BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 68, No. 6, 2020
DOI: 10.24425/bpasts.2020.134651

CIVIL ENGINEERING

Numerical study for evaluation of a vibration based damage index for effective damage detection

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Abstract. An improved damage detection index for a structural component is proposed, using eigenvalues estimated by means of frequency domain decomposition (FDD) and mode contribution subjected to ambient excitation. It is based on vibration measurements obtained from the acceleration data of a simple steel beam. Since the extraction of modal parameters involves practical limitations and, in general, it is difficult to obtain accurate results, therefore in the proposed method a derivative value of the time series acceleration response, termed modal contributing parameter (MCP), is used in combination with eigenfrequencies. The damage is indicated by element stiffness reduction (ESR). Different damage cases for various stiffness reduction values of 1% to 15% were investigated. Damage identification indices for every single damage and multiple damage cases were calculated. The modified MCP damage detection index showed a high index value, even for low-level damage with an element stiffness reduction of as low as 1% over the existing frequency drop and indices based on mode shape change. MCP index derived from the modal response, considering modal contributions to the entire structural response and eigenvalues for damage detection, improved overall sensitivity and reliability of index results. Both single and multiple cases of damage provided equally accurate results based on the MCP index value.

Key words: damage identification, dynamic response, stiffness reduction, ambient excitation, index sensitivity.

1. Introduction

Civil engineering structures have limited design life spans and begin to degrade as soon as they are commissioned despite the correct design methodology having been used during the design and analysis process. During their service life, in addition to human activities, structures are subjected to numerous external loads such as earthquakes, traffic, fire, and vibrations, experiencing damage and degradation over time. This might affect their efficiency, and in the worst case scenario, it can even lead to structural failure. A similar phenomenon is observed in aerospace and mechanical engineering. Therefore, inspection of structural components for damage is important in making decisions about maintaining these structures. Dynamic testing with the aim of detecting damage has become an increasingly common and effective method in damage identification techniques, now known generally as structural health monitoring (SHM) [1, 2]. Vibration-based SHM technologies have the ability to enhance the design and management of structures in many ways, such as: (i) real-time safety evaluation during regular performance of operations, (ii) analyzing recorded structural response data and thus improving the accuracy of structural assessments, and (iii) optimally planning maintenance and repair activities based on accurate details, resulting in cost savings [3, 4].

In SHM, five-level classification is established with increasing difficulty in their determination, namely detection of damage, its severity, its location, the possibility of looking for damage control, and the last level, which uses the aforementioned details and decides on the remaining life of the structure [5–7]. Unsurprisingly, most of the SHM policy work has been inspired by disasters, and more specifically the collapse of bridges. Also, the lowest level of SHM techniques, e.g. damage detection, may be extremely helpful if used to provide an early warning at the very initial stage of low extent structural damage [8].

Over the past 30 years, analyzing the structure's modal properties and their variations due to damage has emerged as a phenomenon of exceptional interest for researchers. An outstanding and detailed review of the research on damage identification can be found in the work of Doebling [1]. Similarly, in-depth reviews of modal testing-based approaches were presented in [9]. Most of the damage assessment algorithms proposed in the literature assume that baseline information on the undamaged state of the structure is availed for its potential direct comparison with a measured response. This can come as a substantial disadvantage as it implies the availability of data on the component being tested in its undamaged state, and the simple notion that any measured change is due solely to damage, and not to the component's changing environmental or boundary conditions [5].

To date, methods for vibration-based identification of damages applied to structural elements have had acceptable reliability in damage identification for the first two stages of SHM. Any quantitative change in global properties (the eigen-

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Manuscript submitted 2020-05-13, revised 2020-06-29, initially accepted for publication 2020-07-28, published in December 2020

frequency, mode shape and damping) of a vibrating structure can indicate the presence of damage. Among all widely used modal parameters, eigenfrequency measurement is easier to use for quantifying changes in structural response but has relatively low sensitivity. The low sensitivity of frequency changes to the damage requires either super-sensitive instrumentation, high extent damage or otherwise describing the damage as the autocorrelation function of the modal coordinate of the mode. Following the idea of uncorrelated modal coordinates, the modal assurance criteria (MAC) are widely used to determine the degree of correlation that exists between undamaged and damaged structures.

Many studies [10–24] have been carried out contributing to the use of natural frequency or mode shape in damage detection. It is widely reported in the literature that the natural frequency is well-related to global structural stiffness but less responsive to the local stiffness change. Alternatively, in relation to changes in the local structural stiffness, the use of mode shape and its derivatives such as curvature and strain energy distributions obtained from spatial integration of modes has been more promising in evaluating damage location [5]. More recent studies have investigated the effects on mode shapes and corresponding curvatures of localized and distributed damage [25, 26]. Such modal-based approaches have low sensitivity, and in detailed structural health information these methods are not sufficiently accurate.

The latest studies [27–31] also show a new trend in the use of the damage evaluation techniques based on wavelet transformation. Wavelet analysis provides a powerful tool to characterize local features of a non-stationary signal. Different engineering fields deal with non-stationary signals for periodic or continuous assessment of structural components. Usually, the assessment techniques are based on dynamic analysis and incurred for the safety assurance of structure throughout its service life. The frequency component is usually considered an attractive parameter for the evaluation process in reaching a conclusion on whether the structure is damaged or not. Hence, it may need the information about the time span for which the different frequency components present in the signal remain the same. These types of time-varying structural parameters predicting the behavior of structures referring to a damaged or undamaged one are well addressed by the methods utilizing wavelet transformations. Even when wavelet transformation has been commonly used in the identification of time-varying parameters or in the detection of damage, it cannot always provide adequate frequency resolution for long-term signal components marked for vibration on civil engineering structures [32].

In vibration-based SHM methods, certain approaches seem promising but their practical use in civil engineering, dealing with huge and complex structures, poses several practical challenges [33]. Today, several new techniques are being developed which aim to eliminate current challenges in the difference between the predicted system response of the model and the measured system response [34]. The main challenge in the field of structural damage identification research is to check the sensitivity of the damage indicator for minor damages in the presence of unavoidable noise from the measurement.

Despite the success in certain aspects, two potential solutions that are based on either frequency drop or mode shape change methods still suffer from low sensitivity in damage detection at an early stage, as reported in [35]. Therefore, the key idea in this study is to provide the theoretical development of an index of damage detection using a cost-effective and reliable technique which has significantly improved sensitivity. In this analysis, a two-stage method has been devised, combining both eigenfrequency and a contributing parameter of the modal derivative. The proposed parameter was estimated from time series data of accelerations when a beam was subjected to ambient excitation. Additionally, the results obtained by using the proposed damage index are compared with the other two indicators.

2. Vibration based damage detection method

Two common indicators found in the literature [35] are summarized in Sections 2.1 and 2.2 and used to compare with the proposed indicator developed for damage detection based on the work of Park and Oh [36], with improved sensitivity. The first indicator is the frequency drop method based index (FDMi), which relies on natural frequency reduction to detect damage; the second is the mode shape change based index (MSCi), which uses modal assurance criteria (MAC). The proposed indicator is now termed a modal contributing parameter based index (MCPi), manifesting improvement thus obtained by utilizing FDD estimated eigenvalues in the calculation of the index value, described in Section 3.3.

2.1. Frequency drop method based index (FDMi). In routine health assessment of a structure, the approach of natural frequency change is potentially useful, due to easy application and global nature of frequencies [37, 38]. The more severe the damage, the greater the drop in natural frequency. Despite limitations, the method of monitoring frequency drops to detect damage associated with change in frequency is adopted for SHM of civil engineering infrastructures [39]. Usually FDMi is effective when the frequency drop for target structure is 5% or above [40–42]. For frequency drops below 5%, alternatively the mode shape modal assurance criterion (MAC) method, discussed in Section 2.2, can be used to quantify the damage.

Flexural stiffness (EI) contributes to a change in natural frequency (f) of the uniform simply supported Bernoulli-Euler beam for transverse free vibration [35, 43]. The frequency drop method based index can be defined as:

$$FDMi = \left[1 - \frac{f_i^d}{f_i^u}\right] \tag{1}$$

where f_i^u and f_i^d are the natural frequencies at i^{th} mode for undamaged and damaged beams, respectively.

