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THE INFLUENCE OF EQUIVALENT RADIUS AND THREAD INCLINATION ANGLE IN A SCREW JOINT ON DISSIPATION OF ENERGY FOR FRICTIONAL MODELS OF BOLTED JOINT

The article presents the problem of structural friction appearing in a screw joint with frictional effects between its elements. In the article, two mathematical models of screw joint are analysed. In the first model, high stiffness of a nut is assumed. In the second model, the influence of both cooperating elements (the screw and nut) is assumed.

1. Introduction

Damping of vibration results from mechanical energy dissipation, which is associated with motion of mechanical assemblies [3,] [4], [5], [19], [20]. The largest intensity of vibrations appears in resonance states, and can cause defective operation of equipment (reduced accuracy of performance of elements). Furthermore, the vibrations can destroy mechanical elements and cause disconnection of joints, too. In many cases, constructors use special dampers of vibrations. Admittedly, it raises costs and causes additional problems with exploitation. Thus, it seems useful to continue works on suppression of vibrations by natural ways, such as structural friction.

Real bodies are not perfectly stiff, and therefore even after application of very small forces to shift two tightened elements springy deformation of some areas of these bodies follows. There are very small slides on the areas. The forces of friction execute work, so they cause dissipation of energy. There are many joints in machines and systems that are motionless

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by definition. Yet, in consequence of the deformations in the joint bodies, the dissipation of energy appears in them. The resistance caused by friction in motionless joints related to elasticity of connected bodies is called structural friction [3], [4], [17], [19].

The article presents the problem of structural friction appearing in a screw joint with frictional effects between its elements (Fig.1). The description of this problem is not simple, therefore, basic laws of mechanics of stress and deformation distribution tension, compression, torsion and shearing are often applied.

The hysteresis loop is the measure of energy dissipation in mechanical arrangements. In the paper, the influence of angle of thread inclination of a screw joint and of equivalent radius on dissipation of energy (hysteresis loop) for two mathematical models of screw joint is presented.

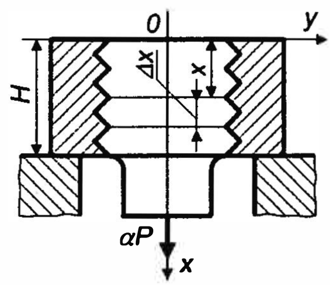

Fig.1. Physical model of the considered screw joint

2. Mathematical models of screw joint

The purpose of this article is to show the influence of angle of thread inclination of a screw joint and equivalent radius on dissipation of energy for the frictional models of bolted joint. First, two models are drawn up. In the first model, high stiffness of a nut is assumed. In the second model, the influence of both cooperating elements (the screw and nut) is assumed.

The considerations in both cases were divided to two stages, analysing first the segment of screw joint (Fig.2, Fig.3), and then solving the value of dislocations and deformations of extreme section of the whole connection (Fig.1).

where:

d_0 – equivalent diameter, d, d_3 – external and internal diameter of cube of surface of segment, d_z – external diameter of screw joint, p – unit pressure, s – height of segment; μ – friction coefficient, σ_{x1}, σ_{x2} – stresses, β – angle of thread inclination of a screw joint, Δr_{01} – absolute extension, u – dislocation

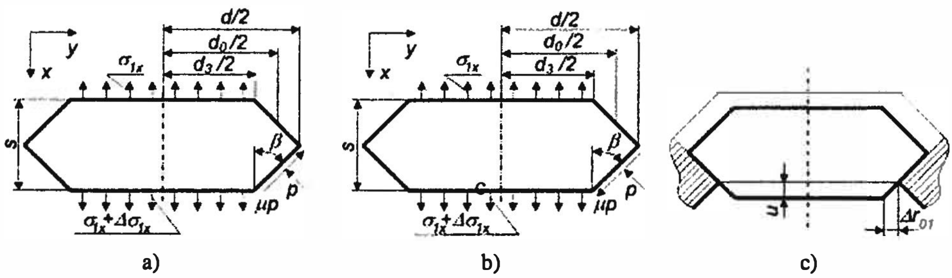

Fig.2. Model 1 – segment of screw joint: a) axial loading; b) axial releasing; c) dislocation of the segment of screw joint.

