ARCHIVE OF MECHANICAL ENGINEERING

Volume 67 2020 Number 3

DOI: 10.24425/ame.2020.131697

Key words: band saw machine, kinetic energy loss, optimization procedure, optimization solution

Boycho Marinov[1]

Energy losses in big band saw machines – analysis and optimization

In this paper, the energy losses in big band saw machines are investigated. These losses are caused by the geometric and angular inaccuracies with which the leading wheels are made. Expressions for calculating the kinetic energy of the mechanical system in the ideal and the real cases are obtained. For this purpose, expressions for calculating the velocities of the centers of the masses in two mutually perpendicular planes are obtained. A dependence for calculation of the kinetic energy losses of the mechanical system in final form is received. Optimization procedure is used to determine the values of the parameters at which these losses have minimum values. The proposed study can be used to minimize energy losses in other classes of woodworking machines.

1. Introduction

Band saw machines are the most common woodworking machines. These machines are used in various technological operations, which leads to the necessity of designing different classes of machines. They can be classified into three groups, based on general principles:

- Band saw machines for sawing logs and prisms. They are also called the log band saw machines or the big band saw machines. The diameters of the leading wheels are between 1100 and 3500 [mm] and the width of the band is from 140 to 360 [mm];
- Deal band saw machines. They are used for sawing prisms, thick boards and covers. These machines have great performance. The diameters of the leading wheels are between 1000 and 1500 [mm] and the width of the band is from 70 to 175 [mm];

✉ Boycho Marinov, e-mails: boicho_marinoff@yahoo.co.uk; boicho_marinoff@imbm.bas.bg

[1]The Institute of Mechanics, Bulgarian Academy of Sciences, Sofia, Bulgaria.

- Ordinary band saw machines. They are used for cutting up and processing curvilinear surfaces. The diameters of the leading wheels are between 400 and 1000 [mm] and the width of the band is up to 40 [mm].

Scientific studies related to energy efficiency of different classes of woodworking machinery are published in the technical literature. Measurement of specific cutting energy (ESP) for evaluating the efficiency of band sawing of different workpiece materials is investigated by Sarwar et al. [1]. It is found that measuring the ESP is a better way for determining the efficiency of the cutting process compared to the other methods, such as determining tool wear, cutting forces, etc. It has been also found that the increase in ESP reflects the degradation of the cutting performance. Mandic et al. [2] have made a comparative analysis of two methods for the power consumption measurement in circular saw cutting of laminated particle board. The features of construction and the shape characteristics of the circular saw determine possible relationship between the power consumption, acoustic emission and the cutting process progress. Kopecký et al. [3] offer an innovative approach to predicting energy effects of wood cutting process with circular-saw blades. In the classical approach, energy effects of wood sawing process are generally calculated on the basis of the specific cutting resistance. In this article, a new computational model using the approach of modern fracture mechanics is presented. Orlowski et al. [4] use fracture mechanics for predicting energy effects in wood sawing. In the classical approach, energy effects of wood sawing process are generally calculated on the basis of the specific cutting resistance. In this paper, it is proved that cutting power models based on modern fracture mechanics can be used for estimation of energy effects of sawing of every kinematics. Iskra et al. [5] investigate the energy balance of the orthogonal cutting process. As a result of their study, the authors reach the following conclusions:

- The contribution of the amount of pure cutting energy in relation to electrical energy consumed during orthogonal machining rises significantly when cutting speed increases.
- Thermal output is the most significant undesirable phenomenon accompanying cutting. Up to 28% of input energy can be dissipated by heating of the tool and chips during orthogonal machining.

In this article, energy losses in big band saw machines are analysed. These machines are widely used in the woodworking industry because of their high productivity. Their work, however, is characterized by high-energy consumption, which is mostly because main links of the machine are very large. Due to this fact, there appear corresponding linear and angular inaccuracies in both wheels, which cannot be avoided.

The purpose of the proposed study is to determine the energy losses in the mechanical system resulting from these inaccuracies. In order to achieve this purpose, the following basic tasks must be fulfilled:

- obtaining expressions for calculating the kinetic energy of the mechanical system in the absence of linear and angular inaccuracies;

- obtaining expressions for calculating the kinetic energy of the mechanical system in the presence of linear and angular inaccuracies;
- obtaining an expression to calculate the kinetic energy loss of the mechanical system in the real case.

Another purpose of this study is to present various optimization solutions that can reduce the actual energy consumption of the band saw machines in the operation mode. In this case, it is necessary to use optimization procedures, as well as to compile the appropriate objective functions. These functions are used by the optimization procedures to calculate the optimal parameter values. In this way, it can be ensured that the band saw machines will operate in the optimal mode with minimum energy losses.

2. Solution principles

In this part, in order to solve the problems posed, a principle scheme of band saw machine and the corresponding dynamic models are presented in Figs.1, 2 and 3. These models are used to determine the kinetic energy of the mechanical system in the absence and in the presence of linear and angular inaccuracies.

2.1. Principle scheme of the band saw machine

The scheme of the band saw machine is shown in Fig. 1 [6]. The following symbols are defined: 1, 2, 5, 6 – belt pulleys, E – electric motor, 3 and 4 – leading wheels, *A* – band-saw blade, 7 and 8 – chain-wheels.

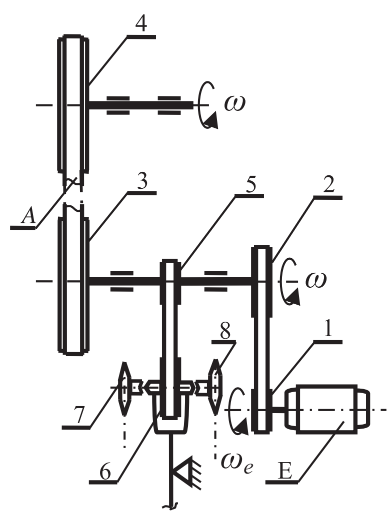

Fig. 1. Principle scheme of band saw machine

2.2. Kinetic energy of the mechanical system in the absence of linear and angular inaccuracies – ideal case

In this case, we use the dynamic model shown in Fig. 2.