2.2. Mode shape change based index (MSCi). Mode shapes include spatial information and they are less influenced by environmental effects than frequencies. This makes mode shapes a desirable tool for damage detection [6, 44]. Structural damages cause variations in the mode shapes, thus in the case of incomplete correlation between mode shapes of damaged and undamaged states, they ensure the presence of damage [45–47]. The MAC criterion matrix is defined as follows:

$$MAC_{j,k} = \frac{\left(\sum_{i=1}^{n} [\varphi_u]_i^j [\varphi_d]_i^k\right)^2}{\sum_{i=1}^{n} \left([\varphi_u]_i^j\right)^2 \left([\varphi_d]_i^k\right)^2} \tag{2}$$

Where $[\varphi_u]$ and $[\varphi_d]$ represent the mode shapes before damage and after damage, respectively. $MAC_{j,k}$ factor indicates the degree of correlation between the j^{th} and k^{th} mode, and n is the number of measurement nodes. In the case of less severe damage, corresponding to a frequency drop of less than 5%, the MAC method can identify damage successfully in higher-order modes. These modes are very sensitive to damages; however, they are challenging to be identified in real-time situations [48]. Usually, bound value 0.9 is known as the MAC rejection level [46] and any diagonal of less than 0.9 means that the structure is damaged. The expression of the mode shape change based index is as follows:

$$MSCi = \left(1 - MAC_{j,k}\right) \tag{3}$$

3. Modal contributing parameter based index (MCPi)

The modal response of a structure is influenced by the mode contribution at each degree of freedom (DOF). Based on the principle of change in the physical properties of the structure due to damage, it is important to consider each modal contribution while analyzing the vibration characteristics of a structure. In this context, the total number of modes that were extracted from the dynamic responses equals the total number of measuring locations or DOFs.

3.1. Extraction of spatial data from dynamic response. A study was performed to establish a correlation between the MCP index and the change in the stiffness of a damaged structure. Inherent physical properties like the natural frequency and mode shape are commonly extracted in existing damage detection techniques, correlating the state of an undamaged and damaged structure. Unlike these techniques, this study promoted the use of acceleration responses directly measured for a structure to extract modal response without the need for any system identification (SI) procedure. The effects of all potentially extracted modes were represented in the MCP. Thus, the MCP value posed an indicative of the modal response for each mode and each DOF, and was obtained in the manner discussed in the following sections.

The frequency-domain of measured responses having spatial information needs to be appropriately analyzed to classify the modal responses without mixing one mode with another, especially if the modes were closely spaced. Therefore, selecting the suitable filtering technique and required bandwidth enables the extraction of the modal responses with significant meaningful information. Thus, modal response extraction is followed by bandpass filtering [49] in this study and moreover, the set-up of pass bandwidth is also presented in this section.

To simulate the ambient vibration, the concept of white noise input was used for dynamic analysis [48]. The ambient vibration was simulated from a band-limited stationary random white-noise spectrum distributed between 0 and 200 Hz with peak ground acceleration of 0.0005 m/s^2, and the vibration time history was 200 s. Natural frequencies were estimated using a simple and well-recognized, widely used technique in the operational modal analysis (OMA), introduced by Brinker [50–52] and known as frequency-domain decomposition (FDD). So, based on FDD results thus obtained, a filter was applied for extraction of modal responses incorporating the acceleration time-history response of all accelerometers attached to the structures, unlike the way it was used in [36, 53], where the extraction of modal response was based on the Fourier transform of any signal obtained from the structure. FDD shows that a number of singular values raising above the noise level in the single plot in any frequency band is most definitely a modal response, at least if it has a peak as a modal response [48].

The selection of the pass bandwidth of measured dynamic responses is described in Fig. 1. A comparison was made between the interval of two consecutive peaks of eigenfrequencies, selected by the peak-picking method, and then the first eigenfrequency was obtained and the minimum value was considered as the unique pass bandwidth expressed in Eq. (5), which is a difference between the value of upper and lower bound frequencies.

$$B_f = \text{Min}\{B_i, 2\times f_1\} \quad i = 1 \text{ to } (n_m - 1) \tag{4}$$

$$B_T = \beta \times B_f \tag{5}$$

where B_f is the selected frequency bandwidth, B_i is the interval between each set of two consecutive eigenfrequencies, f_1 is the first eigenfrequency and n_m is the number of modes correspondingly the same as the number of dynamic response measurement points. In Eq. (5), B_T is the tunable frequency pass bandwidth range of filter and with β being a tuning coefficient for bandwidth adjustment, defined by the user for each case having a value, it must be greater than 0 and ranges up to 1. 0.9 was assumed as the default value for this study. The user can take this value based on any pre-analysis results or a presumption of experiencing very closely spaced modes. An FDD response of an arbitrary case of a steel beam under ambient base excitation, having first three bending modal frequencies of 10.18, 36.76 and 80.04 Hz, respectively, is expressed in Fig. 1. Using these eigenfrequencies, two potential intervals were determined among those values and compared with twice the first eigenfrequency, resulting in the smallest one selected as filter frequency bandwidth. The dynamic time history $y_T^j(t) = y^j(t)\big|_0^T$ responses of j-th DOF, which are measured from a structure with the help

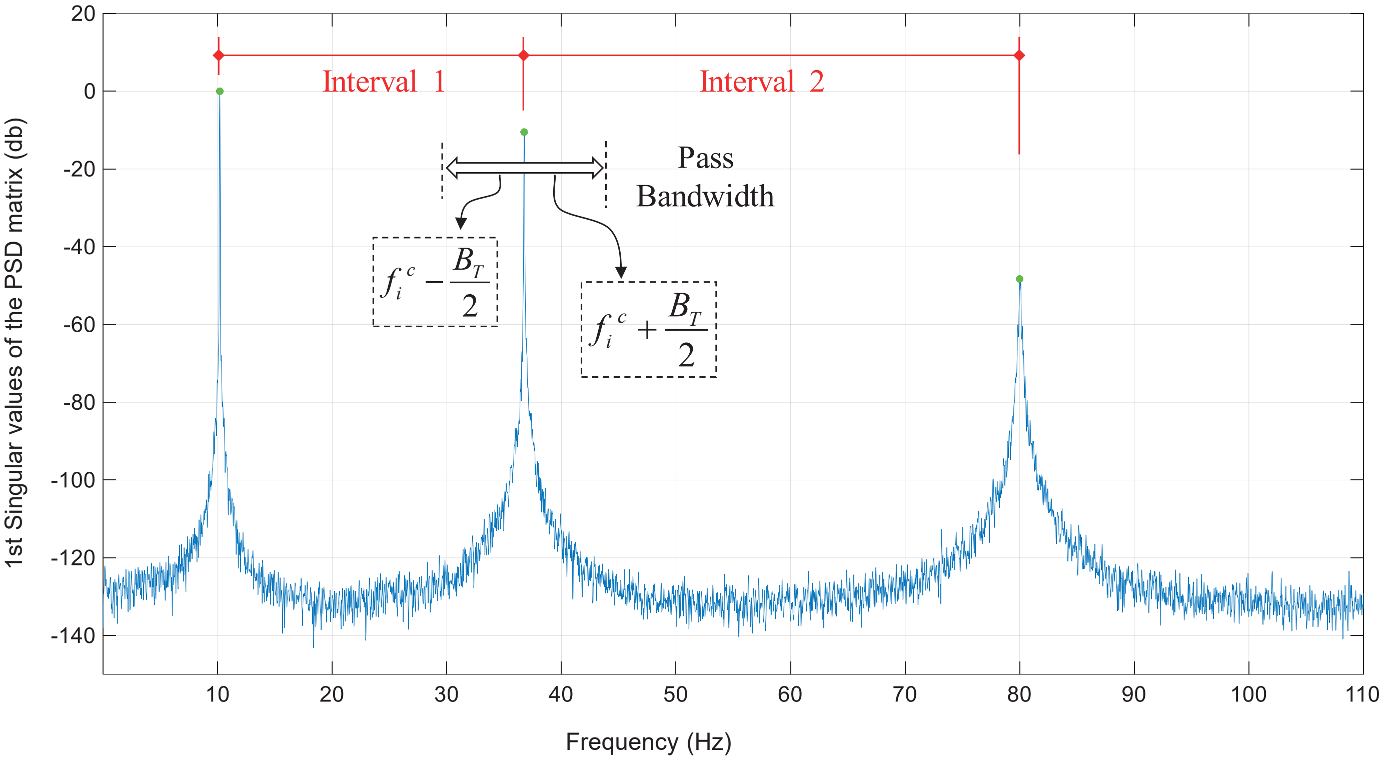


Fig. 1. Selection of pass bandwidth of filter for modal response extraction

of accelerometers installed at n_d different locations for a total time $T > 0$, are separately converted to modal responses with a selected bandwidth using frequency domain analysis. Equations (6–8) represents data processing from the time domain to the frequency domain.