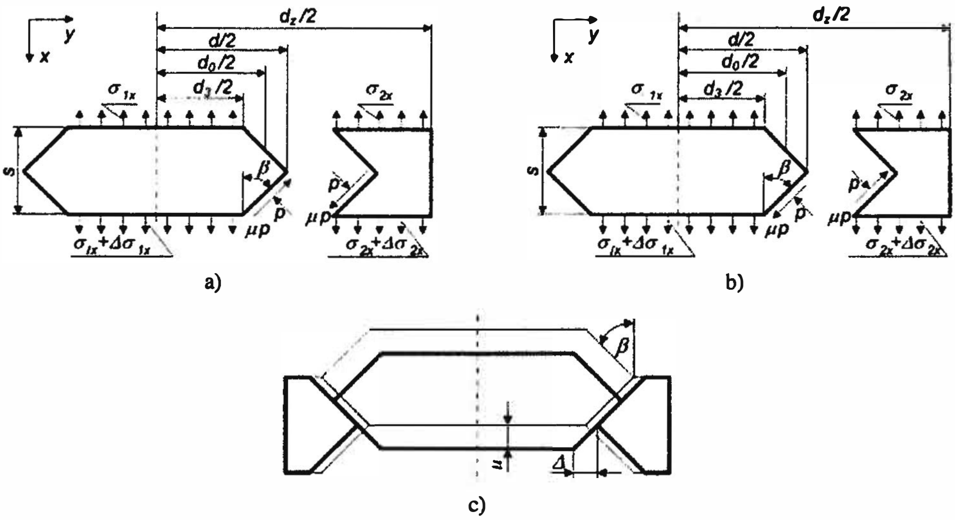

Fig.3. Model 1 – segment of screw joint: axial loading;axial releasing; dislocation of the segment of screw joint.

of the segment of the screw joint, Δ – clearance between cooperating elements.

Assumptions:

1. Distribution of the pressure per unit area between cooperating surface of screw joint is uniform.
2. Joined elements can be characterized by a constant coefficient of friction for any value of the pressure per unit area.
3. Friction forces on the faying surface of mating elements are subject to Coulomb's law.
4. Properties of the material are subject to Hooke's law.
5. Friction is fully developed in the slip zone and amounts to zero outside of it.

6. Changes in the force P are slow, which justifies the omission of inertia forces.
7. Assumption of plane sections (transverse sections are plane and do not deform after stress) holds.
8. In the model, the role of internal friction is not taken intro account.

It is assumed that the screw joint is axially loaded. Therefore, the problem is split into two stages (loading and releasing) and the maximum stresses are in elastic strain of material of screw joint. When the loading increases, the forces of friction on the joined surface increase gradually, too. When the unit pressure p is very high, the elements become permanently joined. The friction forces change their sign from $\mu \cdot p$ to $-\mu \cdot p$. The elements in the system do not get relocated, which results in a permanent radial deformation (segment 2 – Fig.4).

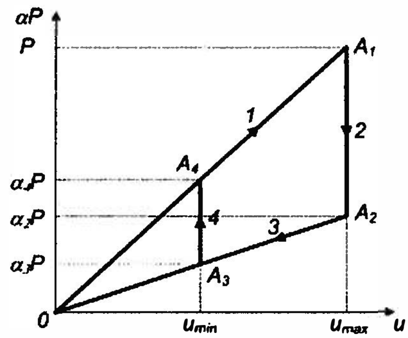

Fig.4. Hysteresis loop of the screw joint.

The system is exposed to normal stresses σ_X and $\sigma_X + \Delta\sigma_X$ distributed evenly on the surfaces of the elements in the system (Fig. 2). The normal stresses occur due to action of the $\alpha x P$ $(0 \leq \alpha \leq 1)$ axial load in the discussed system.

