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Planned Motion Equations of Free-running Grain Mixture Flow

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Summary. In the article the equations have been worked making it possible to model the motion of free-running grain mixture flow on a flat sloping vibrating sieve within the framework of shallow water theory.

Free-running grain mixture is considered as a heterogeneous system consisting of two phases, one of which represents solid particles and the other one gas. The mixture is brought into a state of fluidity by means of high-frequency vibration imposition. Coefficients of internal and external friction and dynamic-viscosity decrease by exponential law as the fluctuation intensity is increased.

When considering grain mixture dynamics, the following assumptions are put forward: we ignore the air presence in space between particles, we consider the density of particles to be constant, the free-running mixture is similar to Newtonian liquid.

The basic system of equations of grain mixture dynamics is due to the laws of continuum mechanics. The equation of continuity is issued from the law of conservation of mass, and the dynamic equations are issued from the law of variation of momentum.

The stress tensor equals to the sum of the equilibrium tensor and the dissipative tensor. The equilibrium part of the stress tensor is represented by the spherical tensor, which is found to conform to Pascal law for liquids, and the dissipative part, which is responsible for viscous force effect and defined by Navier-Stokes law.

Boundary conditions on the surfaces (restricting the capacity of the free-running grain mixture) have been researched. The distributions of apparent density and velocity field are assigned at the inlet and outlet flow sections of the mixture. The normal velocity component of the grain mixture on the side frames and on the sieve becomes zero, which meets the no-fluid-loss condition of the medium through the frame. Beyond that point at this time we satisfy dynamic conditions, which characterize the mixture sliding down the hard frame, motion flow resistance force is represented as average velocity linear dependence. A kinematic condition and two dynamic ones are stipulated on the free surface layer. One of the conditions states mass flow continuity across the free surface, the other one states the stress continuity while passing through the free surface.

The basic premise of planned motion equations is the condition of small size of flow depth in comparison with its width. With the use of shallow water theory the basic principles of the equations of flow dynamics are simplified and for their solving a Cauchy problem can be set.

Key words: shallow water theory, planned flow motion, free-running mixture, stress tensor, strain velocity, viscous friction.

INTRODUCTION

Grain separators feeding including supplying and distribution of the material being processed across the sieve working surface is one of the factors providing the quality and productivity of the separation process. The existing grain separators do not provide with uniform distribution of the grain mixture across the working surface. The deviation from grain supplying average value across the sieve width approaches 20 %. To solve this problem it is vital to research the mixture dynamics on the sieve. Consequently, mathematical modeling of FM flow motion is a priority task.

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

The grain material distribution across the width of separating parts and the influence of the non-uniform distribution on the efficiency of separation have been analyzed in the papers [1, 2].

For loading improvement A.N. Ziulin, Y.I. Bazhenov [3-5] developed the flat sieve shape providing the material uniform distribution across the entire area under separation.

Ye. S. Goncharov [6] has made a study of the nature of unstable motion of the mixture in the segment of the sieve loading. He has also developed recommendations on the decrease of the mixture's unstable motion impact on the process of the sieves' loading.

D.F. Deberdeyev's research showed that the preparative surface applying at the sieve starting point increases the distribution of free-running material.

V.P. Olshanski [8] researched the regularities of flow mixture velocity change across the sieve length while non-uniform supplying. It is stated that under the harmonic pulsation supply the length of the non-uniform motion area increases with the frequency decrease and with the vibration amplitude increase of mixture velocity supply onto the sieve.

The sieve working surface loading is characterized with the material distribution throughout the whole layer volume which demands the consideration of spatial movement of FM. However, in the stated papers FM movement has been considered in one plane.

In the paper [9] the quick motion of granulated media theory has been applied for describing FM flow dynamics. A mathematical model of FM flow motion in three-dimensional space has been worked out. However, the obtained equation system is rather difficult, there are no completed solution algorithms and similarly solved problems do not exist.

In L.N. Tischenko's scientific works hydrodynamic equations have been applied for describing the free-running grain mixture dynamics. The obtained results conform to the experiment to a great extent which raises the possibility of using the shallow water theory in research of FM flow motion on the vibrating sieve.

OBJECTIVE

The work aimed at formulating the planned motion equations of free-running grain mixture flow on the vibrating sieve.

THE MAIN RESULTS OF THE RESEARCH

Let us consider the sieve in the shape of a pan angled at θ angle in relation to the horizon (Fig. 1). Constructively, the pan is divided into segments with the help of partitions A, *B*, *C*, *D*; A_1, B_1, C_1, D_1; A_2, B_2, C_2, D_2, located parallel to each other, oriented down the sieve slope and perpendicular to the pan bottom. The quantity of the corresponding segments may be optional.