Later in this paper, Section 4 additionally covers the discussion on arrangement of accelerometers and different damage zones based on nodal segmentation.

$$Y^j(f) = \int_{-\infty}^{\infty} y^j(t)\, e^{-ift} dt \qquad j = 1 \text{ to } n_d \tag{6}$$

$$\begin{gathered} Y_i^j(f) = Y^j(f) \\ for\ \ i = 1\ to\ n_m,\ f_i^c - \frac{B_T}{2} \le f_i \le f_i^c + \frac{B_T}{2} \end{gathered} \tag{7}$$

$$\begin{gathered} y_i^j(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y_i^j(f)\, e^{ift} dt \\ for\ \ i = 1 \text{ to } n_m,\ j = 1 \text{ to } n_d \end{gathered} \tag{8}$$

where n_d is the number of DOFs under consideration or it may be considered as the number of dynamic response measurement locations, $Y^j(f)$ is the frequency domain function of the measured response, and n_m is the number of modes required. Moreover, frequency domain response was filtered using the value from Eq. (7) and expressed as $Y_i^j(f)$. B_T is the frequency pass bandwidth range, f_i^c represents the eigenfrequency of mode, and $y_i^j(t)$, being inverse to the frequency domain brings back a filtered discrete-time history response for further post-processing. The acceleration responses were measured from a beam for a total time of 200 seconds. The modal responses illustrated in Fig. 2b were obtained from Eq. (8). Figure 2c shows the small segment enlarged view of the modal responses of accelerations.

3.2. Formulation of modal contributing parameter (MCP). The modal contributions for a vibration-based system estimated using [36] was a measure of the contribution of each mode to the structural response instead of contribution of each DOF to a structural response. MCP represents the modal response of a structure, which explains the combination of response contributions of all the modes under consideration. A fundamental task in structural assessments is the quantitative analysis of three-dimensional structures, especially when structures are large and subject to ambient excitation of ground-borne noise.

The root mean square (RMS) is a very common quantity used to describe structural similarity [54], expressed as $RMS = \sqrt{\sum_n \boldsymbol{k}^2/t}$, calculated between identical parameters of two superposed, i.e. damaged and undamaged, structures. Where $\boldsymbol{k}$ is the modal response comprising total n-values and t denotes total measured response time. Afterwards, MCP was determined using the following mathematical expressions.

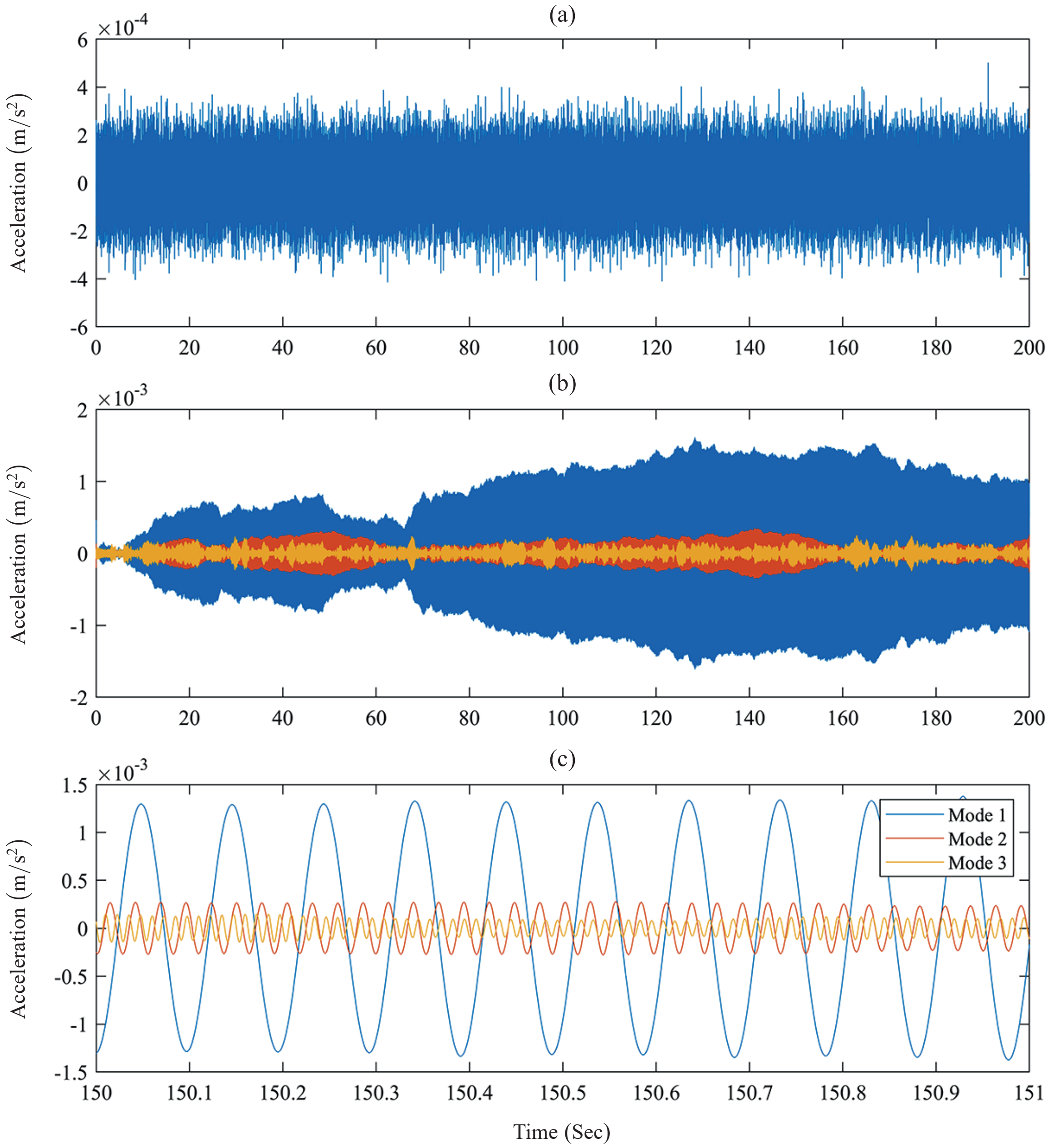


Fig. 2. Dynamic response under ambient base excitation: a) White noise input ambient base excitation, b) Response modal accelerations, c) Enlarged view for modal accelerations for 150 s to 151 s

$$\left(RMS_i^j\right)_{mr} = \sqrt{\frac{\sum_{k=0}^{t} y_i^j(k)^2}{t}} \tag{9}$$

$$MCP_i^j = \frac{\left(RMS_i^j\right)_{mr}}{\sum_{i=1}^{n_m}\sum_{j=1}^{n_d}\left(RMS_i^j\right)_{mr}} \tag{10}$$

where $i = 1\ to\ n_m$, $j = 1\ to\ n_d$ and $\left(RMS_i^j\right)_{mr}$ is the RMS of the structural response while subscript mr stands for modal response and t denotes total time for acceleration measured response of structure.