The value of the average radius is the following [1], [14], [15]:

$$\frac{d_0}{2} = r_0 = \frac{2}{3} \cdot \frac{\left(\frac{d}{2}\right)^3 - \left(\frac{d_3}{2}\right)^3}{\left(\frac{d}{2}\right)^2 - \left(\frac{d_3}{2}\right)^2} \tag{2.1}$$

For the model 1, the process of loading and unloading (segment 1 and 3 – Fig.4) can be described with the following equations:

The equation of the equilibrium of forces operating in the system (in the direction of the x-axis, Fig. 2) is as follows:

– for the screw sector

$$-\sigma_{1x}\cdot\pi\cdot r_0^2-\mu\cdot p\cdot\cos\beta\cdot 2\pi\cdot r_0\cdot\frac{s}{2\cos\beta}-p\cdot\sin\beta\cdot 2\pi\cdot r_0\cdot\frac{s}{2\cos\beta}+ \\ +(\sigma_{1x}+\Delta\sigma_{1x})\cdot\pi\cdot r_0^2=0 \tag{2.2}$$

The equation of the equilibrium of forces operating in the system in the radial direction (y-axis, Fig. 2) takes the following form:

– for the screw sector

$$-\sigma_{1r}+\mu\cdot p\cdot\sin\beta-p\cdot\cos\beta=0 \tag{2.3}$$

$$\varepsilon_{1r}=-\frac{p\cdot\cos\beta\cdot(1-\mu\cdot tg\,\beta)\cdot(1-\nu_1)}{E_1}-\frac{\nu_1\cdot\sigma_{1x}}{E_1} \tag{2.4}$$

$$\Delta r_{01}=-\varepsilon_{1r}\cdot r_0=\frac{p\cdot\cos\beta\cdot r_0\cdot(1-\mu\cdot tg\,\beta)\cdot(1-\nu_1)}{E_1}+\frac{\nu_1\cdot r_0\cdot\sigma_{1x}}{E_1} \tag{2.5}$$

$$u=\frac{\Delta r_{01}}{tg\,\beta} \tag{2.6}$$

After substituting formula (2.5) into formula (2.6), the following formula on the axial displacement is obtained:

$$u=\frac{p\cdot\cos\beta\cdot r_0\cdot(1-\mu\cdot tg\,\beta)\cdot(1-\nu_1)}{E_1\cdot tg\,\beta}+\frac{\nu_1\cdot r_0\cdot\sigma_{1x}}{E_1\cdot tg\,\beta} \tag{2.7}$$

$$\varepsilon_{1x}=\frac{du}{dx}=\frac{r_0^2\cdot\cos\beta\cdot(1-\mu\cdot tg\,\beta)\cdot(1-\nu_1)}{E_1\cdot tg\,\beta\cdot(tg\,\beta+\mu)}\cdot\sigma''_{1x}+\frac{\nu_1\cdot r_0}{E_1\cdot tg\,\beta}\cdot\sigma'_{1x} \tag{2.8}$$

After transformation of formulas 2.4 and 2.8, we obtain:

$$\sigma''_{1x}+\left(\frac{\nu_1\cdot r_0}{tg\,\beta}-\frac{2\nu_1\cdot r_0\cdot\cos\beta\cdot(1-\mu\cdot tg\,\beta)}{tg\,\beta+\mu}\right)\frac{tg\,\beta\cdot(tg\,\beta+\mu)}{r_0^2\cdot\cos\beta\cdot(1-\mu\cdot tg\,\beta)\cdot(1-\nu_1)}\cdot\sigma'_{1x}-$$
$$+\frac{tg\,\beta\cdot(tg\,\beta+\mu)}{r_0^2\cdot\cos\beta\cdot(1-\mu\cdot tg\,\beta)\cdot(1-\nu_1)}\cdot\sigma_{1x}=0 \quad (2.9)$$

$$\sigma''_{1x}+a_0\cdot\sigma'_{1x}-b_0\cdot\sigma_{1x}=0 \quad (2.10)$$

where:

$$a_0=\frac{\nu_1(tg\,\beta+\mu-2\sin\beta+2\cos\beta\cdot\mu\cdot tg^2\beta)}{r_0\cdot\cos\beta\cdot(1-\mu\cdot tg\,\beta)\cdot(1-\nu_1)},\quad b_0=\frac{tg\,\beta\cdot(tg\,\beta+\mu)}{r_0^2\cdot\cos\beta\cdot(1-\mu\cdot tg\,\beta)\cdot(1-\nu_0)}$$

(2.11)

