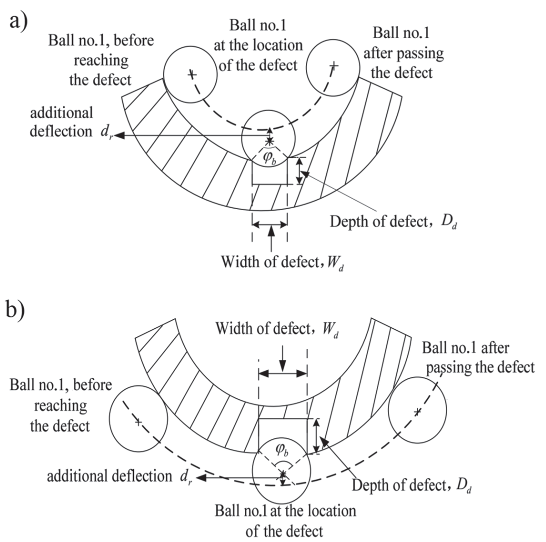

Fig. 4. Modelling of basic defect in a form of a rectangle with sharp edges: a) defect location on outer raceway, b) defect location on inner raceway [49]

The inclusion of a single defect on the outer and/or inner ring raceway in the simulated signal is connected with satisfying conditions derived on the basis of geometrical relations shown in Fig. 4. In the model, element ς_j is responsible for an additional motion induced by the defect. This element can get additional amplitude d_r (when ball falls into the defect) or 0 value (in any other case). Depending on the condition to be satisfied, for both raceways parameter ς_j is:

$$\varsigma_j = \begin{cases} d_r, & \left|\text{mod}\left[\theta_j - \left(\varphi_d + \frac{\varphi_b}{2}\right),\, 2\pi\right]\right| < \frac{\varphi_b}{2}\,, \\ 0, & \text{any other case.} \end{cases} \tag{7}$$

On the other hand, deformation resulting from the location fault in an angular position of the ith rolling element ς_j for the ball is:

$$\varsigma_j = \begin{cases} 0, & j \neq k, \\ d_r, & 0 < \varphi_d < \varphi_b\,,\;\; \pi < \varphi_d < (\pi + \varphi_b),\;\; j = k, \\ 0, & \text{any other case,} \end{cases} \tag{8}$$

where: for the inner raceway $\varphi_d = \omega_s t + \varphi_{d0}$; for the outer raceway $\varphi_d = \varphi_{d0}$; and for the rolling element $\varphi_d = \text{mod}(\omega_s t + \varphi_{d0},\ 2\pi)$; φ_d denotes the angular position of the defect at a given moment; φ_b is the angle related to defect width; k is the number of the ball, on which the defect was modelled; φ_{d0} denotes the initial angular position of the defect; ω_s is the angular velocity of the shaft; t denotes the time.

The demonstrated dynamic model of a rolling bearing, with the defect on the outer raceway and/or the inner raceway and/or the rolling element, can be solved and analyzed, e.g. using the Runge-Kutta numerical method.

4.2. The model including changing defect topography. The second model shows the modification of the standard model, involving inclusion of the changes in the topography of simulated local defects located on the inner and outer raceway. The dynamic model of a ball bearing from 2014, described by Liu and Shao [50] has 2 degrees of freedom. They are related to the displacement of the inner ring with the shaft in two radial directions perpendicular to each other. The outer ring located in the housing is considered immovable and non-deformable. Same as in the case of the model described in 3.1, the work of the ball with raceways is simulated using a non-linear system including a spring and a damper, and elastic deflections are derived from the Hertz theory. The model structure includes the change in defect topography caused by the ball hitting defect edge. Cyclic strokes induce plastic deformations, as a result of which edges of local defects become blunt, changing the sharp edge into small, smooth, and flat surfaces. It is assumed that both defect edges change symmetrically. The process involving defect topography modifications is shown in Fig. 5.