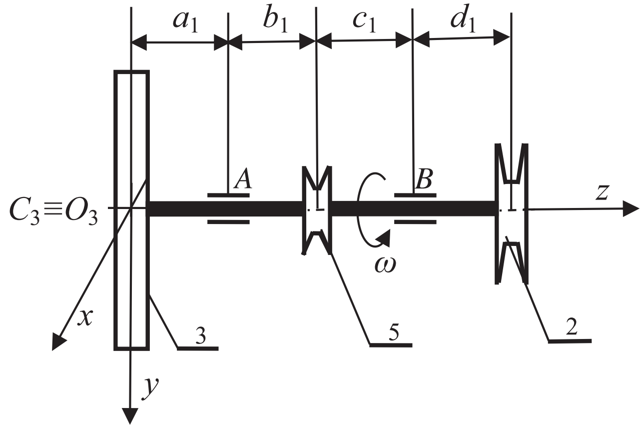

Fig. 2. Dynamic model in absence of linear and angular inaccuracies

The kinetic energy of the mechanical system can be calculated from the following dependence [7–9]:

$$T_p = \frac{1}{2} J_p \omega^2, \tag{1}$$

where J_p is the mass moment of inertia of the system with respect to the axis of rotation, i.e., toward the axis z. We can calculate it from the following expression:

$$J_p = J_2 + J_3 + J_5 + J_{sh} \, . \tag{2}$$

In the above expression, J_2 and J_5 are the mass moments of inertia of the belt pulleys 2 and 5, J_3 is the mass moment of inertia of the leading wheel 3 and J_{sh} is the mass moment of inertia of the main shaft with respect to the axis of rotation. ω is the angular velocity of the mechanical system around the same axis.

2.3. Kinetic energy of the mechanical system in the presence of linear and angular inaccuracies – real case

Due to their large dimensions, the leading wheels are made with the corresponding geometric and angular inaccuracies. In this case, these inaccuracies are: geometric deviation $e = O_3C_3$, where C_3 is the centre of mass of the disc 3 and angular deviation α. These deviations can be seen in Fig. 3 [10].

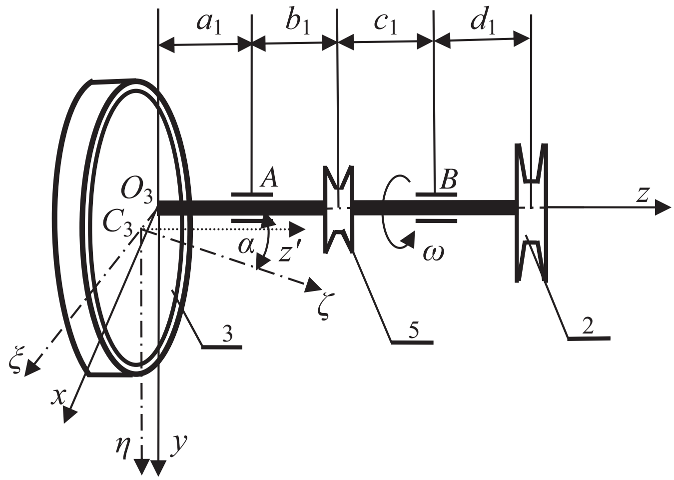

Fig. 3. Dynamic model in presence of linear and angular inaccuracies

The kinetic energy of the mechanical system can be calculated from the following dependence:

$$T_r = \frac{1}{2} J_r \omega^2 + \frac{1}{2} m_3 \left[(\dot{y}_1(z_1,t))^2 + (\dot{x}_1(z_1,t))^2 \right] + \\ + \frac{1}{2} m_5 \left[(\dot{y}_2(z_2,t))^2 + (\dot{x}_2(z_2,t))^2 \right] + \frac{1}{2} m_2 \left[(\dot{y}_3(z_3,t))^2 + (\dot{x}_3(z_3,t))^2 \right], \quad (3)$$

where m_3, m_5 and m_2 are the masses of the three disks. The coordinates z_i $(i = 1 \div 3)$ have the following values: $z_1 = 0$, $z_2 = a_1 + b_1$, $z_3 = a_1 + b_1 + c_1 + d_1 = l$. J_r is the mass moment of inertia of the system in the presence of linear and angular deviations, i.e., in the real case. This moment can be calculated from the following expression:

$$J_r = J_2 + J_3' + J_5 + J_{sh} \, . \quad (4)$$

The mass moment of inertia of the leading wheel with respect to the axis of rotation is denoted by J_3' and can be calculated using the formulas known from technical literature [7].

$$J_3' = J_\zeta \cos^2 \alpha_\zeta + J_\xi \cos^2 \alpha_\xi + J_\eta \cos^2 \alpha_\eta + m_3 \rho_C^2 \, , \quad (5)$$

where $\rho_C = e \cos \alpha$. The angles between the axis z' and the axes ζ, ξ and η are marked with α_ζ, α_ξ, α_η. These angles take the following values: $\alpha_\zeta = \alpha$, $\alpha_\xi = \dfrac{\pi}{2} + \alpha$, $\alpha_\eta = \dfrac{\pi}{2}$. In this way, the expression (5) takes the following form:

$$J_3' = J_\zeta \cos^2 \alpha + J_\xi \sin^2 \alpha + m_3 e^2 \cos^2 \alpha, \quad (6)$$

where J_ζ and J_ξ are the mass moments of inertia of the leading wheel with respect to the principal axes of inertia ζ and ξ.

Other variables used in the expression for the kinetic energy are: $\dot{y}_1(z_1,t)$, $\dot{y}_2(z_2,t)$ and $\dot{y}_3(z_3,t)$ – the velocities of the mass centers of the three disks relative to the axis y, $\dot{x}_1(z_1,t)$, $\dot{x}_2(z_2,t)$ and $\dot{x}_3(z_3,t)$ – the velocities of the mass centers of the three disks relative to the axis x.

These velocities can be determined by expressions for the transverse vibrations of the main shaft in the vertical plane O_3yz and the horizontal plane O_3xz.

