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APPLICATION OF DIFFERENTIAL EVOLUTION ALGORITHM FOR IDENTIFICATION OF EXPERIMENTAL DATA

In the paper, the authors present the approach to modelling of austenitic steel hardening basing on the Frederick-Armstrong's rule and Chaboche elastic-plastic material model with mixed hardening. Non-linear uniaxial constitutive equations are derived from more general relations with the assumption of an appropriate evolution of back stress. The aim of the paper is to propose a robust and efficient identification method of a well known material model.

A typical LCF strain-controlled test was conducted for selected amplitudes of total strain. Continuous measurements of instant stress and total strain values were performed. Life time of a specimen, signals amplitudes and load frequency were also recorded.

Based on the measurement, identification of constitutive equation parameters was performed. The goal was to obtain a model that describes, including hardening phenomenon, a material behaviour during the experiment until the material failure. As a criterion of optimisation of the model least square projection accuracy of the material response was selected.

Several optimisation methods were examined. Finally, the differential evolution method was selected as the most efficient one. The method was compared to standard optimisation methods available in the MATLAB environment. Significant decrease of computation time was achieved as all the optimisation procedures were run parallel on a computer cluster.

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1. Introduction

The essential experimental tools that provide data for parameters identification of constitutive models are cyclic fatigue tests. For the description of many materials, linear constitutive models are not suitable. Very often it is necessary to introduce some nonlinearity to the constitutive relation in order to obtain satisfactory results in modelling of such a material.

On the other hand, having a non-linear constitutive equation, one encounters the problem of finding analytical solution, and numerical methods must be employed. Additionally, in such a case identification of the equation parameters is not a trivial problem especially due to the required numerical differentiation of recorded signals.

2. Constitutive equations

In literature describing cyclic properties of material, the elastc-plastic model with mixed hardening is most frequently assumed [1], [2]. The back stress specifies the center of yield surface and represents the kinematic hardening. In turn, varying radius of this surface represents isotropic hardening or softening (the yield surface expands or shrinks). The yield condition is assumed in a familiar form.

$$F = \sqrt{\frac{3}{2}(s_{ij} - X_{ij})(s_{ij} - X_{ij})} - R = 0 \tag{1}$$

where s_{ij} denotes the stress deviator, X_{ij} is the back stress deviator and R is the radius of the yield surface. Assuming the evolution of back stress to be governed by the hardening and recovery processes, we have

$$\dot{X}_{ij} = \frac{2}{3} C \dot{\varepsilon}^p_{ij} - \gamma X_{ij} \dot{\lambda} \tag{2}$$

ε^p_{ij} denotes the plastic strain tensor and λ is the length of plastic strain trajectory

$$\lambda = \int_0^t \sqrt{\frac{2}{3} \dot{\varepsilon}^p_{ij} \dot{\varepsilon}^p_{ij}}\, dt \tag{3}$$

The equation (2) is the one most often used to describe cyclic properties of a material. In the case of different kinds of irreversible processes considered like creep or ratchetting, the back stress is defined by the following sum:

$$\dot{X}_{ij} = \sum_k \dot{X}_{ij}^{(k)} = \frac{2}{3} \sum_k C_k \dot{\varepsilon}_{ij}^p - \gamma_k X_{ij}^{(k)} \tag{4}$$

The Authors' aim is to verify identification method, so that the simplest formulation of equation (4) is used. The equation (2) is the familiar Frederick-Armstrong rule with constant C and λ. The back stresses than evolves to the limit surface [3]

$$\sqrt{\frac{3}{2} X_{ij} X_{ij}} = \frac{3C}{2\gamma} \tag{5}$$

Similarly, we have limit surface in the stress space

$$\sqrt{\frac{3}{2} s_{ij} s_{ij}} = R + \frac{3C}{2\gamma} \tag{6}$$

In turn, we assume the evolution equation for radius of the yield surface

$$\dot{R} = Q(\lambda)\dot{\lambda} \tag{7}$$

The parameter of isotropic hardening depends on the length of plastic strain trajectory only. The plastic strain tensor can be evaluated assuming, for example, the associated flow rule

$$\dot{\varepsilon}_p = \dot{\lambda} \frac{\partial F}{\partial \sigma_{ij}} = \dot{\lambda} \frac{3(s_{ij} - X_{ij})}{2R} \tag{8}$$