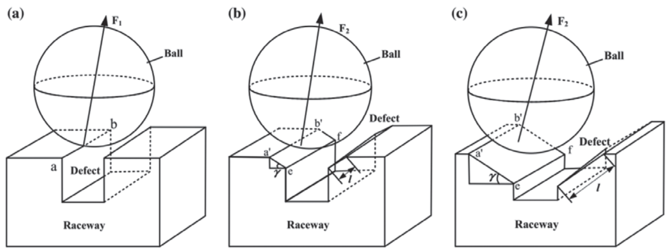

Fig. 5. Different types of ball contact with defect edge: a) sharp edge, b) slightly blunt edge, c) strongly blunt edge [50]

The change in the local defect topography changes both the trajectory of the ball running on race, and the nature of contact between the rolling element and race. In the case of the sound race, deformations are simulated as ball-ball contact. When the rolling element hits still an unmodified defect edge, the contact nature changes into ball-line contact, whereas when the bearing ball rolls on a blunt edge, elastic deformation is simulated as ball-plane contact. In the described model, the type of the ball contact with the defect is being identified on the basis of three geometrical parameters of the defect: ξ_d is the ratio of defect length L (defect size in ball motion direction) to defect width B (defect size in a perpendicular direction to the ball motion route), ξ_{bd} is the ratio of the ball diameter to the smaller of defect sizes: $d/\min(L;B)$ and γ $(0 < \gamma < \pi/2)$ is the defect edge cutting angle. This formulation facilitates the simulation of different defect shapes, e.g. point defect or crack. Different defect shapes and their size designations are demonstrated in Fig. 6.

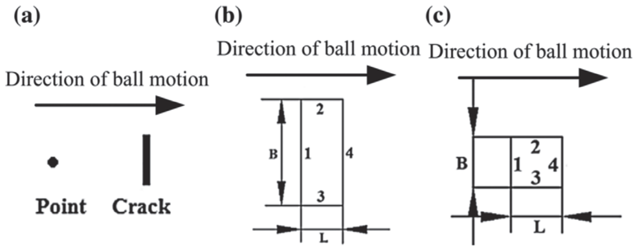

Fig. 6. Simulation of two types of different defects: a) point defect and raceway crack, b) crack simulation, c) point defect simulation [50]

The two equations for the described model motion are as follows:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + K\sum_{j=1}^{Z}\lambda_j\delta_j^{3/2}\cos\theta_j = F_x\,, \tag{9}$$

$$m_1\ddot{y}_1 + c_1\dot{y}_1 + K\sum_{j=1}^{Z}\lambda_j\delta_j^{3/2}\sin\theta_j = F_y\,. \tag{10}$$

Total deformation resulting from the contact of j-th ball set at the angle θ_j:

$$\delta_j = x_1 cos\theta_j + y_1\sin\theta_j - C_r - H', \tag{11}$$

where: λ_j denotes the loading zone parameter for loading generated by j-th rolling element (its value can be either 1 or 0, depending if δ_j is a positive value, or less or equal to 0). The other symbols are the same as those described for the first model shown in 3.1. Element H is a time-varying function of forced displacement caused by simulated defects. Value H depends on the values of coefficients ξ_d and ξ_{ed}, and takes four different forms, which are a function of the following geometrical parameters of the model:

$$H_1,\ H_2,\ H_3,\ H_4 = f\left(\gamma, l, D_o, D_i, H,\ B,\ d\right), \tag{12}$$

where: l denotes the blunt edge surface length; D_i is the inner raceway diameter; D_o is the outer raceway diameter; H denotes the defect depth and d is the ball diameter.

The authors of the publication simulate the vibration acceleration signal by way of solving the demonstrated equations using the Runge–Kutta fourth-order method with a constant time step.

4.3. The model including outer ring deformation. The third analyzed model presents a modification of the standard model, involving the inclusion of the deformation of the outer ring, built using finite elements. Finite elements are of two-node type and none of them is stiffly blocked. As a result, the outer ring is fully deformable in radial direction. Tadina and Boltezar are the authors of model [51] developed in 2011. The model shown in Fig. 7 has four degrees of freedom, related to the outer ring displacement in two radial directions perpendicular to each other, the ball motion, and outer ring deformation.

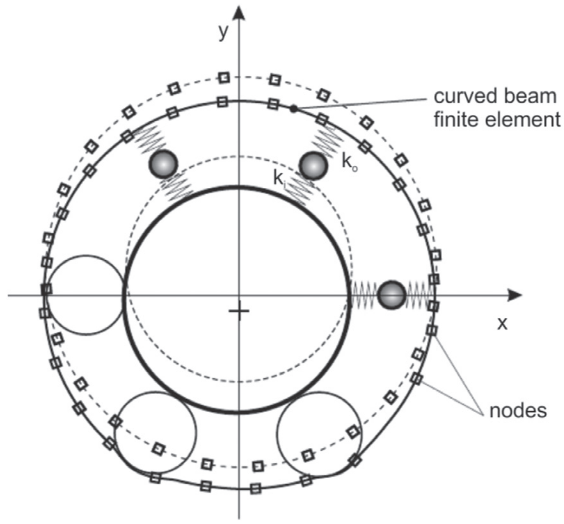

Fig. 7. The model of bearing vibrations considering deformation of an outer ring consisting of two-node finite elements [51]