With respect to the fixed coordinates z_i $(i = 1 \div 3)$, these expressions can be written as follows:

$$\begin{aligned} y_1(z_1,t) &= Z_{21}\sin\omega t, & x_1(z_1,t) &= Z_{11}\cos\omega t, \\ y_2(z_2,t) &= Z_{22}\sin\omega t, & x_2(z_2,t) &= Z_{12}\cos\omega t, \\ y_3(z_3,t) &= Z_{23}\sin\omega t, & x_3(z_3,t) &= Z_{13}\cos\omega t. \end{aligned} \tag{7}$$

The above expressions include the variables Z_{2i} and Z_{1i} $(i = 1 \div 3)$. These variables can be determined by the following dependencies:

$$\begin{aligned} Z_{21} &= \left(\bar{R}_1 + \bar{V}_1\right), \qquad Z_{11} = \left(\bar{L}_1 + \bar{P}_1\right), \\ Z_{22} &= \left(\bar{R}_2\cos\omega_0 z_2 + \bar{S}_2\sin\omega_0 z_2 + \bar{V}_2\cosh\omega_0 z_2 + \bar{W}_2\sinh\omega_0 z_2\right), \\ Z_{12} &= \left(\bar{L}_2\cos\omega_0 z_2 + \bar{M}_2\sin\omega_0 z_2 + \bar{P}_2\cosh\omega_0 z_2 + \bar{Q}_2\sinh\omega_0 z_2\right), \\ Z_{23} &= \left(\bar{R}_3\cos\omega_0 z_3 + \bar{S}_3\sin\omega_0 z_3 + \bar{V}_3\cosh\omega_0 z_3 + \bar{W}_3\sinh\omega_0 z_3\right), \\ Z_{13} &= \left(\bar{L}_3\cos\omega_0 z_3 + \bar{M}_3\sin\omega_0 z_3 + \bar{P}_3\cosh\omega_0 z_3 + \bar{Q}_3\sinh\omega_0 z_3\right), \end{aligned} \tag{8}$$

where $\bar{R}_i$, $\bar{S}_i$, $\bar{V}_i$, $\bar{W}_i$, $\bar{L}_i$, $\bar{M}_i$, $\bar{P}_i$ and $\bar{Q}_i$ $(i = 1 \div 3)$ are the constants of integration. ω_0 is a parameter that can be calculated from a dependency known from technical literature [11].

The expressions for the velocities of the mass centers can be written in the final form.

$$\begin{aligned} \dot{y}_1(z_1,t) &= Z_{21}\omega\cos\omega t, & \dot{x}_1(z_1,t) &= -Z_{11}\omega\sin\omega t, \\ \dot{y}_2(z_2,t) &= Z_{22}\omega\cos\omega t, & \dot{x}_2(z_2,t) &= -Z_{12}\omega\sin\omega t, \\ \dot{y}_3(z_3,t) &= Z_{23}\omega\cos\omega t, & \dot{x}_3(z_3,t) &= -Z_{13}\omega\sin\omega t\,. \end{aligned} \tag{9}$$

Taking into account the fact that the constants of integration for the two planes are equal to one another, i.e., $\bar{R}_i = \bar{L}_i$, $\bar{S}_i = \bar{M}_i$, $\bar{V}_i = \bar{P}_i$ and $\bar{W}_i = \bar{Q}_i$ $(i = 1 \div 3)$, we conclude that the variables Z_{2i} and Z_{1i} $(i = 1 \div 3)$ are also equal, i.e., $Z_{21} = Z_{11}$, $Z_{22} = Z_{12}$, $Z_{23} = Z_{13}$.

The constants of integration can be calculated from the following dependencies:

$$\bar{R}_i = \frac{\Delta_{\bar{R}_i}}{\Delta_B}, \quad \bar{S}_i = \frac{\Delta_{\bar{S}_i}}{\Delta_B}, \quad \bar{V}_i = \frac{\Delta_{\bar{V}_i}}{\Delta_B}, \quad \bar{W}_i = \frac{\Delta_{\bar{W}_i}}{\Delta_B}, \quad (i = 1,\ 2,\ 3), \tag{10}$$

where Δ_B, $\Delta_{\bar{R}_i}$, $\Delta_{\bar{S}_i}$, $\Delta_{\bar{V}_i}$ and $\Delta_{\bar{W}_i}$ $(i = 1 \div 3)$ are determinants that take different values at different values of the linear and geometric inaccuracies, i.e., at different values of e and α.

Δ_B is the determinant of the matrix $\mathbf{B}$, which has the following form [12]:

Columns 1 through 7

$$
\mathbf{B} = \left[\begin{array}{ccccccc}
0 & -1 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 \\
\cos\omega_0 a_1 & \sin\omega_0 a_1 & \cosh\omega_0 a_1 & \sinh\omega_0 a_1 & 0 & 0 & 0 \\
-\sin\omega_0 a_1 & \cos\omega_0 a_1 & \sinh\omega_0 a_1 & \cosh\omega_0 a_1 & \sin\omega_0 a_1 & -\cos\omega_0 a_1 & -\sinh\omega_0 a_1 \\
-\sin\omega_0 a_1 & \cos\omega_0 a_1 & -\sinh\omega_0 a_1 & -\cosh\omega_0 a_1 & \sin\omega_0 a_1 & -\cos\omega_0 a_1 & \sinh\omega_0 a_1 \\
0 & 0 & 0 & 0 & \cos\omega_0 a_1 & \sin\omega_0 a_1 & \cosh\omega_0 a_1 \\
0 & 0 & 0 & 0 & \cos\omega_0(a_1+b_1+c_1) & \sin\omega_0(a_1+b_1+c_1) & \cosh\omega_0(a_1+b_1+c_1) \\
0 & 0 & 0 & 0 & -\sin\omega_0(a_1+b_1+c_1) & \cos\omega_0(a_1+b_1+c_1) & -\sinh\omega_0(a_1+b_1+c_1) \\
0 & 0 & 0 & 0 & -\sin\omega_0(a_1+b_1+c_1) & \cos\omega_0(a_1+b_1+c_1) & \sinh\omega_0(a_1+b_1+c_1) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} \right.
$$