The axial displacement of the whole screw joint during loading of the system for the boundary conditions (Fig.1):

$$for\ \ x=0\Rightarrow\sigma_{1x}=0$$
$$for\ \ x=H\Rightarrow\sigma_{1x}=\frac{\alpha\cdot P}{\pi\cdot r_0^2} \quad (2.12)$$

where:
α – coefficient which changes from *0* to *1* $0\le\alpha\le1$ (*Fig.4*)
can be described:

$$u|_{(x=H)}=\frac{\alpha\cdot P}{\pi\cdot E_1\cdot tg\,\beta}\cdot\left(\frac{\cos\beta\cdot(1-\mu\cdot tg\,\beta)\cdot(1-\nu_1)}{2(tg\,\beta+\mu)}\left(\frac{\sqrt{a_0{}^2+4b_0}}{th\left[\frac{H}{2}\sqrt{a_0{}^2+4b_0}\right]}-a_0\right)+\frac{\nu_1}{r_0}\right)$$

(2.13)

where:
H – height of screw joint, u – axial displacement, P – axial force, p – pressures per unit area; μ – coefficient of friction; E_i – Young's modulus of elasticity.

Similarly, the dislocation of the extreme cross-section in stage three is solved, and it has the following form:

$$u\big|_{(x=H)}=\frac{\alpha\cdot P}{\pi\cdot E_1\cdot tg\,\beta}\cdot\left(\frac{\cos\beta\cdot(1+\mu\cdot tg\,\beta)\cdot(1-\nu_1)}{2(tg\,\beta-\mu)}\cdot\left(\frac{\sqrt{a_1^2+4b_1}}{th\left[\frac{H}{2}\cdot\sqrt{a_1^2+4b_1}\right]}-a_1\right)+\frac{\nu_1}{r_0}\right)$$

(2.14)

where:

$$a_1=\frac{\nu_1(tg\,\beta-\mu-2\sin\beta-2\cos\beta\cdot\mu\cdot tg^2\beta)}{r_0\cdot\cos\beta\cdot(1+\mu\cdot tg\,\beta)\cdot(1-\nu_1)},\quad b_1=\frac{tg\,\beta\cdot(tg\,\beta-\mu)}{r_0^2\cdot\cos\beta\cdot(1+\mu\cdot tg\,\beta)\cdot(1-\nu_1)}$$

(2.15)

The energy dissipation for one cycle of stress (Fig. 4) equals:

$$\psi=\frac{P^2\cdot\alpha_1^2\cdot\nu_1}{2\pi\cdot E_1\cdot tg\,\beta\cdot r_0}\cdot(n_1+1)\cdot\left(1-\frac{n_1+1}{n_2+1}\right)-\frac{P^2\cdot\alpha_3^2\cdot\nu_1}{2\pi\cdot E_1\cdot tg\,\beta\cdot r_0}\cdot(n_2+1)\cdot\left(\frac{n_2+1}{n_1+1}-1\right)$$

(2.16)

where:

$$n_1=\frac{r_0}{\nu_1}\cdot\frac{\cos\beta\cdot(1-\mu\cdot tg\,\beta)\cdot(1-\nu_1)}{2(tg\,\beta+\mu)}\cdot\left(\frac{\sqrt{a_{0}^2+4b_0}}{th\left[\frac{H}{2}\cdot\sqrt{a_{0}^2+4b_0}\right]}-a_0\right)$$

$$n_2=\frac{r_0}{\nu_1}\cdot\frac{\cos\beta\cdot(1+\mu\cdot tg\,\beta)\cdot(1-\nu_1)}{2(tg\,\beta-\mu)}\left(\frac{\sqrt{a_1^2+4b_1}}{th\left[\frac{H}{2}\cdot\sqrt{a_1^2+4b_1}\right]}-a_1\right)$$

(2.17)