3.3. Theoretical development: modal contributing parameter based index (MCPi). Dynamic equilibrium equation for free vibration can be expressed as:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = 0 \tag{11}$$

where **K** and **M** are stiffness and mass matrix, respectively. The development of MCPi was set up into two levels. The first level simply restates the computation of the eigenfrequency matrix and mode matrix for Eq. (12), as can be found in references [10, 55–61]. Equation (11) would lead the form for any a solution that corresponds to the equal number of natural modes, say n, as shown below:

$$\mathbf{K}\mathbf{\Phi}_n = \mathbf{M}\mathbf{\Phi}_n\mathbf{E}' \qquad n = 1 \text{ to } a, \text{ solutions} \tag{12}$$

where:
n = mode number equals to a solution,
$\mathbf{K} = [\square]_{a\times a}$, square stiffness matrix,
$\mathbf{M} = [\square]_{a\times a}$, square mass matrix,
$\mathbf{\Phi} = [\square]_{a\times a}$, mode or indicator matrix,
$\mathbf{E}' = [\ddots]_{a\times a}$, eigenfrequency matrix.

It would be reasonable to take the number of solutions/eigenfrequencies equal to the number of DOFs or the number of acceleration measured points and get the n-by-n invertible matrix. Thus, n eigenfrequencies indicate n modes. The indirect modal contributing parameter based matrix $\mathbf{\Phi}^{MCP} = [\square]_{a\times a}$ was formed as appeared in Eq. (13). For the general case of n modes, the indicator matrix $\mathbf{\Phi}^{MCP}$ and eigenfrequency matrix $\mathbf{E}'$ are expressed as:

$$\mathbf{\Phi}^{MCP} = \begin{bmatrix} D_1, M_1 & \cdot & \cdot & D_1, M_n \\ \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot \\ D_n, M_1 & \cdot & \cdot & D_n, M_n \end{bmatrix} \tag{13}$$

$$\mathbf{E}' = \begin{bmatrix} \omega_1^2 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \omega_n^2 \end{bmatrix}. \tag{14}$$

The indictor matrix in Eq. (13) was filled using the MCP values calculated using Eq. (10) of the measured modal responses for each DOF and each mode. Equation (12) can be further stated as:

$$\mathbf{K} = \mathbf{M}\mathbf{\Phi}^{MCP}\mathbf{E}'\left(\mathbf{\Phi}^{MCP}\right)^{-1}. \tag{15}$$

A comparison between the stiffness matrix of both undamaged and damaged structures was required, hence it was assumed that no mass reduction will take place for the damaged case and so $\mathbf{M}$ remained constant. The following equations are stated for undamaged and damaged cases:

$$\mathbf{K}_u = \mathbf{\Phi}_u^{MCP}\mathbf{E}_u'\left(\mathbf{\Phi}_u^{MCP}\right)^{-1} \tag{16}$$

$$\mathbf{K}_d = \mathbf{\Phi}_d^{MCP}\mathbf{E}_d'\left(\mathbf{\Phi}_d^{MCP}\right)^{-1} \tag{17}$$

where subscript letter u and d, respectively, represent damaged and undamaged states. The modal contributing parameter based index was then defined in each matrix cell by:

$$MCP_i^j = \left[1 - \frac{(K_d)_i^j}{(K_u)_i^j}\right]. \tag{18}$$

Singular value of the damage index is needed to quantify the occurrence and severity of damage in the structure. Equation (18) would lead to square matrix of separate cell values of MCP for each DOF and each mode. The single MCPi index value was found using Eq. (20).

$$\left(MCP_i^j\right)_{normal} = \frac{MCP(i,j)}{\max\left|MCP(i,j)\right|_{a\times a}} \tag{19}$$

$$\mathrm{MCPi} = \frac{\sum_{i=1,j=1}^{i=a,j=a}\left(MCP_i^j\right)_{normal}}{a\times a} \tag{20}$$

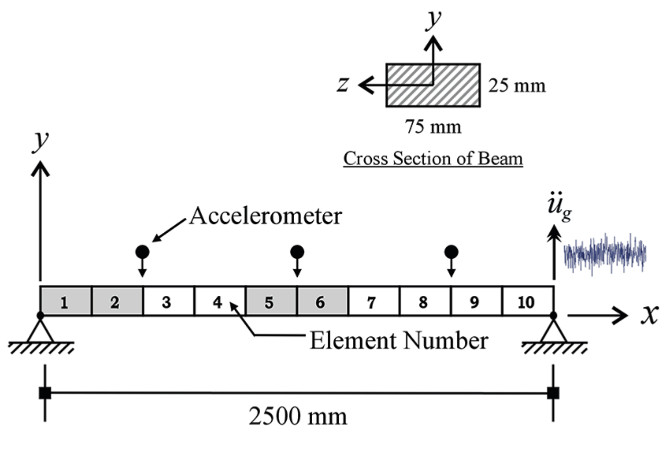
where:
MCPi = single value of damage index,
i = number of mode,
j = number of DOF,
a = total number of eigenfrequencies or modes.

4. Validation of MCPi through numerical simulation

The dynamic response of a 2500 mm long, simply supported steel beam has been studied to investigate the physical relevance of MCPi. Time series white noise was used to simulate the dynamic load and the first three natural modes were considered. The acceleration response to the excitation of white noise was used to extract the modal parameters. It is certainly more difficult than the traditional impact testing for the extraction of modal parameters but found practical for large structures of civil engineering. This example could be seen in two steps. First, it appears to validate the performance of using FDD to extract modal parameters according to their predicted theoretical values. When these parameters were obtained within bound values and with fair enough accuracy, their use in subsequent damage index calculation was considered in the second step. Ultimately, this study will be summarized to show the effectiveness and sensitivity of the proposed damage index called MCPi. Element-stiffness reduction (ESR) has been adopted for the extent of damage and can be measured as:

$$\mathrm{ESR} = \left(1 - \frac{E_d}{E_u}\right).100\% \tag{21}$$

where E_d and E_u stand for the modulus of elasticity of damaged and undamaged beam, respectively. Four groups of damage, named control beam, low level, medium level and higher level were initially classified based on the severity of the damage. Each group was subdivided into two additional cases of damage of increasing stiffness reduction, except for the control beam, with E equal to 200 GPa and the corresponding ESR as 0%. Low-level damage was the lowest at which the ESR was 1% and 3%. The next group was represented as medium-level with two



Geometry of Simply Supported Beam

Damage %	Case 1: Mid Span			Case 2: Near Support		
	Label	EN		Label	EN	
1%	ES1M	5	6	ES1E	1	2
3%	ES3M	5	6	ES3E	1	2
6%	ES6M	5	6	ES6E	1	2
9%	ES9M	5	6	ES9E	1	2
12%	ES12M	5	6	ES12E	1	2
15%	ES15M	5	6	ES16E	1	2

	Label	EN									
		1	2	3	4	5	6	7	8	9	10
Case 3: Multiple Location Damage	MD1	5%	5%	–	–	5%	5%	–	–	–	–
	MD2	10%	10%	–	–	10%	10%	–	–	–	–
	MD3	3%	3%	–	–	5%	5%	–	–	–	–
	MD4	10%	10%	–	–	5%	5%	–	–	–	–

EN = Element Number

Fig. 3. Schematic view of beam model with different damage locations

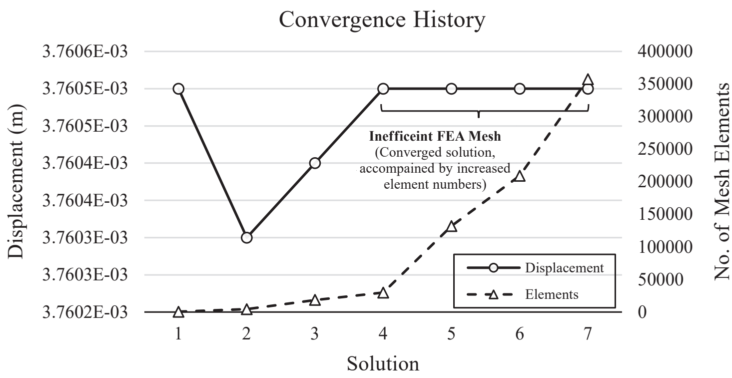
cases of ESR as 6% and 9%. Similarly, the damage cases of the higher-level group were ESR as 12% and 15%. Initially, mid-span and near-end support locations were selected for damage investigation for each case of damage. Two additional damage cases representing the multiple damage case were also considered. Fig. 3 displays all damage cases and the ESR values for the beam which were introduced in this analysis. Primarily, a total of 17 cases were adopted, comprising different levels of ESR and different locations of damage.