The hardening modulus H can be determined from the consistency condition

$$\frac{\partial F}{\partial \sigma_{ij}} \dot{\sigma}_{ij} - \dot{\lambda} H = 0, \tag{9}$$

where

$$H = C - \frac{3(s_{ij} - X_{ij}) X_{ij}}{2R} + Q(\lambda) \tag{10}$$

The model parameters C and λ may be functions of the length of plastic strain trajectory as well as the functions of distance δ between the yield surface and corresponding limit surface. The distance δ is the length of the interval determined by the actual point on the yield surface and the point on the limit surface with the same normal vector. In uniaxial state equations (1)–(5) took respectively the following form

$$|\sigma - X| = R$$
$$\dot{X} = C\dot{\varepsilon}_p - \gamma(\lambda)X\left|\dot{\varepsilon}_p\right|$$
$$X_{\text{lim}} = \frac{C}{\gamma(\lambda)}$$
$$\lambda = \int_0^t \left|\dot{\varepsilon}_p\right| dt$$

The function $\gamma(\lambda)$ must be a function of positive values and asymptotically convergent to a specified finite value. The function may be used to describe material cyclic hardening or softening. If the function is decreasing, the radius of the limit surface grows and hardening of the material occurs. On the other hand, if the function increases, the limit surface shrinks, and softening of the material occurs.

3. Experimental data

Experimental data were obtained in cyclic tests on typical round specimens (acc. to PN-74-H-04327) of diameter of 7.5 mm made of AISI 304 steel. Mechanical properties of steel are given in Table 1. The tests were performed on MTS machine with the total strain signal amplitude being controlled and the frequency of the signal kept at 0.2 Hz.

Table 1.

Mechanical properties of AISI 304 steel

E [Mpa]	HRB	$R_{0.2}$ [MPa]	R_t [MPa]	R_a [MPa]	A5 [%]
1.93e5	216	593	752	65	34

It was assumed that the total strain may be decomposed into plastic and elastic parts and therefore the plastic strain value was computed in a following way

$$\varepsilon_p = \varepsilon_t - \frac{\sigma}{E} \tag{11}$$

During the experiment, the load and the signal of material response were recorded with a time index for every one of 80 points per cycle. There were two types of experiments realised. In the first one, the amplitude of the control signal was changed starting from 0.0005 to 0.007 with a step of 0.0005. There were 100 cycles per amplitude level realized. In the second one, typical fatigue experiments with constant amplitude of total strain were performed.

For materials exhibiting distinct hardening (see Fig. 1), like austenitic steel AISI304, identification based on the data acquired in a step test leads to underestimated results for hardening, although this type of experiment is a good test bed for an identification procedure development and verification. It is also useful for a fast estimation of parameters ranges that are further necessary as an input for the identification procedure.

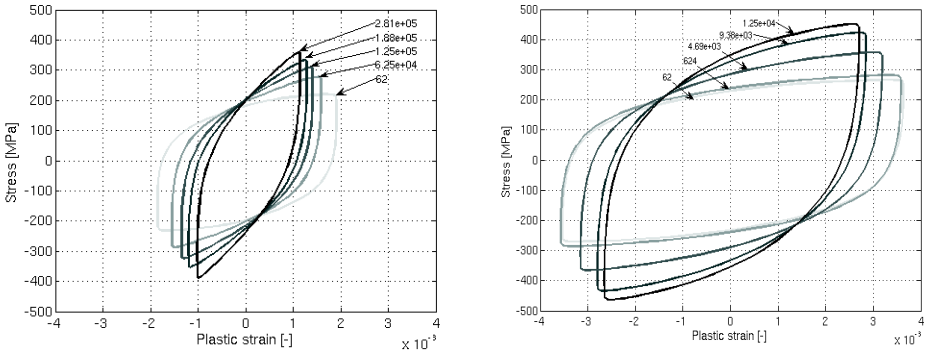

Fig. 1. Hysteresis loops with time stamps in the constant total strain amplitude experiments plotted in plastic strain domain. On the left hysteresis for $\square_t = 0.003$, on the right for $\square_t = 0.005$

The approach to data acquisition that requires recording the whole spectra of input and output signals, is different than that presented in [6] but enables identification of a complete parameter vector in one course instead of dividing the identification process into several stages. Another advantage of having complete signal recorded is the ease of the identification procedure programming and then its deployment while one has to supply structured files with time-indexed measured data and parameters for the identification procedure only.