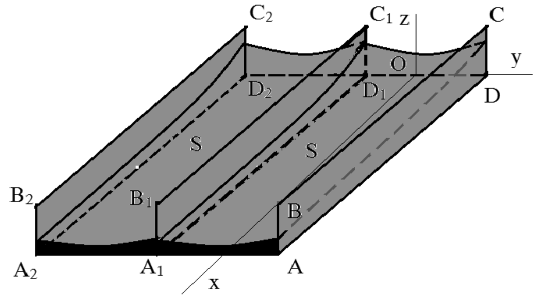

Fig. 1. Flat pan

We choose the coordinate system so that its origin lies in the plane CDD_2C_2. Let us direct the axis *Ox* down the sieve slope parallel to partitions, dividing the sieve into segments. Let us denote the distance between the partitions via l_1. The axis *Ox* is considered to be lying in the middle between the partitions *ABCD* and $A_1B_1C_1D_1$. The direction of the other axes is understandable from Fig. 1. Since the partitions completely isolate grain flows in the segments from each other, FM movement is sufficient to be considered in one of the segments.

Grain flow comes on the sieve through the inlet section CDD_2C_2, corresponding $x=0$, and comes out of the pan when $x=l$.

Let us consider the problem of FM dynamics reviewed in the paper [9]. In accordance with the quick motion of granulated media theory, the stress tensor has non-linear dependence on the strain velocity tensor which complicates the problem solving to a great degree. Let us hold the opinion of FM being similar to Newtonian liquid [12, 13]. In this case the equilibrium part of stress tensor $\hat{\sigma}^r$ appears to be a spherical tensor which corresponds to the fulfilling of Pascal law for liquids

$$\sigma_{ik}^{r} = -p\delta_{ik}, \tag{1}$$

and non-equilibrium part $\hat{\sigma}^d$ which is responsible for viscous force actions is defined by Navier-Stokes law

$$\sigma_{ik}^{d} = \left(\lambda - \frac{2}{3}\mu\right)\left(\frac{\partial \mathrm{v}_1}{\partial x_1} + \frac{\partial \mathrm{v}_2}{\partial x_2} + \frac{\partial \mathrm{v}_3}{\partial x_3}\right)\delta_{ik} + \mu\left(\frac{\partial \mathrm{v}_k}{\partial x_i} + \frac{\partial \mathrm{v}_i}{\partial x_k}\right), \tag{2}$$

where: *p* - pressure; δ – second-rank identity tensor; λ, μ – dynamic coefficients of volume and shearing viscosity; $x_1=x$, $x_2=y$, $x_3=z$ – cartesian reference system coordinates; $\mathrm{v}_1=u$, $\mathrm{v}_2=\mathrm{v}$, $\mathrm{v}_3=w$ – velocity vector movement components of continuous medium.

For many media it is considered that $\lambda = 2/3\,\mu$.

Then FM dynamics equations coincide with the viscous compressible medium equations. If you direct the gravitation force down at an angle θ relative to the vertical line, that is to define components of a unit vector pointing the direction of the gravitation force $\zeta_i = (\sin\theta, 0, -\cos\theta)$, we shall have the continuity equation and motion equations as follows:

$$\frac{\partial}{\partial t}\rho + \left(\frac{\partial}{\partial x}\rho\right)u + \rho\frac{\partial}{\partial x}u + \left(\frac{\partial}{\partial y}\rho\right)v + \rho\frac{\partial}{\partial y}v + \frac{\partial}{\partial z}(\rho w) = 0 \quad (3)$$

$$\rho\left(\frac{\partial}{\partial t}u + u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u + w\frac{\partial}{\partial z}u\right) - -\frac{\partial}{\partial x}\sigma_{xx} - \frac{\partial}{\partial y}\sigma_{xy} - \frac{\partial}{\partial z}\sigma_{xz} - \rho g\sin(\theta) = 0, \quad (4)$$

$$\rho\left(\frac{\partial}{\partial t}v + u\frac{\partial}{\partial x}v + v\frac{\partial}{\partial y}v + w\frac{\partial}{\partial z}v\right) - -\frac{\partial}{\partial x}\sigma_{xy} - \frac{\partial}{\partial y}\sigma_{yy} - \frac{\partial}{\partial z}\sigma_{yz} = 0, \quad (5)$$

$$\rho\left(\frac{\partial}{\partial t}w + u\frac{\partial}{\partial x}w + v\frac{\partial}{\partial y}w + w\frac{\partial}{\partial z}w\right) - -\frac{\partial}{\partial x}\sigma_{xz} - \frac{\partial}{\partial y}\sigma_{yz} - \frac{\partial}{\partial z}\sigma_{zz} + \rho g\cos(\theta) = 0, \quad (6)$$

where:
$\rho=\gamma v$ – medium density with account of interstitial spaces; γ – particle density of the mixture; v - volumetric particle density; t - time.