4.1. Finite element model of beam. To show the validity of the proposed index, a simple finite element (FE) beam model was built using Workbench, Ansys Inc., representing undamaged and other damaged cases based on displacement convergence criteria. The schematic model of the beam selected for the study is shown in Fig. 3.

The 3D finite element model (FEM) was constructed using an 8-node stress element. Since the total number of elements used in a mesh must be optimized before running a full simulation; the computational power and time required for generation of grid-independent solutions is minimized.

Thus, the opted mesh for the model consists of a total of 612 Hex type elements and 4211 nodes. This was achieved following a mesh interdependency study. The results of the mesh independence study are shown in Fig. 4.

The maximum theoretical deflection of a simply supported beam was calculated as 0.003761 m (37.61 mm). The maximum total deformation (0.003798 m) obtained from the FE model, for the self-weight of the beam, was found well-relating with theoretical results with an error difference of just 0.9%. Maximum displacement from seven mesh sizes of different mesh densities was plotted in Fig. 4 for convergence analysis condition of allowable change in displacement set to 0%. The last four solutions' results are trending towards exceedingly in element numbers and reduction in element size, making the number of nodes range up to approx. 0.5 million in number. The displacement results line becomes horizontal and shows the mesh independence and convergence regime. The result of the displacement with Solution-1 was also well-satisfying for the convergence regime results. Thus, for reducing the computational cost and analysis time, the FE model has meshed with hex-elements, resulting only in 4211 nodes.



Solution	Total Deformation (m)	Change (%)	Nodes	Elements
1	3.7605E-03	–	4211	612
2	3.7603E-03	–3.0958E-05	8613	4457
3	3.7604E-03	2.6624E-03	31207	18598
4	3.7605E-03	5.8820E-04	48939	29988
5	3.7605E-03	3.0958E-04	198966	132024
6	3.7605E-03	1.3002E-04	308443	209121
7	3.7605E-03	1.3002E-04	514360	357535

Fig. 4. Mesh independence study using displacement convergence criteria

The geometric details of the FE beam model were illustrated in Fig. 3. The material properties exhibited by the control beam were Poisson's ratio of 0.3, 7850 kg/m^3 mass density, and E of 200 GPa, while E varied from one damage case to another in the manner presented in Fig. 3. By taking standard gravitational acceleration as 9.81 m/s^2, the self-weight was estimated in the negative y-direction.

4.2. Comparison of dynamic parameter identified using different methods. To verify the FM model being the true

Table 1
Comparison of dynamic characteristic of different full stiffness reductions of beam

Damage Cases	**Frequency** Number	**Analytical** Frequencies (Hz)	**Experimental**	
			Frequencies (Hz) Eigenvalue Analysis	Frequencies (Hz) FDD Method
Undamaged	1	9.16	9.07	10.18
	2	36.62	36.28	36.75
	3	82.40	81.62	80.04
Stiffness reduction 1%	1	8.97	8.87	10.14
	2	35.88	35.89	36.53
	3	80.74	80.75	79.56
Stiffness reduction 3%	1	8.81	8.81	10.03
	2	35.22	35.23	35.91
	3	79.25	79.26	78.17
Stiffness reduction 5%	1	8.61	8.61	9.89
	2	34.45	34.46	35.20
	3	77.52	77.53	76.67

representative of a physical system, three damage scenarios of full stiffness reduction of the beam, i.e.1%, 3% and 5% of the control beam, were investigated. Initially, eigenvalue analyses were performed so that modal parameters could be approximated for the control beam model. The results were compared for the first three higher participation modes of cumulative participation of 93.44%, as shown in Table 1. It is observed that the obtained natural frequencies using FDD have good accuracy, where percentage errors of less than 14.8%, 2.2%, and 2.8% were obtained from modes 1 to 3, respectively.

A review of the outcomes obtained through different identification procedures is presented below. Figure 5 displays the

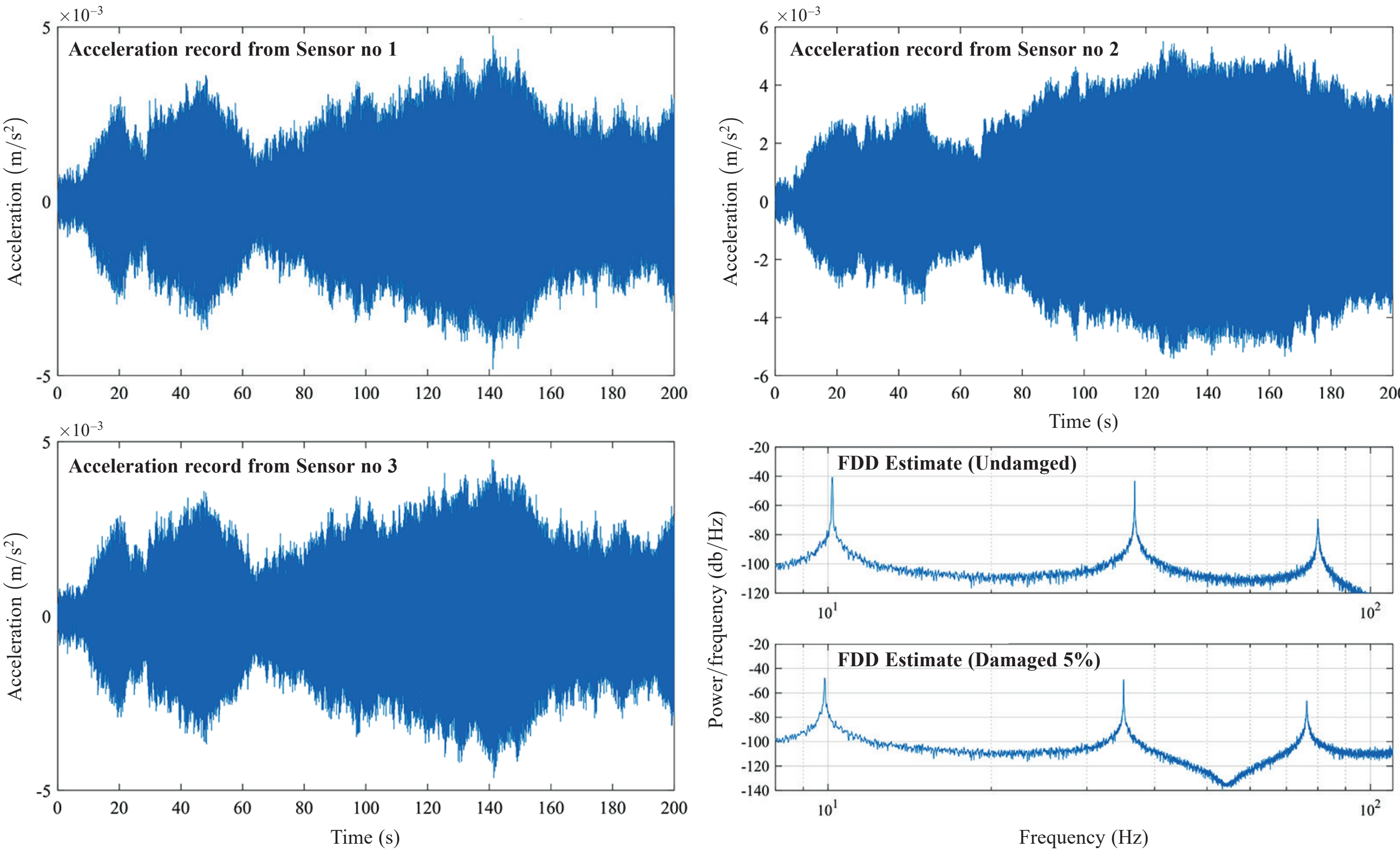


Fig. 5. Acceleration records of undamaged beam and the FDD estimates for undamaged and damaged 5% case of beam