Moreover, the simulation includes the centrifugal force of balls, sliding between the working surfaces and flexibility of housing, which can undergo an unsymmetrical deformation. In the demonstrated model, the ball is modelled as a separate bearing component, which can work both with the inner and outer raceway. In the two previous models, the ball was simulated as an interaction between two raceways connected in parallel by a damper and spring. However, the model does not embody a whole range of factors, including: lubrication, the ball rotation around own axis, changes in the ball motion path during operation, temperature changes (change in grease viscosity, expansion of rolling elements and raceway, and reduced material strength) as well as the interaction between the basket and other bearing elements.

$$m_1\ddot{x}_1 + k_1\sum_{j=1}^{Z}\delta_{1j}^{3/2}\frac{\rho_j cos\theta_j + x_2 - x_1}{\chi_j} = W_s\cos\theta_s\,, \tag{13}$$

$$m_1\ddot{y}_1 + k_1\sum_{j=1}^{Z}\delta_{1j}^{3/2}\frac{\rho_j\sin\theta_j + y_2 - y_1}{\chi_j} = W_s\sin\theta_s\,, \tag{14}$$

$$m_b\ddot{\rho}_j - \rho_j\omega^2 + k_2\delta_{2j}^{3/2} - k_i\delta_{i,j}^{3/2}\frac{\beta}{\chi_j} = - m_b g\sin\theta_j - m_b\left(\ddot{x}_2\cos\theta_j + \ddot{y}_2\sin\theta_j\right), \tag{15}$$

$$M\ddot{\Delta} + C\dot{\Delta} + K\Delta = F_m, \tag{16}$$

where: $\beta = \rho_j + (x_2 - x_1)\cos\theta_j + (y_2 - y_1)\sin\theta_j$; k_1, k_2 are the radial flexibility coefficient for the inner $\{1\}$ and outer $\{2\}$ raceway; δ_{1j}, δ_{2j} are the deformations of the ball – inner ring $\{1\}$ and the ball – outer ring $\{2\}$ contact; ρ_j denotes the the distance of j-th ball from the outer ring centre in radial direction; χ_j is the distance of the ball centre from the inner raceway in the radial direction; θ_s denotes the angular position of the inner raceway relative to the outer raceway centre; m_b is the ball mass; ω denotes the angular velocity of the shaft; g denotes the gravitation acceleration; M is the matrix of masses; C is the damping matrix; K is the rigidity matrix; Δ denotes the vector of displacement; F_m is the vector of node forces.

The main purpose of the authors' model was to study the bearing behaviour during operation at a time-varying rotational speed (bearing run-down). The demonstrated differential equations of motion were solved numerically using the modified Newmark integration method.

4.4. The model including waviness of rings and lubrication. In the models discussed so far, the raceways had homogeneous surfaces and time-varying elastic deformations were based on the sinusoidal function. In 2015, Liu and Shao demonstrated a bearing model embodying both race surface waviness and work of a rolling element with a raceway in a lubricating medium [52]. The model facilitates a simulation of waviness, which is the same along the entire perimeter, but it may differ on the inner and outer races. As a result of cyclic changes in the raceway radii of curvature, the nature of the rolling element and race work undergoes considerable changes. Moreover, the changes in the raceway radii of curvature generate changes in time of lubricating oil film thickness. The discussed model concerns a rolling bearing, but it can be also effectively used to predict ball bearing vibrations.

Figure 8 demonstrates how contact rigidity may change depending on the angular position of the outer ring.

The system of equations consists of two dynamic equations of a motion, and the degrees of freedom are related to the axial displacement of the outer ring in two perpendicular directions.

$$m_1\ddot{x}_1 + c\dot{x}_1 + K'\sum_{j=1}^{Z}\lambda_j\delta_j^{10/9}\cos\theta_j = F_x, \tag{17}$$

$$m_1\ddot{y}_1 + c\dot{y}_1 + K'\sum_{j=1}^{Z}\lambda_j\delta_j^{10/9}\sin\theta_j = F_y, \tag{18}$$

K' – total flexibility coefficient for the contact between rolling element and races (smooth and with waviness). Coefficient K' includes all of the following: a total flexibility coefficient for the contact between one rolling element and two non-lubricated smooth races, a total flexibility coefficient for the contact between one rolling element and two non-lubricated races characterised by certain waviness, a total flexibility coefficient for the contact between one rolling element and two lubricated smooth races, a total flexibility coefficient for the contact between one rolling element and two lubricated races characterised by certain waviness.