4. Identification

4.1. General identification scheme

The lest square method is used as a hysteresis fitting criterion and therefore the objective function may take the following form

$$\bar{Q} = \int_0^{t_z} (\sigma(t) - \tilde{\sigma}(t))^2 dt \tag{12}$$

Since both components – measurements and simulation data – of the function Q are discrete ones, the function should be written in a summation form

$$Q = \sum_{i=0}^{N} (\sigma(t_i) - \tilde{\sigma}(t_i))^2 \tag{13}$$

where $\sigma(t_i)$ denotes measured material response at instant t_i and $\tilde{\sigma} = \tilde{\sigma}(t_i; C, R, X_0, \gamma(\lambda(t_i)))$ – computed material response at instant t_i. Additionally, basing on the results presented in [6], one assumes that yield surface radius R is constant, and parameter γ is a function of the length of plastic strain trajectory. The function $\gamma(\lambda)$ was assumed as

$$\gamma(\lambda) = \mathrm{e}^{a\lambda+b} + c. \tag{14}$$

Finally, the parameter vector consisted of 6 parameters: C, R, X_0, a, b, c among which X_0 is an initial condition technical parameter introduced to enable start of a numerical solver. It denotes an initial location of the yield surface and is unique for every recorded load and response spectrum.

The core of the identification procedure is a numerical solver of the nonlinear equation (2). It is based on the well known Dormand-Prince formula for solving ordinary differential equations. The solver is programmed in the MATLAB/Simulink environment and utilises fixed step ODE5 implementation of Dormand-Prince method provided by this environment.

By means of the solver, the material response, the essential part of the objective function (12), is computed during identification procedure. The function (12) is minimized by the differential evolution algorithm. All optimisation procedures are run parallel on a computer cluster, the result of which is a significant reduction of processing time.

4.2. Differential evolution method

The differential evolution method (DE) was developed by Price and Storn [4], [5]. It is a simple and fast, population based stochastic function minimizer. DE method is widely applied in filters and radio design, multiprocessor synthesis, neural networks learning and various optimization cases. The source (Java, C, and Matlab) code of the DE algorithm is presented on Authors' web page [5].

The DE method fulfils several requirements, and has the following advantages [4]:

– the ability to handle non-differentiable, nonlinear and multimodal cost functions,
– parallelizability,
– ease of use, i.e. few control variables,
– good convergence properties.

At the beginning of an optimization procedure, an initial population of parameter vectors is supplied. Every value of a vector parameter is randomly chosen from a given interval. The interval is independently specified for every parameter. Determination of the interval requires some information about the parameter range as the DE method is the most efficient if the optimal parameter value is located within given interval.

Then, evolution mechanism is applied to parameter vectors. To the next iteration, or in terms of genetic algorithms generation, only those parameter vectors of the procedure "survive" that produce the least value of an objective function. The evolution mechanism consists simply of adding a weighted difference between two vectors while creating new one (this procedure is called "mutation") and comparing a newly created vector to the one from an existing population. In order to increase the diversity of the perturbed ("muted") vectors, one introduces the "crossover". Crossover procedure generates trial vector by randomly mixing the perturbed vector with the last target vector. If the trial vector produces a lower value of an objective function than that compared to, the new vector replaces it in the population ("selection") [4], [5].

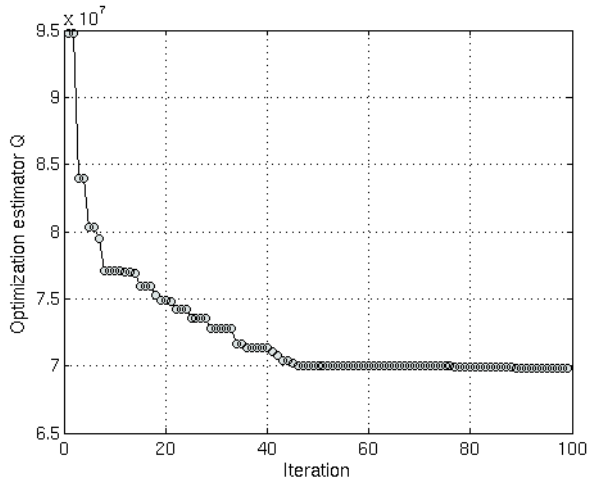

Fig. 2. Typical plot of changes of objective function value obtained in the DE identification procedure

The differential evolution algorithm outperforms [4] others (Adaptive Simulated Annealing, the Annealed Nelder and Mead approach, the Breeder Genetic Algorithm, the EASY Evolution Strategy and the method of Stochastic Differential Equations). In most instances, DE outperformed all of the above minimizations approaches in terms of required number of function evaluations necessary to locate a global minimum of the test functions. The

algorithm is very simple and straightforward and DE is also very easy to use as it requires only few robust control variables.