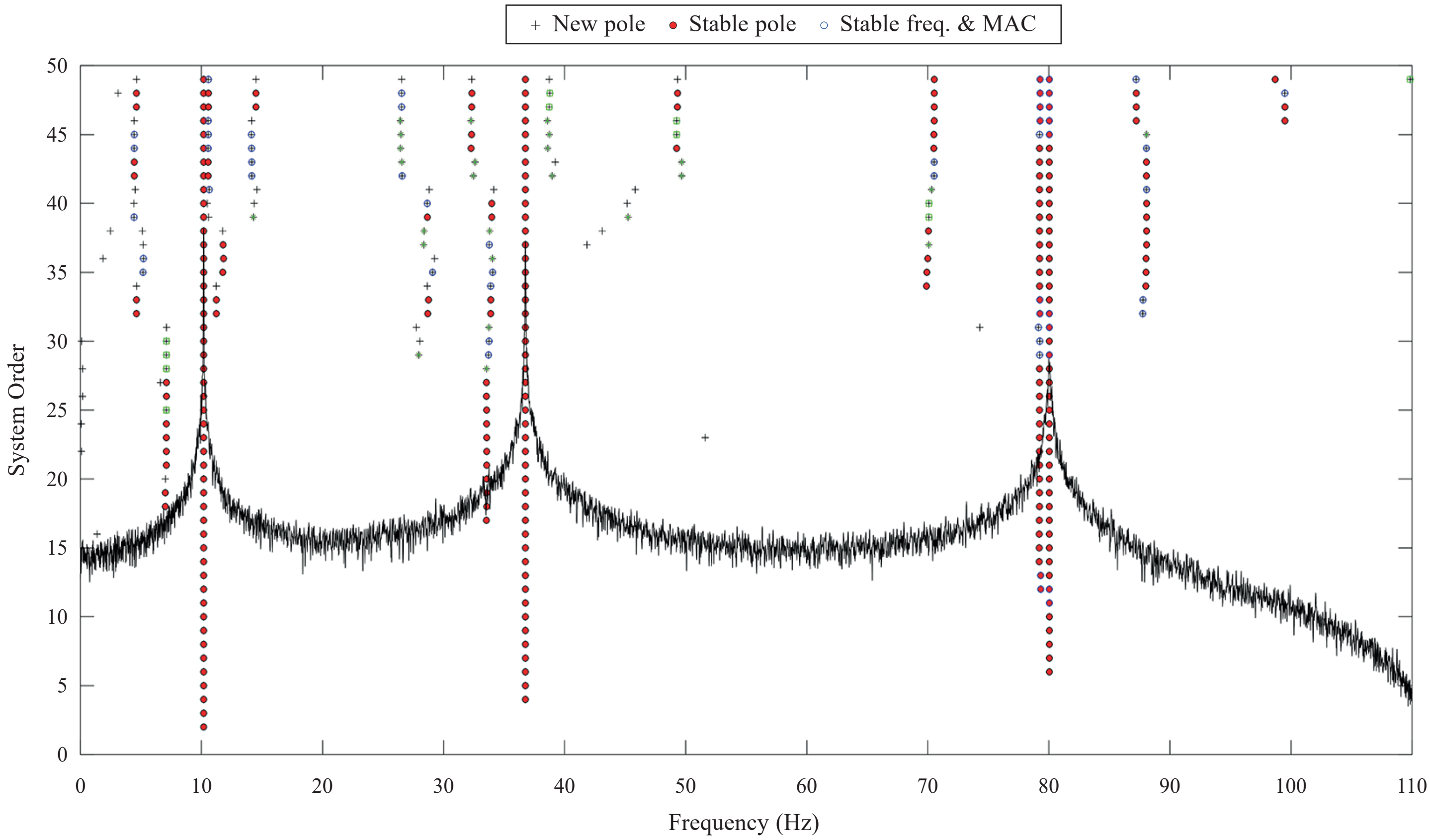


Fig. 6. Stabilization diagram for control beam

acceleration records and frequency-varying peaks of the highest singular value selected during the application of the FDD technique for the undamaged and damaged case of 5% stiffness reduction. A stabilization diagram is presented in Fig. 6, where the frequencies which now maintain stability are identified by red circles, superimposed on the singular value spectrum in the background for increasing orders of the representative stochastic model. Thus, the structure's three vibration modes for an undamaged case can be reliably identified as stable modes. This makes fair sense when taking FDD as a method of baseline to identify the structure's natural frequencies in its operating state. The three lowest vibration modes can be observed to become stable at 10.18, 36.75 and 80.04 Hz, giving a clear indicator of the same three modes.

5. Numerical results and discussion

Now following careful selection of the beam FE model, six different damage levels were investigated on the beam model, corresponding to varying moduli of elasticity at the mid-span, near support and multi-damage locations. All damage scenarios were already discussed in Fig. 3. Eigenvalue analysis was again performed in each damaged case and, subsequently, the modal parameters were obtained. Finally, indices FDMi, MSCi, and MCPi were determined for the first three bending modes. MCPi has been determined as described in Section 3.3.

5.1. Element stiffness reduction on adjacent elements. The results of FDMi, MSCi, and MCPi calculations are presented and discussed in this section. Six different damaged cases were separately observed in the mid-span and near-support of the beam as 1%, 3%, 6%, 9%, 12% and 15%, based on reduction in *E* value. Figure 7 and Fig. 8 show how the FDMi, MSCi, and MCPi values are compared at different levels and locations of damage.

Based on the results for the first three modes of the FDM and MCS indices, Fig. 7a and Fig. 7b indicate that the first and third mode was sensitive to the system of detecting damage while the damage occurred in the mid-span. Similarly, when the damage was near the end of the support, the third mode, which is higher than the others, was found the most sensitive. For both damage locations, FDMi values were at the maximum level of damage at 15% ESR. It is also observed that mode 1 and mode 3 hold the highest damage value, when the damage is at mid-span and near the end of the support, respectively. Fig. 7c and Fig. 7d represent a comparison of the MSCi values for different damage locations. The MSCi index values observed were very low and found negligible for the less severe damages such as ESR of 1% and 3%. This shows the lower sensitivity of MSCi than that of FDMi. Meanwhile in mode 3, the MSCi index value grew to a maximum of 0.0027 for near-support damage case at 15% ESR. Figure 8 illustrates MCPi values for different damage locations at various damage levels.