Columns 8 through 12

$$\left.\begin{matrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-\cosh\omega_0 a_1 & 0 & 0 & 0 & 0 \\
\cosh\omega_0 a_1 & 0 & 0 & 0 & 0 \\
\sinh\omega_0 a_1 & 0 & 0 & 0 & 0 \\
\sinh\omega_0(a_1+b_1+c_1) & 0 & 0 & 0 & 0 \\
-\cosh\omega_0(a_1+b_1+c_1) & \sin\omega_0(a_1+b_1+c_1) & -\cos\omega_0(a_1+b_1+c_1) & \sinh\omega_0(a_1+b_1+c_1) & \cosh\omega_0(a_1+b_1+c_1) \\
\cosh\omega_0(a_1+b_1+c_1) & \sin\omega_0(a_1+b_1+c_1) & -\cos\omega_0(a_1+b_1+c_1) & -\sinh\omega_0(a_1+b_1+c_1) & -\cosh\omega_0(a_1+b_1+c_1) \\
0 & \cos\omega_0(a_1+b_1+c_1) & \sin\omega_0(a_1+b_1+c_1) & \cosh\omega_0(a_1+b_1+c_1) & \sinh\omega_0(a_1+b_1+c_1) \\
0 & -\cos\omega_0(a_1+b_1+c_1+d_1) & -\sin\omega_0(a_1+b_1+c_1+d_1) & \cosh\omega_0(a_1+b_1+c_1+d_1) & \sinh\omega_0(a_1+b_1+c_1+d_1) \\
0 & \sin\omega_0(a_1+b_1+c_1+d_1) & -\cos\omega_0(a_1+b_1+c_1+d_1) & \sinh\omega_0(a_1+b_1+c_1+d_1) & \cosh\omega_0(a_1+b_1+c_1+d_1)
\end{matrix}\right] . \tag{11}$$

$\Delta_{\bar{R}_i}$ $\Delta_{\bar{S}_i}$, $\Delta_{\bar{V}_i}$ and $\Delta_{\bar{W}_i}$ $(i = 1 \div 3)$ are determinants of the matrices that are formed by the matrix **B**. In these matrices, the corresponding columns of matrix **B** are replaced by the column vector **b**.

$$\mathbf{b} = \left[\frac{m_3 e\omega^2\cos\alpha}{EJ\omega_0^3} \quad \frac{M_{bm}}{EJ\omega_0^2} \quad 0 \quad 0 \quad \frac{A_{bm}}{EJ\omega_0^3} \quad 0 \quad 0 \quad \frac{B_{bm}}{EJ\omega_0^3} \quad 0 \quad 0 \quad 0 \quad 0\right]^T . \tag{12}$$

For each determinant, the corresponding substitutions are made, which are listed below.

$\Delta_{\bar{R}_i}$ $(i = 1 \div 3)$ – the first, the fifth, or the ninth column; $\Delta_{\bar{S}_i}$ $(i = 1 \div 3)$ – the second, the sixth or the tenth column;

$\Delta_{\bar{V}_i}$ $(i = 1 \div 3)$ – the third, the seventh or the eleventh column; $\Delta_{\bar{W}_i}$ $(i = 1 \div 3)$ – the fourth, the eighth or the twelfth column.

For example, the determinant $\Delta_{\bar{R}_1}$ is written in the following way:

Columns 1 through 7

$$\Delta_{\bar{R}_1} = \left[\begin{array}{ccccccc}
\dfrac{m_3 e\omega^2 \cos\alpha}{EJ\omega_0^3} & -1 & 0 & 1 & 0 & 0 & 0 \\
\dfrac{M_{bm}}{EJ\omega_0^2} & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & \sin\omega_0 a_1 & \cosh\omega_0 a_1 & \sinh\omega_0 a_1 & 0 & 0 & 0 \\
0 & \cos\omega_0 a_1 & \sinh\omega_0 a_1 & \cosh\omega_0 a_1 & \sin\omega_0 a_1 & -\cos\omega_0 a_1 & -\sinh\omega_0 a_1 \\
\dfrac{A_{bm}}{EJ\omega_0^3} & \cos\omega_0 a_1 & -\sinh\omega_0 a_1 & -\cosh\omega_0 a_1 & \sin\omega_0 a_1 & -\cos\omega_0 a_1 & \sinh\omega_0 a_1 \\
0 & 0 & 0 & 0 & \cos\omega_0 a_1 & \sin\omega_0 a_1 & \cosh\omega_0 a_1 \\
0 & 0 & 0 & 0 & \cos\omega_0(a_1+b_1+c_1) & \sin\omega_0(a_1+b_1+c_1) & \cosh\omega_0(a_1+b_1+c_1) \\
\dfrac{B_{bm}}{EJ\omega_0^3} & 0 & 0 & 0 & -\sin\omega_0(a_1+b_1+c_1) & \cos\omega_0(a_1+b_1+c_1) & -\sinh\omega_0(a_1+b_1+c_1) \\
0 & 0 & 0 & 0 & -\sin\omega_0(a_1+b_1+c_1) & \cos\omega_0(a_1+b_1+c_1) & \sinh\omega_0(a_1+b_1+c_1) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right.$$