The dislocation of the extreme cross-section and energy dissipation for one cycle of stress for model two are solved similarly and they have the following form:

$$u\big|_{(x=H)}=\frac{\alpha\cdot P}{\pi\cdot r_0^2}\cdot\left(\frac{l_1\cdot r_0}{tg\,\beta}-\frac{a\cdot l_0}{2\,tg\,\beta}+\frac{l_0\cdot\sqrt{a_{0}^2+4b_0}}{2\,tg\,\beta\cdot th\left[\frac{H}{2}\sqrt{a_{0}^2+4b_0}\right]}\right) \quad (2.18)$$

$$u|_{(x=H)}=\frac{\alpha\cdot P}{\pi\cdot r_0^2}\cdot\left(\frac{l_1\cdot r_0}{tg\,\beta}-\frac{a_1\cdot l_2}{2\,tg\,\beta}+\frac{l_2\cdot\sqrt{a_1^{\,2}+4b_1}}{2\,tg\,\beta\cdot th\left[\frac{H}{2}\sqrt{a_1^{\,2}+4b_1}\right]}\right) \tag{2.19}$$

where:

$$a_0=\frac{-m_0\cdot tg\,\beta+l_1\cdot r_0}{l_0};\quad b_0=\frac{tg\,\beta}{E_1\cdot l_0} \tag{2.20}$$

$$a_1=\frac{-m_1\cdot tg\,\beta+l_1\cdot r_0}{l_2};\quad b_1=\frac{tg\,\beta}{E_1\cdot l_2} \tag{2.21}$$

$$l_1=-\frac{\nu_2\cdot r_0^2}{E_2\cdot\left(r_{z0}^2-r_0^2\right)}+\frac{\nu_1}{E_1} \tag{2.22}$$

$$\begin{aligned}m_0&=\frac{2\nu_1\cdot r_0\cdot\cos\beta\cdot(1-\mu\cdot tg\,\beta)}{E_1\cdot(\mu+tg\,\beta)}\\ m_1&=\frac{2\nu_1\cdot r_0\cdot\cos\beta\cdot(1+\mu\cdot tg\,\beta)}{E_1\cdot(tg\,\beta-\mu)}\end{aligned} \tag{2.23}$$

Energy dissipation for one cycle of stress (Fig.4) equals:

$$\psi=\frac{P^2\cdot\alpha_1^2\cdot m_2}{2\pi\cdot r_0^2}\cdot\left(1-\frac{m_2}{m_3}\right)-\frac{P^2\cdot\alpha_3^2\cdot m_3}{2\pi\cdot r_0^2}\cdot\left(\frac{m_3}{m_2}-1\right) \tag{2.24}$$

where:

$$\begin{aligned}m_2&=\frac{l_1\cdot r_0}{tg\,\beta}-\frac{a_0\cdot l_0}{2\,tg\,\beta}+\frac{l_0\cdot\sqrt{a_0^{\,2}+4b_0}}{2\,tg\,\beta\cdot th\left[\frac{H}{2}\sqrt{a_0^{\,2}+4b_0}\right]}\\ m_3&=\frac{l_1\cdot r_0}{tg\,\beta}-\frac{a_1\cdot l_2}{2\,tg\,\beta}+\frac{l_2\cdot\sqrt{a_1^{\,2}+4b_1}}{2\,tg\,\beta\cdot th\left[\frac{H}{2}\sqrt{a_1^{\,2}+4b_1}\right]}\end{aligned} \tag{2.25}$$

A detailed analysis of this problem was described in earlier articles by the author [10], [11], [12], [13].

3. Simulation results

Simulations have been conducted with the help of professional software (Mathematica 4.1). The main purpose of the simulations was to show the influence of the angle of thread inclination of a screw joint and of the equivalent radius on dissipation of energy for the frictional models of bolted joint.

The values of the loop field of hysteresis and the values of displacement for selected angles (metric and unified screw thread) of thread inclination were taken down in Table 2 and Table 3.

Simulations show (Fig. 5 and Fig. 6) that the dependence between the angle of thread inclination of a screw joint and the field of hysteresis is not linear. Similarly, the dependence between the angle of thread inclination of a screw joint and the displacement of the extreme cross-section is not linear either.

Moreover, it can be easily noticed that both values decrease with the growth of value of inclination angle of screw thread. It can be proven that in inch screw joints (thread angle amounts to 55 degrees) the natural dissipation of energy is larger than in metric screw joint (thread angle amounts to 60 degrees (Table 2). The screw connection should be self-locking and the thread angle has to meet condition ($tg\beta \leq \mu$).