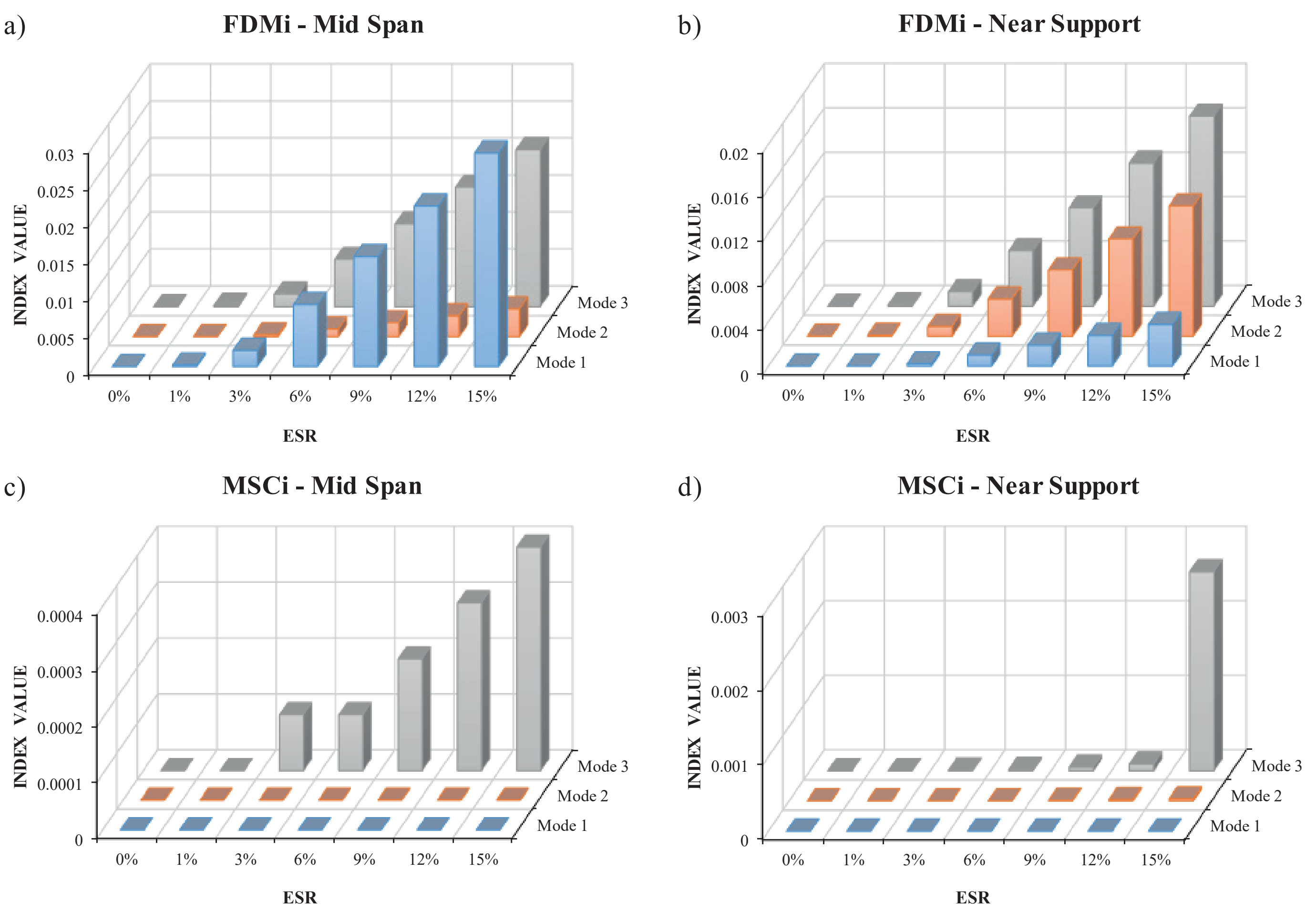


Fig. 7. FDMi and MSCi – comparison for different damage levels and locations

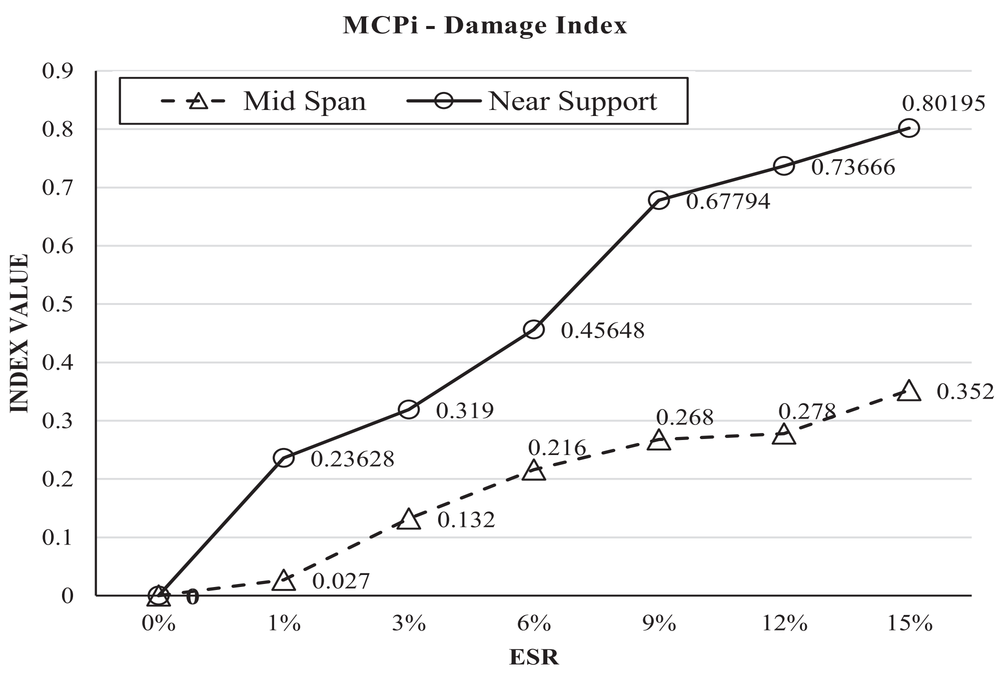


Fig. 8. Comparison of MCPi values for different levels and locations of damage

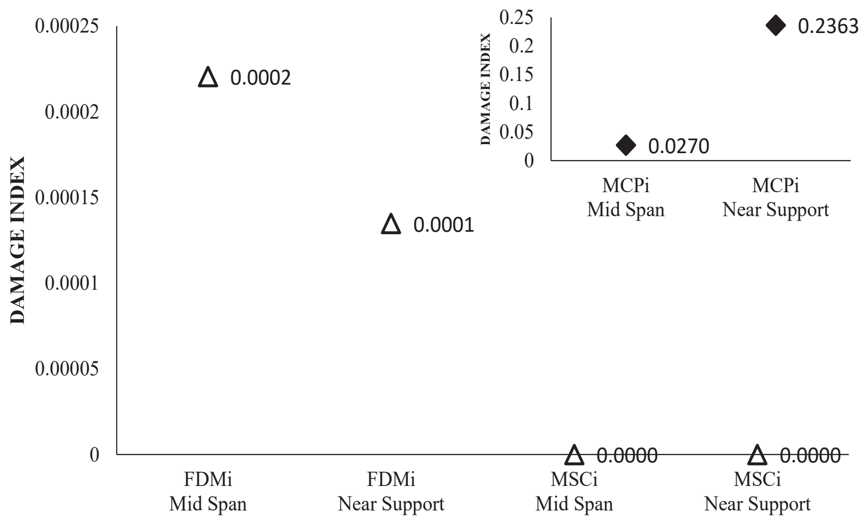


Fig. 9. MCPi comparison with FDMi and MSCi for damage at different locations for 1% ESR

The results of this work obtained here suggest that MCPi is a promising alternative to other damage indices with much greater notable sensitivity. MCPi has the value of 0.8019 for near-support damage at ESR of 15%, which is significantly greater than the index value obtained from FDMi and MSCi. This also indicates that the sensitivity of the index has increased, which relates to the growth in damage. This argument is consistent with the literature and reinforces the general belief that damage index value increases with the growth of damage. Furthermore, it would be also interesting to compare all indices at different locations of damage. Figure 9 shows a comparison of indices for ESR of 1%, being the lowest damage among all other presented damage levels.

The observations of this section outline the results and show that even for the lowest degree of damage (ESR of 1%) the MCPi index values are much greater than the FDMi and MSCi, regardless of the location of the damage. The results also show that MCPi was more sensitive when the damage was situated elsewhere than in the middle region of the beam.

Moreover, the MAC values for the calculation of MSCi were mainly observed between 0.9 and 1 for all the variations introduced in terms of element stiffness reduction either in single damage cases or multiple damage cases (discussed in Section 5.2). This indicates a high correlation between the set of undamaged and damaged beam mode shapes. Thus, the least sensitivity of the MSCi was therefore observed throughout all damage cases.

The average FDMi and MSCi values were compared for mid-span damage cases, as shown in Fig. 10. It was observed that the MCPi was effective over traditional indices. Weak