Columns 8 through 12

$$\left.\begin{matrix}
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
-\cosh\omega_0 a_1 & 0 & 0 & 0 & 0\\
\cosh\omega_0 a_1 & 0 & 0 & 0 & 0\\
\sinh\omega_0 a_1 & 0 & 0 & 0 & 0\\
\sinh\omega_0(a_1{+}b_1{+}c_1) & 0 & 0 & 0 & 0\\
-\cosh\omega_0(a_1{+}b_1{+}c_1) & \sin\omega_0(a_1{+}b_1{+}c_1) & -\cos\omega_0(a_1{+}b_1{+}c_1) & \sinh\omega_0(a_1{+}b_1{+}c_1) & \cosh\omega_0(a_1{+}b_1{+}c_1)\\
\cosh\omega_0(a_1{+}b_1{+}c_1) & \sin\omega_0(a_1{+}b_1{+}c_1) & -\cos\omega_0(a_1{+}b_1{+}c_1) & -\sinh\omega_0(a_1{+}b_1{+}c_1) & -\cosh\omega_0(a_1{+}b_1{+}c_1)\\
0 & \cos\omega_0(a_1{+}b_1{+}c_1) & \sin\omega_0(a_1{+}b_1{+}c_1) & \cosh\omega_0(a_1{+}b_1{+}c_1) & \sinh\omega_0(a_1{+}b_1{+}c_1)\\
0 & -\cos\omega_0(a_1{+}b_1{+}c_1{+}d_1) & -\sin\omega_0(a_1{+}b_1{+}c_1{+}d_1) & \cosh\omega_0(a_1{+}b_1{+}c_1{+}d_1) & \sinh\omega_0(a_1{+}b_1{+}c_1{+}d_1)\\
0 & \sin\omega_0(a_1{+}b_1{+}c_1{+}d_1) & -\cos\omega_0(a_1{+}b_1{+}c_1{+}d_1) & \sinh\omega_0(a_1{+}b_1{+}c_1{+}d_1) & \cosh\omega_0(a_1{+}b_1{+}c_1{+}d_1)
\end{matrix}\right] . \tag{13}$$

In expression (12), the following symbols are used: E is the modulus of elasticity, $J = J_x = J_y$ is the axial moment of inertia of the main shaft. A_{bm}, B_{bm} and M_{bm} can be calculated from the dependencies shown below.

$$\begin{aligned}
A_{bm} &= \frac{1}{2(b_1+c_1)}\Big[\omega^2(J_\xi - J_\varsigma)\sin 2\alpha - 2\omega^2 m_3(a_1+b_1+c_1+e\sin\alpha)e\cos\alpha \\
&\quad - \left(R_b^n + R_\Sigma\right)e\cos\alpha\Big], \\
B_{bm} &= \frac{1}{2(b_1+c_1)}\Big[\left(R_b^n + R_\Sigma\right)e\cos\alpha + 2\omega^2 m_3(a_1+e\sin\alpha)e\cos\alpha \\
&\quad - \omega^2(J_\xi - J_\varsigma)\sin 2\alpha\Big], \\
M_{bm} &= \frac{1}{2}\left[\left(R_b^n + R_\Sigma\right)e\cos\alpha - \omega^2\left(J_\xi - J_\varsigma - m_3e^2\right)\sin 2\alpha\right],
\end{aligned} \tag{14}$$

where R_Σ is the total resistance force acting between the workpiece and the band saw machine. This force is calculated for each individual case. R_b^n is the normal force loading the workpiece. This force can be calculated from expressions that are available from various sources [6, 12] and written below.

$$R_b^n = \frac{mK_{\Delta(\lambda)}bHu}{V}, \tag{15}$$

where V is the cutting speed, $H$is the thickness of the workpiece, u is the feeding speed, b is the width of the cutter. m is a coefficient that changes within the following limits $0 \leqslant m \leqslant 1$. $K_{\Delta(\lambda)}$ is the specific work of the cutting. It is determined from the expressions below [13].

$$K_\Delta = k + \frac{a_\rho p}{u_{z\Delta}} + \frac{\alpha_\Delta H}{b}, \qquad K_\lambda = k + \frac{a_\rho ps}{bu_{z\lambda}} + \frac{\alpha_\lambda H}{b}. \tag{16}$$

The symbol $\alpha_{\Delta(\lambda)}$ (α_Δ for stage-set teeth and α_λ for part-set teeth) is the friction intensity of the shavings on the cutting walls, a_ρ is a coefficient of the blunt teeth, s is the thickness of the band saw blades. The fictitious pressure on the front side of the teeth is marked with k and p is the fictitious specific force on the back side of the teeth. The feeding of one tooth is marked with $u_{z\Delta}$, $u_{z\lambda}$.

In order to obtain an expression for the kinetic energy of the mechanical system in the final form, we substitute the expressions (9) in expression (3), taking into account the explanations given above.

$$T_r = \frac{1}{2}J_r\omega^2 + \frac{1}{2}m_3Z_{21}^2\omega^2 + \frac{1}{2}m_5Z_{22}^2\omega^2 + \frac{1}{2}m_2Z_{23}^2\omega^2. \tag{17}$$

We write this expression in its final form.

$$T_r = \frac{1}{2}\left(J_r + m_3Z_{21}^2 + m_5Z_{22}^2 + m_2Z_{23}^2\right)\omega^2. \tag{18}$$

2.4. Kinetic energy loss of the mechanical system resulting from linear and angular inaccuracies

The loss of kinetic energy can be calculated from the following expression:

$$ {}_{\Delta}T = T_r - T_p \,. \tag{19}$$

We replace T_r and T_p with their equal expressions written in (18) and (1) and get the following expression:

$$ {}_{\Delta}T = \frac{1}{2}\left(J_r + m_3 Z_{21}^2 + m_5 Z_{22}^2 + m_2 Z_{23}^2\right)\omega^2 - \frac{1}{2} J_p \omega^2. \tag{20}$$

We transform the above expression and get the following dependency:

$$ {}_{\Delta}T = \frac{1}{2}\left[\left(J_3' - J_3\right) + m_3 Z_{21}^2 + m_5 Z_{22}^2 + m_2 Z_{23}^2\right]\omega^2. \tag{21}$$

We record expression (21) in a more detailed form.

$$ \begin{aligned} {}_{\Delta}T = \frac{1}{2}\Big[&\left(J_\zeta \cos^2\alpha + J_\xi \sin^2\alpha + m_3 e^2 \cos^2\alpha - J_3\right) + m_3 Z_{21}^2 \\ &+ m_5 Z_{22}^2 + m_2 Z_{23}^2\Big]\omega^2. \end{aligned} \tag{22}$$

In the above expression, we take into account the fact that $J_3 \equiv J_\zeta$, when there are no linear and angular inaccuracies, i.e., when $e = 0$ and $\alpha = 0$. Thus, we can write the expression for kinetic energy loss in the presence of linear and angular inaccuracies in the following form:

$$ {}_{\Delta}T = \frac{1}{2}\left[\left(J_\xi - J_\zeta\right)\sin^2\alpha + m_3 e^2 \cos^2\alpha + m_3 Z_{21}^2 + m_5 Z_{22}^2 + m_2 Z_{23}^2\right]\omega^2. \tag{23}$$