Table 1

No.	Parameter [unit]	Value
1.	Maximum axial force P [N]	8000
2.	Parameter α_1	1
3.	Parameter α_3	0.125
4.	Poisson ratio ν	0.32
5.	Young's modulus E [N/m^2]	2.1×10^{11}
6.	Coefficient of friction μ	0.17
7.	Angle of thread inclination of a screw joint β [°]	55;60
8.	Full diameter of screw d [m]	10×10^{-3}
9.	Inside diameter of screw d_3 [m]	8.026×10^{-3}
10.	Equivalent diameter of screw joint d_z [m]	13×10^{-3}
11.	Height of screw joint H [m]	20.03×10^{-3}
12.	Pitch of thread s [m]	1.5×10^{-3}
13.	Equivalent radius r_0 [m]	4.525×10^{-3}

Table 2

Angle β[°]	Loop field of hysteresis ψ [Nm]		In percentages [%]	
	Model 1	Model 2	Model 1	Model 2
60	0.0014177	0.0017854	79.40	100
55	0.0018024	0.0022564	79.88	100
Angle β[°]	Displacement u [mm]		In percentages [%]	
	Model 1	Model 2	Model 1	Model 2
60	0.0011412	0.00139738	81.67	100
55	0.0015071	0.0018562	81.19	100

Table 3

Model	Loop field of hysteresis ψ [Nm]		In percentages [%]	
	β=55 [°]	β=60 [°]	β=55 [°]	β=60 [°]
Model 1	0.0018024	0.0014177	100	78.66
Model 2	0.0022564	0.0017854	100	79.13
Model	Displacement u [mm]		In percentages [%]	
	β=55 [°]	β=60 [°]	β=55 [°]	β=60 [°]
Model 1	0.0015071	0.0011412	100	75.72
Model 2	0.0018562	0.00139738	100	75.28

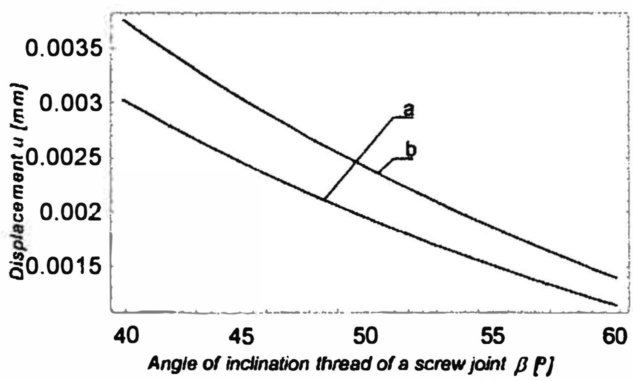

Fig.5. Dependence of angle of thread inclination of a screw joint u(β) on displacement: a) model 1; b) model 2

It can be stated that while designing machines, devices and constructions in which one intends to have the best utilization of natural properties of damping of vibrations of thread joints, one should use screw threads with the smallest angle of screw thread β (Fig. 6) – taking all geometrical parameters and the property of screw joint (e.g. self-locking condition) into consideration.

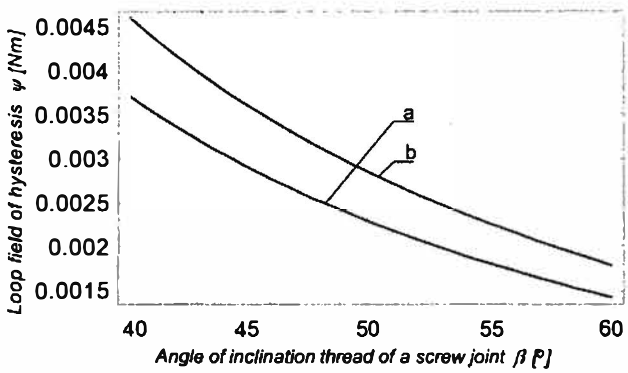

Fig.6. Dependence of angle of thread inclination of a screw joint ψ(β) on energy losses: a) model 1; b) model 2

The influence of equivalent radius on displacement of the extreme cross-section for the frictional models of screw joint is not linear (Fig.7). Similarly, the influence of radius on dissipation of energy is not linear, either (Fig.8). Calculations have been conducted with professional software (Mathematica 41). The program allows for changing all geometrical parameters of screw joint.

Graphs 7 and 8 show that, with the growth of value of equivalent radius, both values (energy losses and displacement) decrease.