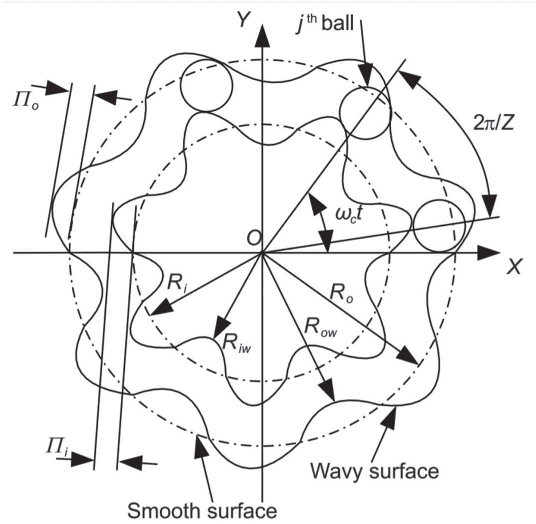

Fig. 8. Diagram of a rolling bearing model that includes raceway surface waviness [52]

All other symbols have already appeared in previous descriptions; however, in the case of a problem formulated in this way, total deformation resulting from the contact of j-th rolling element set at the angle θ_j relative to the horizontal axis is:

$$\delta_j = x\cos\theta_j + y\sin\theta_j - C_r + \Pi_j + h_{ij} + h_{oj}, \tag{19}$$

where: h_{ij} is the central thickness of the film between the rolling element and inner race place of j-th roller, h_{oj} is the central thickness of the film between the rolling element and outer race in place of j-th roller. Time-varying function driving dislocation caused by a given case of waviness in place of the j-th rolling element Π_j is given by the following formula:

$$\Pi = \sum_{s=1}^{N_w}\Pi_{ws}\sin\left(\frac{2L_{ws}}{\lambda_{ws}}\right), \tag{20}$$

where: N_w denotes the number of waves, Π_{ws} amplitude of s-th is the wave, L_{ws} denotes the angular position of wave, average length of s-th wave.

The demonstrated equations are solved using the Runge–Kutta fourth-order method with constant time step.

4.5. Dynamic model of a damaged bearing considering changes in the viscosity damping coefficient. The last of the analyzed models was presented by Kong, Huang, Jiang, Wang, and Zhao in 2019. The model predicts the operation of ball bearings with a localized defect on the outer race and facilitates the examination of the impact of the damping variation on the vibrations generated by the faulty bearing [53].

The damping of vibration in a rolling bearing depends mainly on the internal clearance, applied force, rotational speed of the shaft and also the type of grease used. For other models, the damping coefficient is determined empirically and has a constant value. The authors of this paper presented a concept in which the damping coefficient may change its value. The presented model allows for the prediction of the value of the empirical damping coefficient depending on various bearing operating conditions, e.g. the shaft rotational speed or applied load. This model has 5 degrees of freedom and does not consider some very important elements such as: ball slipping, axial forces, or uneven distance between the balls. In addition, the authors used the assumption that a lubricating film always exists. Figure 9 shows a scheme of the model presented in the discussed paper.

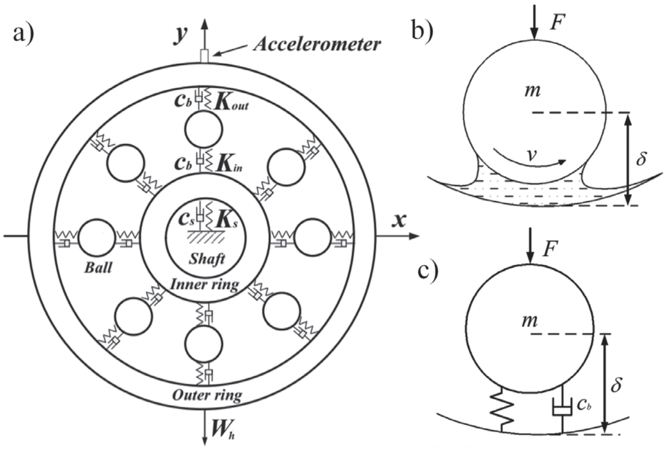

Fig. 9. a) The vibration model of the bearing system with 5 degrees of freedom, b) Contact model of the ball and the raceway, c) a spring-damping model [53]