Stress tensor components $\hat{\sigma}$

$$\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix}, \quad (7)$$

coincide with matrix elements of the degree (3×3)

$$\begin{pmatrix} -p+2\mu\frac{\partial u}{\partial x} & \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) & \mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) \\ \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) & -p+2\mu\frac{\partial v}{\partial y} & \mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) \\ \mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) & \mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) & -p+2\mu\frac{\partial w}{\partial z} \end{pmatrix}. \quad (8)$$

The FM layer takes up the area V, restricted by a flat bottom Σ_0 (AA_1D_1D), side panels Σ_1 $(A_1B_1C_1D)$ and Σ_2 $(ABCD)$ of the segment, by the inlet Σ_3 (AA_1B_1B) and outlet Σ_4 (CC_1D_1D) sections . At the top the FM layer is restricted by the surface Γ which changes its shape and moves according to the FM movement when the medium is in motion. The surfaces of that kind are called "free" in hydrodynamics and their form is defined in process of the problem solving. It should be noted that the presence of the medium free surface complicates the mathematical problem setting, the problem becomes non-linear and its solving is to be defined in the area unknown beforehand.

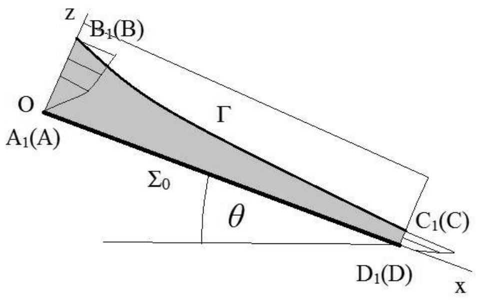

Fig. 2. Side frame of the sieve segment

Let us denote the layer depth counted along the normal to the pan bottom up to the free surface by $h=h(t,x,y,z)$, l the pan length, l_1 the segment width. Then the areas stated in the previous passage can be defined by the further relations

$$\begin{aligned} &V = \{0<x<l, -l_1/2<y<l_1/2, 0<z<h\} \\ &\Sigma_0 = \{0<x<l, -l_1/2<y<l_1/2, z=0\} \quad (z=0) \\ &\Sigma_1 = \{0<x<l, y=-l_1/2, 0<z<h\} \quad (y=-l_1/2) \\ &\Sigma_2 = \{0<x<l, y=l_1/2, 0<z<h\} \quad (y=l_1/2) \\ &\Gamma = \{0<x<l, -l_1/2<y<l_1/2, z=h\} \quad (z=h) \\ &\Sigma_3 = \{x=0, -l_1/2<y<l_1/2, 0<z<h\} \quad (x=0) \\ &\Sigma_4 = \{x=l, -l_1/2<y<l_1/2, 0<z<h\} \quad (x=l) \end{aligned} \quad (9)$$

To find the single-valued solution of the equation system (3-6) it is necessary to use the boundary and initial conditions.

The normal velocity component of FM on the side panels and the sieve equals to null. It conforms to the medium no-fluid-loss condition through the frame.

$$v_n|_\Sigma = 0 \quad (10)$$

where: V_n appears to be the normal velocity component of FM on the panel.

Beyond that point at this time we satisfy the dynamic conditions, which characterize FM sliding down the hard smooth frame. In hydraulics, when researching the liquid motion along the river stream, the resisting force $\vec{T}$ is taken into consideration forcing on the wet perimeter of the stream cross section in the form of its linear dependence on the stream average velocity $\vec{u}_m$ [14]:

$$\vec{T} = -C\,\vec{u}_m. \quad (11)$$

The stated relation may be given as follows:

Let us consider that $\vec{n} = (n_1, n_2, n_3) = (n_x, n_y, n_z)$ represents the unit external normal relative to volume V to the surface Σ, and $\vec{\tau} = \{\tau_1, \tau_2, \tau_3\}$ - unspecified unitary vector tangent to Σ. In this case the tangential stress $\vec{p}_\tau$ onto Σ is defined in accordance with Cauchy relation [15, 16]:

$$\vec{p}_\tau = n_k \sigma_{ki} \tau_i \vec{e}_i = -C_s \, \mathrm{v}_k \, \tau_k \tau_i \vec{e}_i \qquad (12)$$

where: C_s – phenomenological coefficient, analogous to Chezy's velocity factor; $\vec{e}_i$ - of the coordinate basis vector of the cartesian reference system.