Moreover, displacement and energy losses are larger for model 2 Fig.5 and Fig.6.

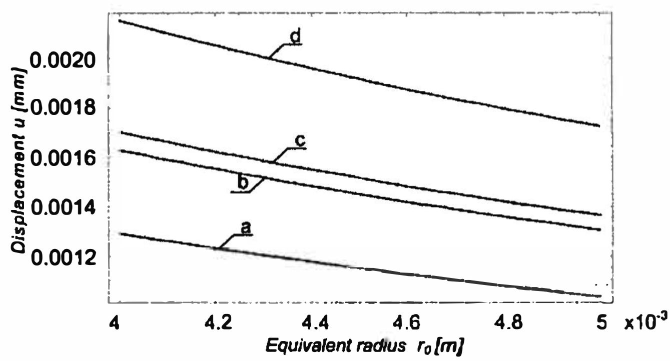

Fig.7. Dependence of equivalent radius of a screw joint $u(r_0)$ on displacement: a) model 1 (β=60°); b) model 2 (β=60°); c) model 1 (β=55°); d) model 2 (β=55°)

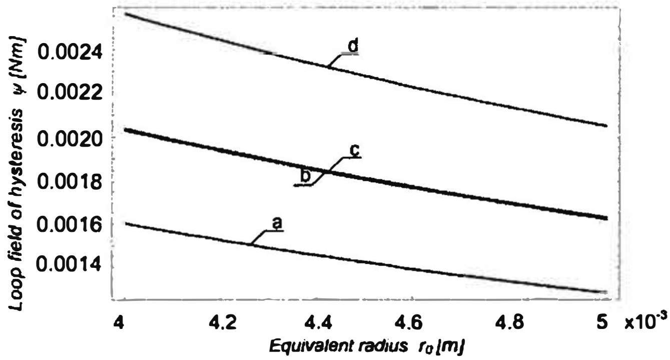

Fig.8. Dependence of equivalent radius of a screw joint $\psi(r_0)$ on energy losses: a) model 1 (β=60°); b) model 2 (β=60°); c) model 1 (β=55°); d) model 2 (β=55°)

4. Conclusion

The article presents a problem of dissipation of energy in screw joints with frictional interaction between its elements. The structural friction in screw joints can be used to damp vibrations in dynamic systems. Thus, it seems useful to continue works on suppression of vibrations by natural ways, such as structural friction. Basic laws of mechanics have been applied during the analysis of the distribution of stresses and deformations at tension and compression. The calculations have been conducted with professional software (Mathematica 4.1), which made it possible to analyze them quickly.

The article presents two mathematical models of screw joints. In the first model, high stiffness of a nut is assumed. In the second model, the influence of both cooperating elements (the screw and nut) is assumed.

The results of the simulation show that geometrical parameters of screw joint (Table 1) have large influence on the dissipation of energy (Fig.6 and Fig.8) and the dislocation of the extreme cross-section of screw joint (Fig.5 and Fig.7). The value of the loop field of hysteresis and the maximum values of displacement for selected angles of thread inclination were taken down in Table 2. Simulations show (Fig.5 and Fig.6) that the dependence between the angle of thread inclination of a screw joint and the energy losses is not linear. Similarly, the dependence between the angle of thread inclination of a screw joint and the displacement of the extreme cross-section is not linear, either.

Moreover, the growth of value of inclination angle of screw thread both values decrease. It can be proven that in inch screw joints (β=55°) the natural dissipation of energy is larger than in metric screw joint (β=60°).

Mathematical models were created on the basis of an equivalent radius (formula 2.1). The value of the equivalent radius depends on character of pressure per unit area. In the article, it has been assumed that the distribution of the pressure per unit area between cooperating surface of screw joint is even (2.1-2.25). This is not true. In real mechanical systems, the distribution of pressure per unit area can have different shapes (triangular, parabolic etc.) and that is why the value of equivalent radius is different too.

Graphs 7 and 8 show that with the growth of value of equivalent radius both values (energy losses and displacement) decrease non-linearly. Simulations show that in model 2, the values of dissipation of energy and displacement are larger than in model 1.