Let's consider an example that is based on a model presented in the current paper and let's go through one generation of an optimization procedure. First, one shall start from providing ranges for parameters that set up the parameter vector $x_i = [C, R, X_0, a, b, c]$. The ranges are given by additional two vectors:

$x_{max} = [C_{max}, R_{max}, X_{0\max}, a_{max}, b_{max}, c_{max}]$

and

$x_{min} = [C_{min}, R_{min}, X_{0\min}, a_{min}, b_{min}, c_{min}]$.

Then, first generation of $i = n$ vectors is randomly generated. Their elements vary within the previously given intervals. Based on the generated vectors, another vector v named trial vector is created according to relation

$$v = x_{I_1,G} + F\left(x_{I_2,G} - x_{I_3,G}\right) \tag{15}$$

where $I_1, I_2, I_3 \in [0, n-1]$ and are mutually different integers, G is the number of current generation and F denotes real, constant amplification factor. Further, in order to increase diversity among parameter vectors, vector u is generated in the so-called crossover process [4]. Elements of vector u are combination of v and one of $x_{i,G}$ vectors. Finally, vector u is compared to one of $x_{i,G}$ vectors and if it yields a lower value of objective function, it becomes a member of generation $G+1$, if not $x_{i,G}$ is passed to this generation. The basic DE1 scheme of differential evolution was presented above. Other evolution schemes are also available [4], [5]. Fundamentally, they are similar to the one described above and mainly differ on the manner of the trial vector generation.

4.3. Identification results

The progress of an objective function value changes in the DE procedure iterations as presented in Fig. 2. The value close to minimum is reached in 50 iterations (generations). The maximum number of generations was arbitrarily reduced to 100. In comparison to the conjugate gradient method, DE gives the value of an objective function lower by an order of magnitude for the same identification data set within less than 100 generations.

Table 2.

The best fitted parameter vector for the step load test experimental data

Parameter	C	R	a	b	c
Value	408599	72.49	−0.50	7.32	1938

In Table 2, the best-fitted parameter vector for the step load experimental data is presented. The plots of mechanical hysteresis loops (plastic strain – stress) simulated with parameters obtained by the identification method and compared to experimental results are presented in Fig. 3.

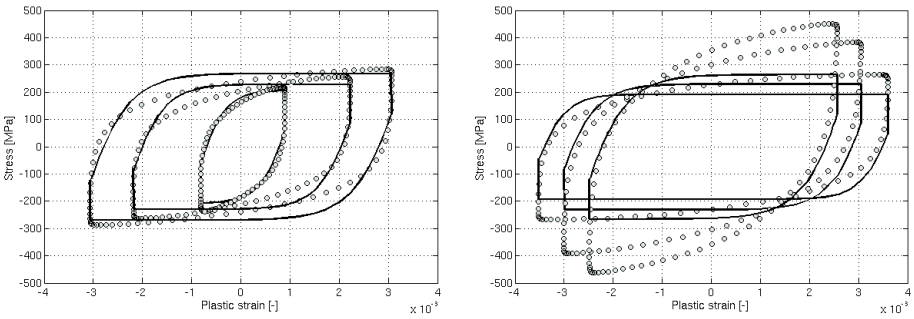

Fig. 3. Sample experimental (dots) and simulated (continuous lines) mechanical hysteresis loops for the step load test on the left. On the right, experimental data for a constant total strain amplitudes and simulation for that case on the basis of parameters values given in Table 2

Table 3.