We replace the dependencies Z_{2i} $(i = 1{-}3)$ with their equivalents and obtain the expression for the loss of kinetic energy in the presence of linear and angular inaccuracies in the final form. This expression is presented below as the dependence (24).

$$ \begin{aligned} {}_{\Delta}T = &\frac{1}{2}\left[\left(J_\xi - J_\zeta\right)\sin^2\alpha + m_3 e^2\cos^2\alpha + m_3\left(\frac{\Delta_{\bar{R}_1}}{\Delta_B} + \frac{\Delta_{\bar{V}_1}}{\Delta_B}\right)^2\right]\omega^2 + \\ &+ \frac{1}{2}\left[m_5\left(\frac{\Delta_{\bar{R}_2}}{\Delta_B}\cos\omega_0 z_2 + \frac{\Delta_{\bar{S}_2}}{\Delta_B}\sin\omega_0 z_2 + \frac{\Delta_{\bar{V}_2}}{\Delta_B}\cosh\omega_0 z_2 + \right.\right. \\ &\left.\left. + \frac{\Delta_{\bar{W}_2}}{\Delta_B}\sinh\omega_0 z_2\right)^2\right]\omega^2 + \frac{1}{2}\left[m_2\left(\frac{\Delta_{\bar{R}_3}}{\Delta_B}\cos\omega_0 z_3 + \frac{\Delta_{\bar{S}_3}}{\Delta_B}\sin\omega_0 z_3 + \right.\right. \\ &\left.\left. + \frac{\Delta_{\bar{V}_3}}{\Delta_B}\cosh\omega_0 z_3 + \frac{\Delta_{\bar{W}_3}}{\Delta_B}\sinh\omega_0 z_3\right)^2\right]\omega^2. \end{aligned} \tag{24}$$

This expression allows calculating the loss of kinetic energy at different values of the linear and angular deviations, as well as different values of the mass, and geometric and kinematic characteristics of the mechanical system.

3. Optimization solutions

3.1. Optimization solutions reducing energy losses

As can be seen from the expression (24), the loss of kinetic energy of big band saw machines depends on different parameters. The purpose of this part of the study is to propose optimization solutions that minimize these losses. We use optimization procedure with constraints-*fmincon* [14]. This procedure looks for the minimum of a multidimensional function with the corresponding constraints. In this case, the interval of constraints in which the parameters e and α change must be determined. These constraints are expressed by the following inequalities: $lb \leqslant [e, \alpha] \leqslant ub$, where lb and ub are the lower and upper bounds. It is also necessary to determine the initial approximation $x0$ around which the function has a local minimum. If the initial approximation is inappropriate, the procedure selects a new initial approximation. The way in which the optimization procedure is applied is shown below. In order to start the procedure, it is necessary to create an *objective function*, which uses Eq. (24).

```
function F_def
x0 = [Initial Approximation];
lb = [Lower Bounds];
ub = [Upper Bounds];
ops = optimset('LargeScale','off');
[x,fval,exitflag,output]=fmincon(@def1,x0,A,b,Aeq,beq,lb,ub,nonlcon,ops),
function F1=def1(x)
.
objective function
.
.
F1 = [expression for calculating the loss of kinetic energy – see Eq. (24)].
```

3.2. Optimization procedure – application

Due to the large dimensions of the leading wheels, they are always produced with corresponding inaccuracies. The magnitudes of these inaccuracies depend on many factors, such as different manufacturing technologies, different assembly technologies, different operating conditions, and more. For this reason, we use the allowable technological boundaries in which the parameters e and α are enclosed, i.e., $e_{\min} \leqslant e \leqslant e_{\max}$, $\alpha_{\min} \leqslant \alpha \leqslant \alpha_{\max}$. These boundaries determine the zone in which the band saw machine can operate. This zone is called the working zone. The area under the working zone is called the initial zone and the area above the working zone is called the final zone. We can apply the optimization procedure for the three zones. However, this is not always necessary. The most important area in which we look for an optimization solution is the working zone. In order to obtain the optimal values of the parameters e and α, in which the loss of kinetic energy is minimal, we use the following input data [6, 13, 15–17]:

$$\begin{gathered}
\omega = 50\ [\mathrm{s}^{-1}], \quad V = 40\ [\mathrm{m/s}], \quad u = 0.5\ [\mathrm{m/s}], \quad \omega_0 = 0.6026\ [1/\mathrm{m}], \\
\beta = 0.2\ [\mathrm{rad}], \quad \gamma = 0.1\ [\mathrm{rad}], \quad H = 0.42\ [\mathrm{m}], \quad a_1 = 0.6\ [\mathrm{m}], \\
b_1 = 0.6\ [\mathrm{m}], \quad c_1 = 0.6\ [\mathrm{m}], \quad d_1 = 0.4\ [\mathrm{m}], \quad z_1 = 0\ [\mathrm{m}], \\
z_2 = 1.2\ [\mathrm{m}], \quad z_3 = 2.2\ [\mathrm{m}], \quad r_1 = 0.12\ [\mathrm{m}], \quad r_2 = 0.25\ [\mathrm{m}], \\
r_3 = r_4 = 0.8\ [\mathrm{m}], \quad r_5 = 0.1\ [\mathrm{m}], \quad r_6 = 0.16\ [\mathrm{m}], \quad b = 2.5\ [\mathrm{mm}], \\
g = 9.81\ [\mathrm{m/s}^2], \quad m_2 = 145\ [\mathrm{kg}], \quad m_3 = 810\ [\mathrm{kg}], \quad m_5 = 85\ [\mathrm{kg}], \\
J_\xi = 171\ [\mathrm{kgm}^2], \quad J_\varsigma = 336\ [\mathrm{kgm}^2], \quad m = 0.5, \\
J = J_x = J_y = 121 \times 10^{-8}\ [\mathrm{m}^4], \quad \rho = 7850\ [\mathrm{kg/m}^3], \quad N_e = 43.4\ [\mathrm{kW}], \\
K_\Delta = 80 \cdot 10^6\ [\mathrm{J/m}^3], \quad E = 2.06 \times 10^{11}\ [\mathrm{Pa}], \quad R_\Sigma = 2400\ [\mathrm{N}], \quad R_b^n = 525\ [\mathrm{N}].
\end{gathered}$$