The five equations representing the described model are as follows:

$$m_1\ddot{x}_1 + c_s\dot{x}_1 + K_s x_1 + F_{1x} + F_{d1y} = 0, \tag{21}$$

$$m_1\ddot{y}_1 + c_s\dot{y}_1 + K_s y_1 + F_{1y} + F_{d1y} = -m_1 g, \tag{22}$$

$$m_2\ddot{x}_2 - F_{2x} - F_{d2x} = 0, \tag{23}$$

$$m_2\ddot{y}_2 - F_{2y} - F_{d2y} = -m_2 g - W_h, \tag{24}$$

$$m\ddot{\delta}_{2j} = \left(F_{1j} + F_{d1j}\right)\cos(\theta_{1j} - \theta_{2j}) - (F_{2j} + m_1) + m_b(2\pi f_c)^2 r, \tag{25}$$

c_s is the damping of the shaft; K_s denotes the elastic coefficient of the rotating shaft F_{1x}, F_{1y}, F_{2x}, F_{2y} are the sum of contact forces between all the balls and the inner ring $\{1\}$ and the outer ring in the horizontal direction $\{x\}$ and vertical direction $\{y\}$; F_{d1x}, F_{d1y}, F_{d2x}, F_{d2y} are the sum of viscosity damping forces between all balls and the inner ring $\{1\}$ and the outer ring in the horizontal direction $\{x\}$ and vertical direction $\{y\}$; W_h is the radial load applied to the housing; F_{1j}, F_{2j} are the contact forces of a j-th ball between the inner $\{1\}$ and outer $\{2\}$ raceways; θ_{1j} and θ_{2j} are the angular positions of the j-th ball on the raceways, respectively; f_c is the rotation frequency of the ball; r is the radius of the ball.

The viscosity damping forces appearing in the model are proportional to the c_b coefficient, which is the viscosity damping coefficient between the ball and the raceway. Its value is affected by the viscosity of the lubricant and may change, depending on the operating conditions of the bearing.

5. Conclusions

The result of the industrial measurement of rolling bearing vibrations carried out as part of quality control process at industrial plants is of unknown value and very difficult to determine accurately. There are no reference bearings for which the volume of the generated vibrations would be precisely known to provide the basis for determining the tested device efficiency. Moreover, there are no systems for measuring the rolling bearing vibrations which would show the correct value for the tested bearing with very high accuracy, and which would be the reference for other systems. It is possible to calibrate a measurement chain of a sensor, to measure the pusher eccentricity or the run-out of the measuring system spindle, but representing the correct value for the vibration level in the tested bearing will never be guaranteed. Consequently, an attempt can be made to develop a theoretical model of a newly built rolling bearing which, considering a considerable number of factors, would allow for predicting the vibration level for a real bearing with known parameters. The analysis shown in the article indicates that there is no model that would embody all highly complicated factors. The completed analyses show that in most cases the following factors are excluded: the inhomogeneous real geometrical structure of the raceway surface, the unsymmetrical shape of the simulated defect, the impact of temperature on material properties of bearing components and grease, the geometrical structure and faults in rolling elements, the deformations in all mating elements of a bearing, a change in time of the existing deviations and properties of the surfaces of raceways and rolling elements, the impurities generated during production processes and grease fouling, defect propagation in time, the interaction of bearing elements with a basket or gaskets, sliding, change of a rolling element trajectory, etc.

In principle, the article is an overview demonstrating selected mathematical models of rolling bearing vibrations, which may possibly be used for industrial purposes. However, as it turns out, most analytical models focus on a bearing with an obvious defect, which may appear only after a long time in service or in wrong service conditions. Therefore, the proposed models are not versatile. Moreover, thorough analysis of these models allows for stating that they are developed using far-reaching simplifications and refer to only a few factors affecting bearing vibrations. There is no mathematical model, which would include the vast majority of real factors. The article facilitates finding a new viewpoint and deriving the following conclusion: in order to create a versatile experimental model, it is required to carry out tests considering all factors affecting the vibration level and using multicriteria statistics.

The sought-after model should include discrete imperfections that affect vibrations (deviations of shape, size and position, excessive waviness, micro-waviness and roughness, or impurities), resulting directly from the production process and possible technological errors. Moreover, the authors believe that the final conclusion constitutes their original contribution.

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