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REFERENCES

[1] Den Hartog J.P.: Forced vibrations with combined viscos and coulumb damping. Phil. Magaz. a J. of sci, vol. 9, 1930, nr 59, Suppl.
[2] Dietrych J.: Podstawy konstrukcji maszyn. WNT, Warszawa 1975.
[3] Giergiel J.: Problemy tarcia konstrukcyjnego w dynamice maszyn. Zeszyty Naukowe AGH, Kraków, 1971, z.44.
[4] Giergiel J.: Problemy tarcia konstrukcyjnego w nieruchomych połączeniach. Zeszyty Naukowe AGH z.30, 1969.
[5] Giergiel J.: Tłumienie drgań mechanicznych. PWN, 1989.
[6] Grudziński K. i inni: Opracowanie metod komputerowych do projektowania połączeń elementów maszyn z uwzględnieniem zjawisk kontaktowych. Sprawozdanie R.P.I.06.,1.20, Warszawa, 1998, 1990.
[7] Grudziński K., Kawiak R.: Zagadnienie modelowania i obliczania połączeń śrubowych metodą elementów skończonych. Sympozjum, Modelowanie w mechanice PTMTS, Beskid Śląski 1981, pp. 179÷188.
[8] Grudziński K.: Niektóre zagadnienia mechaniki kontaktu w budowie maszyn. Prace naukowe Politechniki Szczecińskiej, 339, 1987.
[9] Kaczmarek W.: Analiza czysto-tarciowego modelu połączenia gwintowego. Biuletyn WAT, Vol. LII, Nr 5-6 (2003), pp. 123÷139.
[10] Kaczmarek W.: Analysis of a Bolted Joint with Elastic and Frictional Effects Occurring Between its Elements. Machine Dynamics Problems, PW 2003, Vol. 27, No 1.
[11] Kaczmarek W.: Badanie modelu połączenia gwintowego z czysto-tarciowym oddziaływaniem elementów. Biuletyn WAT, 2001, 12(592), pp. 127÷145.
[12] Kaczmarek W.: Badanie rozpraszania energii w modelu połączenia gwintowego za pomocą maszyny MTS. Biuletyn WAT, Vol. LII, Nr 5-6 (2003), pp. 141÷159.
[13] Kaczmarek W.: Rozpraszanie energii w połączeniu gwintowym z czysto-tarciowym oddziaływaniem elementów. Biuletyn WAT, 2001, sierpień, 8(588), pp. 89÷107.

[14] Kaczmarek W.: Rozpraszanie energii w połączeniu gwintowym ze sprężysto-tarciowym oddziaływaniem elementów. Prace Instytutu Podstaw Budowy Maszyn Politechniki Warszawskiej, 2001, z.21, pp. 83÷101.
[15] Kaczmarek W.: Analysis of a Bolted Joint with Elastic and Frictional Effects Occurring Between its Elements. Machine Dynamics Problems, PW 2003, Vol. 27, No 1.
[16] Kalinin N. G., Liebiediew Ju. A., Liebiediewa W. I., Panowko Ja. G., Strachow G. I., Konstrukcjonnoje diempfirowanije w niepodwiżnych sojedinienjach, Riga: A N. Łotew. SRR., 1960.
[17] Kalinin N.G., Panowko J.G., Lebedev V.J: Konstrukcjonnoje dempfirowanije w niepodwiżnych soedinieniach. Ryga, 1961.
[18] Niezgodziński M.: Wytrzymałość materiałów. PWN, Warszawa 1981.
[19] Osiński Z.: Tłumienie drgań mechanicznych. PWN, Warszawa 1986.
[20] Osiński Z.: Tłumienie drgań. PWN, Warszawa 1997.
[21] Poradnik Mechanika. WNT, Warszawa 1984.

Wpływ kąta pochylenia gwintu i promienia ekwiwalentnego na rozpraszanie energii dla czysto- tarciowych modeli połączenia gwintowego

Streszczenie

W artykule przedstawiono problem tarcia konstrukcyjnego w aspekcie rozpraszania energii w połączeniu gwintowym. Rozpatrzono dwa modele matematyczne połączeń gwintowych: model czysto – tarciowy z założeniem dużej sztywności nakrętki oraz model czysto-tarciowy z uwzględnieniem oddziaływania obu współpracujących elementów (śruby i nakrętki).