We can calculate the values for the change of kinetic energy using the obtained expression (24), as well as the input data shown above. In this case, the parameter e changes from 0 to 0.0015 with step 0.0001 and the parameter α changes from 0 to 0.025 with step 0.0025. The obtained values are presented below as a matrix.

$$_\Delta T = 10^3 \times$$

$$\begin{bmatrix}
0.0000 & 0.0249 & 0.0996 & 0.2241 & 0.3984 & 0.6224 & 0.8962 & 1.2197 & 1.5929 & 2.0157 & 2.4882 \\
0.0003 & 0.0302 & 0.1099 & 0.2395 & 0.4188 & 0.6479 & 0.9267 & 1.2552 & 1.6335 & 2.0613 & 2.5388 \\
0.0010 & 0.0360 & 0.1208 & 0.2553 & 0.4397 & 0.6738 & 0.9577 & 1.2913 & 1.6745 & 2.1074 & 2.5899 \\
0.0023 & 0.0423 & 0.1321 & 0.2717 & 0.4611 & 0.7003 & 0.9892 & 1.3278 & 1.7160 & 2.1540 & 2.6415 \\
0.0040 & 0.0491 & 0.1439 & 0.2886 & 0.4830 & 0.7272 & 1.0211 & 1.3648 & 1.7581 & 2.2010 & 2.6936 \\
0.0063 & 0.0564 & 0.1563 & 0.3060 & 0.5054 & 0.7547 & 1.0536 & 1.4023 & 1.8006 & 2.2486 & 2.7462 \\
0.0091 & 0.0642 & 0.1691 & 0.3239 & 0.5284 & 0.7826 & 1.0866 & 1.4403 & 1.8437 & 2.2967 & 2.7993 \\
0.0124 & 0.0725 & 0.1825 & 0.3423 & 0.5518 & 0.8111 & 1.1201 & 1.4789 & 1.8873 & 2.3453 & 2.8529 \\
0.0162 & 0.0814 & 0.1964 & 0.3612 & 0.5757 & 0.8401 & 1.1541 & 1.5179 & 1.9313 & 2.3944 & 2.9070 \\
0.0204 & 0.0907 & 0.2107 & 0.3806 & 0.6002 & 0.8695 & 1.1886 & 1.5574 & 1.9759 & 2.4440 & 2.9617 \\
0.0252 & 0.1005 & 0.2256 & 0.4005 & 0.6251 & 0.8995 & 1.2237 & 1.5975 & 2.0210 & 2.4941 & 3.0168 \\
0.0305 & 0.1109 & 0.2410 & 0.4209 & 0.6506 & 0.9300 & 1.2592 & 1.6380 & 2.0666 & 2.5447 & 3.0725 \\
0.0363 & 0.1217 & 0.2569 & 0.4418 & 0.6765 & 0.9610 & 1.2952 & 1.6791 & 2.1127 & 2.5958 & 3.1286 \\
0.0426 & 0.1330 & 0.2732 & 0.4632 & 0.7030 & 0.9925 & 1.3317 & 1.7207 & 2.1593 & 2.6475 & 3.1853 \\
0.0495 & 0.1449 & 0.2901 & 0.4851 & 0.7299 & 1.0245 & 1.3688 & 1.7627 & 2.2064 & 2.6996 & 3.2424 \\
0.0568 & 0.1572 & 0.3075 & 0.5076 & 0.7574 & 1.0570 & 1.4063 & 1.8053 & 2.2540 & 2.7522 & 3.3001
\end{bmatrix}.$$

The obtained values show that the loss of kinetic energy changes significantly when the two parameters change. For this reason, we are looking for a solution in which the dynamic system will work in optimal mode. For this purpose, we apply the optimization procedure.

• **working zone**

For this zone, we define the bounds within which the parameters e and α change. It is also necessary to determine the lower and upper bounds lb and ub as well as the initial approximation $x0$.

$$x0 = [0.0125 \;\; 0.00075];$$
$$lb = [0.005 \;\; 0.0003];$$
$$ub = [0.020 \;\; 0.0012].$$

The following results are obtained after executing the procedure.

$$e = 4 \times 10^{-4} \text{ [m]}, \quad \alpha = 0.00524 \text{ [rad]}, \quad {}_{\Delta}T = 155.42 \text{ [J]}.$$

• **initial zone**

In this zone, the parameters e and α change within the following boundaries: $0 \leqslant e \leqslant 0.0015$, $0 \leqslant \alpha \leqslant 0.0050$. We can define the lower and upper bounds lb and ub. It is necessary to determine the initial approximation around which the function ${}_{\Delta}T$ has a local minimum. The results for this zone are presented below.

$$e = 0, \quad \alpha = 0, \quad {}_{\Delta}T = 0.$$

• **final zone**

For this zone, the bounds within which the parameters change are: $0 \leqslant e \leqslant 0.0015$, $0.020 \leqslant \alpha \leqslant 0.025$. We can determine the lower and upper bounds lb and ub as well as the initial approximation $x0$. The kinetic energy losses are the largest in this case. For this reason, the dynamic system should not operate in this zone. The minimum and maximum values of ${}_{\Delta}T$ are written below.

$$e = 0, \quad \alpha = 0.020 \text{ [rad]}, \quad {}_{\Delta}T = 1.5929 \times 10^{3} \text{ [J]},$$
$$e = 1.5 \times 10^{-3} \text{ [m]}, \quad \alpha = 0.025 \text{ [rad]}, \quad {}_{\Delta}T = 3.3001 \times 10^{3} \text{ [J]}.$$

4. Analysis of the obtained results

The calculated numerical data show the change of the function ${}_{\Delta}T$ depending on the change of linear and angular inaccuracies. These parameters depend on the operating conditions under which the two leading wheel work. The most important zone is the working zone. The minimum energy loss for this zone is ${}_{\Delta}T = 155.42$ [J]. This value is calculated by the optimization procedure at the corresponding values of the two parameters. These values are the optimization values and can be used in the operating mode. Having technological capabilities for measuring the real values of e and α, one can determine how far these values differ from the optimal ones. It can also be determined to what extent the real value of the function ${}_{\Delta}T$ differs from its minimum value. This allows determining whether the real machine is operating in optimal mode with minimum energy loss. If the difference is very large, technological solutions can be proposed to reduce this difference.