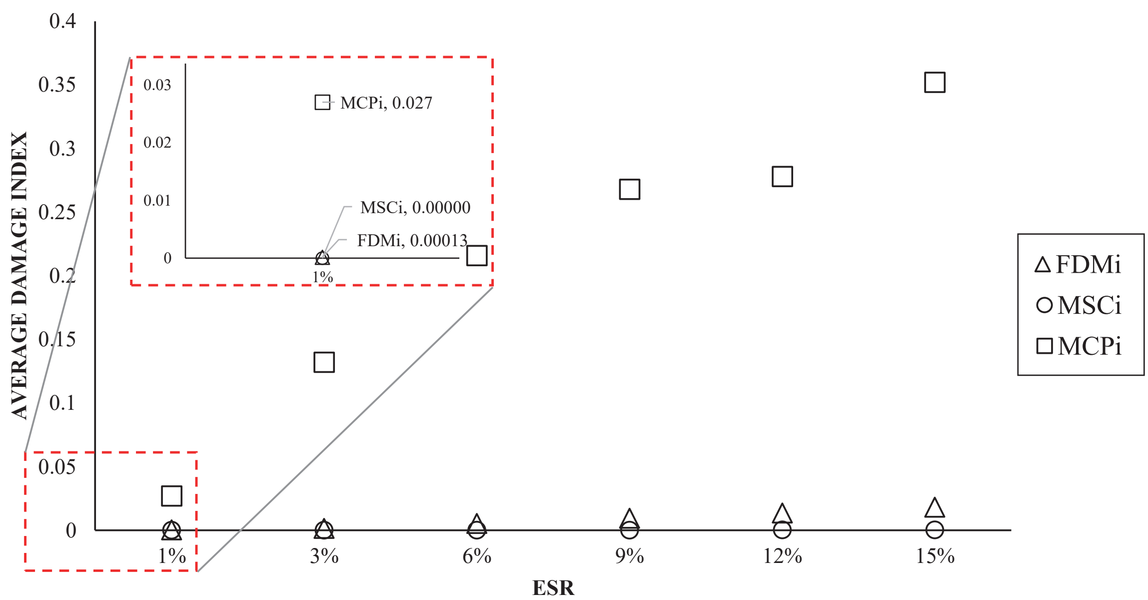


Fig. 10. Average damage index values for mid span damage cases

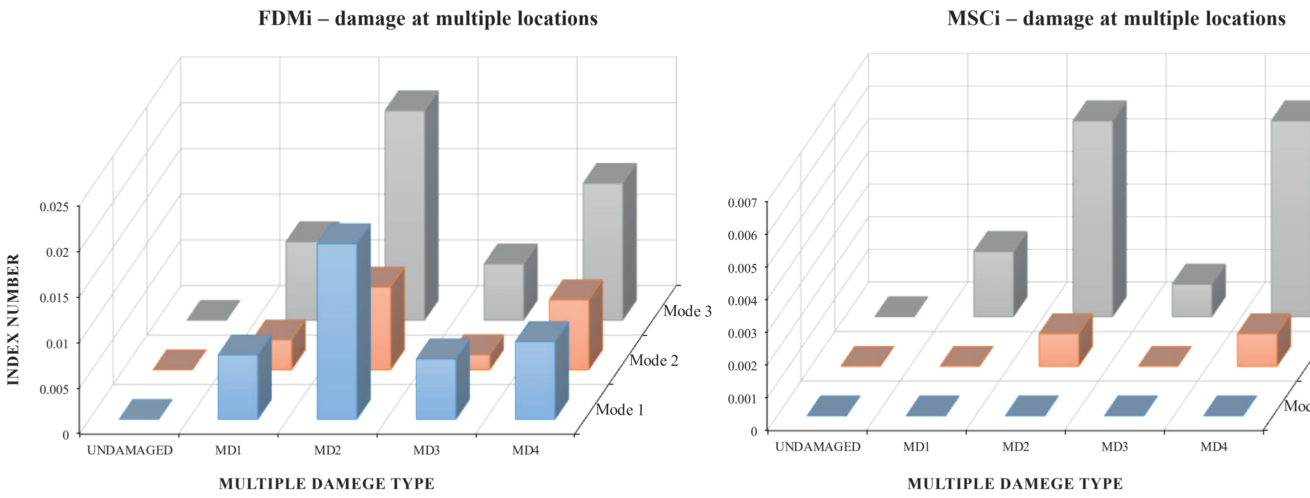


Fig. 11. FDMi and MSCi values comparison for different damage levels and multiple locations

damage with a reduction in stiffness of as little as 1% MCPi showed the high index value, which is an increase in sensitivity of at least 95%.

5.2. Multiple damage: Element stiffness reduction. In this section, the results of FDMi, MSCi, and MCPi calculations are presented and discussed for four different multiple damage levels as MD1 (5%–5%), MD2 (10%–10%), MD3 (3%–5%) and MD4 (10%–5%). The detail of a reduction in *E* value was already presented in Fig. 3. Figure 11 and Fig. 12 shows the FDMi, MSCi, and MCPi values when measured for multiple damage locations and at various damage levels. It is also clear from the data observed, the MCPi is the most sensitive among the other mentioned indices and works equally well in multiple damage cases as it worked for the single damage cases.

It is clear from Fig. 11 and Fig. 12, when comparing the results, that the index value of MCPi for the MD1 case has the highest value as compared with the FDMi index value for mode-1 (being the most sensitive among other modes) and MSCi index value for mode-3 (being the most sensitive among other modes). Similarly, the other results were also in good agreement with the working of the MCPi index as compared to other indices. Figure 12 displays the MCPi index values for four multiple damage cases. MD3, being the least severe damage case in the category of multiple damages considered in this study, shows the increased value of all damage indices for all modes. Again, among all these indices the MCPi has the highest value.

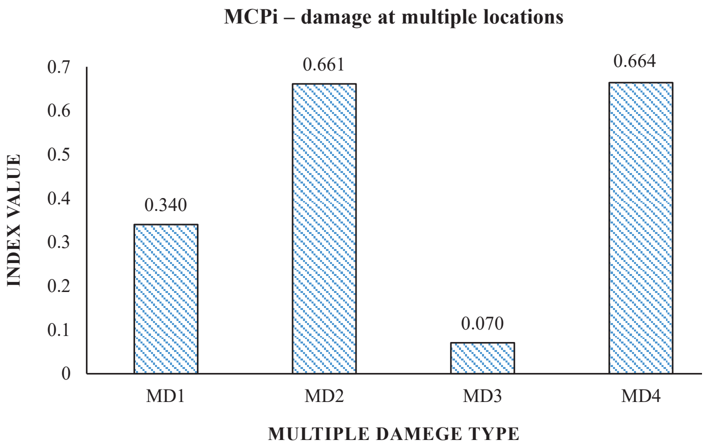


Fig. 12. MCPi index values comparison for different multiple damages

6. Conclusions

The paper proposes a more feasible method of effective damage detection in beams based on acceleration measurements under ambient excitation. It uses global results of eigenvalues and modal contributions for each mode. The main benefit of the suggested method is the simple mathematical computations that were used in damage detection calculations. The following observations are based on the numerical simulations performed on a simply supported beam with different locations and various levels of damage:

– MCPi value is zero for the structural response that implied the undamaged state of a structure. Its value increases as the two responses of the same structure become more distinct. Thus, MCPi value has been observed as an effective indicator of damage identification.
– The proposed MCPi index is effective over traditional indices. At low-level damage with element stiffness reduction of as little as 1%, MCPi showed a high index value, with a 95% rise in sensitivity.
– The outcomes of the analysis also indicate the ability of MCPi to identify the presence of damage at early stages irrespective of the location of damage throughout the span length of the beam.
– The results also show that MCPi was more sensitive when the damage was situated elsewhere than in the middle region of the beam.
– Compared to FDMi and MSCi, the calculation method of MCPi, independent of DOF, helped increase the sensitivity

of detecting damage at various locations and of different severities.
– In addition to MCPi sensitivity in damage detection, it eliminates the need for techniques like FE model updating of physical problems due to the use of data from direct sensors, as input and post-processing involved FDD makes this approach simple to deploy and robust.

The proposed method is generally effective in detecting damage, even for very low-level damage. The sensitivity of the damage index is substantially increased as compared with other assessment procedures. This index is not limited to single beam-type structures but is also expected to be effective for structural components of full scale in SHM. The performance of existing structures is typically affected by fixed joint linked elements that may contribute to the stiffness of the entire structure. In such a case, this global method can estimate the changes in stiffness, regardless of damage location.

Using the proposed global approach, once damage is detected, damage localization is needed. The authors are already working on experimental research on shear-type structures of steel and RCC frames to develop the methodology for analyzing the damage location in detail. Those will be submitted promptly.

Acknowledgements. Other than expressing our gratitude to all the people who have contributed to this research, we would like to extend some special thanks to Professor Kypros Pilakoutas (University of Sheffield) for his support in this research project, as supported by Dr. Sohaib Ahmed. We also wish to thank sincerely for the guidance of Dr. Muhammad Altaf (Mathematics Department) from the University of Engineering and Technology, Taxila, Pakistan. This work was supported by the National Research Program for Universities (NRPU) from the Higher Education Commission (HEC) of Pakistan under project number NRPU 3820.

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