The best fitted parameter vector for a constant amplitude experiments data

Parameter	C	R	a	b	c
Value	488670	88	−0.0652	7.42	1317

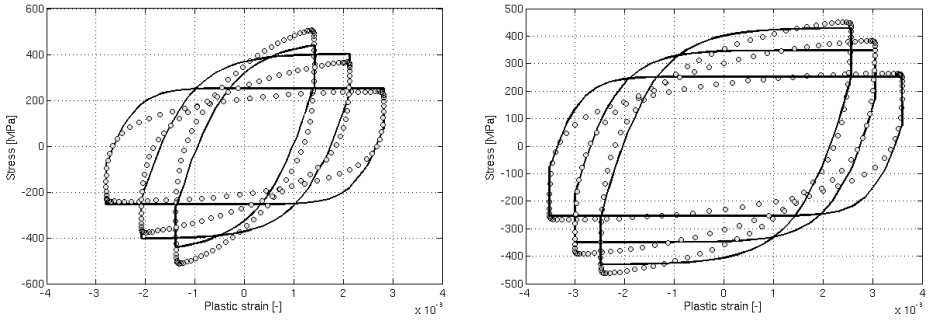

Fig. 4. Sample experimental (dots) and simulated (continuous lines) mechanical hysteresis loops for the constant amplitude $\square_t = 0.004$ on the left and $\square_t = 0.005$ on the right

Identification results for data obtained in the constant amplitude experiments are presented in Table 3, and in Fig. 4. New values of the best fitted parameter vector shown in Table 3, as it was said before, differ from those given in Table 2. One should notice significant change of the parameter a value in the comparison to changes of other parameters. It results in better

fitting of simulated curves to experimental data within the range of total strain amplitudes from 0.003 to 0.005.

5. Conclusions

In the paper, the Authors assumed the model of Friderick-Armstrong, which was accepted and commonly used to the description of the cyclical plasticity. The experimental data were obtained for symmetrical cycles. Therefore, the parameters of this model can be constant. For unsymmetrical cycles, the dependence of these parameters on the length of the trajectory of the plastic deformation or on the distance from the limit surface may be introduced additionally. Experimental investigations were carried out at controlled plastic strain amplitude (no ratchetting) and one could receive good approximation of the dependence between the plastic deformation and the stress, the consequence of which was good approximation of dissipated energy.

Based on a one selected function, the identification method of non-linear differential equation parameters was examined. The proposed constitutive equation approximates hysteresis loops for the range of 0.003 to 0.005 of total strain value for AISI 304 steel. The steel exhibits distinct hardening, and therefore the data obtained in a step load experiment are not suitable for identification, although simulated curves fit data obtained in this experiment well. For the identification, the data obtained in constant total strain amplitudes experiments are more applicable. The constitutive relation may be modified by adding the distance parameter between yield surface and its limiting surface. The identification method would in this case be the same, but the numerical solver should be programmed again.

The differential evolution algorithm finds its application for the identification of non-linear differential equation parameters where a solution of the equation is obtained by a numerical method. It minimizes the objective function much better than the procedure described in [6] that incorporates gradient methods. Utilisation of a computer cluster results in further increase of the presented approach efficiency.

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Zastosowanie algorytmu ewolucji różnicowej do identyfikacji danych eksperymentalnych

S t r e s z c z e n i e

W pracy autorzy przedstawiają sposób matematycznego opisu procesu umocnienia stali austenitycznej w oparciu o modele Fredericka-Armstronga i sprężysto-plastyczny Chaboche z mieszanym umocnieniem. Nieliniowe jednoosiowe równania konstytutywne są wyprowadzane z bardziej ogólnej zależności przy założeniu odpowiedniej postaci równania ewolucji środka powierzchni plastyczności. Celem pracy jest zaproponowanie wydajnej metody identyfikacji dla dobrze znanego modelu.

Dla wybranych amplitud odkształcenia całkowitego przeprowadzono typowe próby niskocyklowe. Rejestrowano czasy życia próbek a także amplitudy i częstotliwości sygnału sterującego (odkształcenia całkowitego) i sygnału odpowiedzi materiału (naprężenia).

W oparciu o wyniki badań eksperymentalnych przeprowadzono identyfikację parametrów równania konstytutywnego. Celem identyfikacji było uzyskanie modelu opisującego zachowanie się materiału w trakcie próby aż do jego zniszczenia. Za kryterium optymalizacji przyjęto estymator otrzymany metodą najmniejszych kwadratów.

Przetestowano kilka metod optymalizacji. Ostatecznie, jako najbardziej wydajną, wybrano metodę ewolucji różnicowej. Metodę porównano ze standardowymi metodami dostępnymi w pakiecie MATLAB. Znaczną redukcję czasu identyfikacji uzyskano dzięki równoległemu uruchamianiu procedur optymalizacyjnych klastrze obliczeniowym.