The initial zone is the most desirable zone in which the band saw machine can operate. In this case, the minimum energy loss is ${}_{\Delta}T = 0$. This value is obtained in the absence of linear and angular inaccuracies. Unfortunately, this case is unreal and cannot be applied. The band saw machine can operate in this area assuming high-grade work of the two leading wheels, as well as very precise technology of their assembling. However, this is difficult to achieve and often the above mentioned parameters have arandom character.

The last zone in which the band saw machine can operate is the final zone. This is the most undesirable area because the energy loss is much greater than that in the other two zones. In this case, the calculated values change in the following interval: $1.5929 \times 10^3 \leqslant {}_{\Delta}T \leqslant 3.3001 \times 10^3$ [J]. Such conditions may occur after prolonged operation of the band saw machine. In the operating mode, large dynamic and shock forces and moments occur that exert loads on the two leading wheels. These loads can change the magnitudes of the linear and angular inaccuracies, i.e., of the parameters e and α. If the values of these parameters exceed the maximum permissible values determined in the design of the machine, it is necessary to propose technological solutions for their reduction. In this way, one can guarantee the work of the band saw machine in the work zone with minimal energy loss.

5. Conclusion

In this article, we examined the loss of kinetic energy in big band saw machines. The main objectives of the study are formulated and the corresponding main tasks are solved. Expressions have been obtained to calculate the kinetic energy of the mechanical system in the ideal and the real case. With the help of these expressions, the final dependence for determining the energy losses for the studied class of machines was obtained. This dependence shows the influence of linear and angular inaccuracies, i.e., of the parameters e and α. A number of optimization solutions have been proposed that allow to calculate the values of both parameters that ensure a minimum value of energy loss.

The results obtained in the study may find application in the design of new band saw machines. The function ${}_{\Delta}T$ may take different values for each specific choice of the corresponding mass, inertial, geometric and kinematic characteristics. This means that the designers could change the characteristics of the main links of the machines, in particular: masses, mass moments of inertia, axial moments of inertia, linear dimensions as well as cutting speeds, feeding speeds and angular velocities, so that the working operation can be carried out with minimum energy consumption.

In conclusion, this research can be seen as a contribution to the contemporary knowledge on energy conservation and design of energy-saving machines. It may contribute to better understanding of these problems and prove useful for many researchers. In particular, the proposed approach can be used in the design of other classes of big woodworking machines, as well as in the study of energy losses for this class of machines.

Manuscript received by Editorial Board, June 15, 2020;
final version, September 03, 2020.

References

[1] M. Sarwar, M. Persson, H. Hellbergh, and J. Haider. Measurement of specific cutting energy for evaluating the efficiency of band sawing different workpiece materials. *International Journal of Machine Tools and Manufacture*, 49(11-12):958–965, 2009. doi: 10.1016/j.ijmachtools.2009.06.008.

[2] M. Mandic, S. Svrzic, and G. Danon. The comparative analysis of two methods for the power consumption measurement in circular saw cutting of laminated particle board. *Wood Research*, 60(1):125–136, 2015.

[3] Z. Kopecký, L. Hlaskova, and K. Orlowski. An innovative approach to prediction energetic effects of wood cutting process with circular-saw blades. *Wood Research*, 59(5):827–834, 2014.

[4] K. Orlowski, T. Ochrymiuk, A. Atkins, and D. Chuchala. Application of fracture mechanics for energetic effects predictions while wood sawing. *Wood Science and Technology*, 47(5):949–963, 2013. doi: 10.1007/s00226-013-0551-x.

[5] P. Iskra, C. Tanaka, and T. Ohtani. Energy balance of the orthogonal cutting process. *Holz Als Roh- und Werkstoff*, 63:358–364, 2005. doi: 10.1007/s00107-005-0021-8.

[6] P. Obreshkov. *Woodworking Machines*. Publishing House “BM”, 1995. (in Bulgarian).

[7] A. Pisarev, Ts. Paraskov, and C. Bachvarov. *Course in Theoretical Mechanics. Second part – Dynamics*. State Publishing House Technics, 1988. (in Bulgarian).

[8] R.M. Dreizler, and C.S. Lüdde. *Theoretical Mechanics: Theoretical Physics 1*. Springer, Berlin, Heidelberg, 2010. doi: 10.1007/978-3-642-11138-9.

[9] F. Scheck. *Mechanics. From Newton's Laws to Deterministic Chaos*. 5th edition, Springer, Berlin, Heidelberg, 2010.

[10] B. Marinov. *Dynamic and Shock Processes in Some Classes of Woodworking Machines. Analysis and Optimization*. Omniscriptum Publishing Group-Germany/LAP LAMBERT Academic Publishing, 2018.

[11] B. Cheshankov. *Theory of the Vibrations*. Publishing House in TU, 1992. (in Bulgarian).

[12] B. Marinov. Spatial deformations in the transmissions of certain classes of woodworking machines. *Mechanism and Machine Theory*, 82:1–16, 2014. doi: 10.1016/j.mechmachtheory.2014.07.010.

[13] Zh. Gochev. *Handbook for Exercise of Wood Cutting and Woodworking Tools*. Publishing House in LTU, 2005. (in Bulgarian).

[14] Yo. Tonchev. *Matlab, Part 3*. Publishing House Technique, 2009. (in Bulgarian).

[15] R. Peters. *Band Saw Fundamentals: The Complete Guide*. Hearst Communications Inc, 2006.

[16] L. Bird. *The Bandsaw Book*. Taunton Press Inc, 2000.

[17] W. Turner. *A Comprehensive Handbook on Uses and Applications of the Band Saw and Jig Saw*. Literary Licensing LLC, 2013.