In the projections of the last vector equation on the tangent direction which is defined by the vector $\vec{\tau}$:

$$n_k \sigma_{ki} \tau_i = -C_s \, \mathrm{v}_k \, \tau_k \tau_i \, . \qquad (13)$$

It is compulsory to meet the inequation

$$p_n \equiv n_k \sigma_{ki} n_i < 0 \, , \qquad (14)$$

implying that the normal stress on the panel should be compressing.

Distributions are assigned at the inlet section of the sieve:

$$\rho_0 = \rho(t,0,y,z), \; u_0 = u(t,0,y,z), \; \mathrm{v}_0 = \mathrm{v}(t,0,y,z), \\ w_0 = w(t,0,y,z), \; h_0 = h(t,0,y) \, . \qquad (15)$$

The corresponding characteristics are sampled either from the experimental data or supplementary theoretical research.

Similar conditions are necessary to be assigned at the outlet section Σ_4. The number of these conditions on this boundary depends on the equation type describing the flow dynamics [17]. Since the dynamics equations possess second derived velocity fields across spatial variables, it is necessary at least to assign conditions for the velocity field at $x=l$. The corresponding conditions are to be assigned here as carried out in computational method problems for viscous liquid dynamics [18, 19]:

$$\left.\frac{\partial u(t,x,y,z)}{\partial x}\right|_{x=l} = 0, \quad \left.\frac{\partial \mathrm{v}(t,x,y,z)}{\partial x}\right|_{x=l} = 0, \quad \left.\frac{\partial w(t,x,y,z)}{\partial x}\right|_{x=l} = 0 \, . \qquad (16)$$

Two types of boundary conditions are set on the free surface of the grain layer Γ – kinematic and dynamic ones. The first one is stated as following: let us assume that the equation of the free surface takes the form of:

$$z - h(t,x,y) = 0 \, . \qquad (17)$$

Let us consider the medium velocity field to be known and x, y, z are coordinates of a medium particle.

Hence:

$$\frac{dx}{dt} = u, \; \frac{dy}{dt} = \mathrm{v}, \; \frac{dz}{dt} = w \, .$$

Differentiating both equation parts on time (17), we will acquire the wanted kinematic condition:

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + \mathrm{v}\frac{\partial h}{\partial y} = w \, . \qquad (18)$$

There are two types of dynamic conditions on Γ: the first one follows the law of conservation of mass and reflects mass flow continuity through Γ:

$$\langle \rho (\mathrm{v}_n - W) \rangle = 0 \, ,$$

where: W- normal velocity movement component of the surface breaking medium Γ; v_n - normal velocity particle component on Γ, angular brackets denote the corresponding function jump on Γ [15].

The Γ surface is a surface of contact breaking for which the values of normal velocity component are $\mathrm{v}_n^+, \mathrm{v}_n^-$ above and below the Γ surface equal W. Consequently, the specified condition is satisfied by itself.

Another dynamic condition implies stress continuity when going through Γ and follows from motion equations. This condition consists of three scalar relations:

$$\left. \begin{aligned} n_k \sigma_{ki}^- n_i &= n_k \sigma_{ki}^+ n_i \\ n_k \sigma_{ki}^- \tau_{1i} &= n_k \sigma_{ki}^+ \tau_{1i} \\ n_k \sigma_{ki}^- \tau_{2i} &= n_k \sigma_{ki}^+ \tau_{2i} \end{aligned} \right\} \; (\Gamma) \, . \qquad (19)$$

Here $\tau_{mi} (m = 1,2)$ unitary noncollinear tangents to the surface of Γ vector, index '-' means that the values of the function are selected on the surface on the side of the area from which the normal '+' is directed – for the values of the function on the surface points from that side where the normal is directed.

Over the surface Γ the free-running medium is not present but there is a certain perfect medium (gas) for which Pascal law is applicable:

$$\hat{\sigma}^+ = -P_0 \, \hat{\delta} \, , \qquad (20)$$

where: P_0 - pressure, exerted on the layer surface of the FM by this medium, along with this there should be $P_0 > 0$ to eliminate the tension strains in the layer; $\hat{\delta} = \delta_{ik} \vec{e}_i \vec{e}_k$ - unit tensor; δ_{ik} - Kronecker symbols.

Lowering the superior index '-' in relations (19), we shall rewrite them as following:

$$\begin{aligned} n_k \sigma_{ki} n_i &= -P_0 \\ n_k \sigma_{ki} \tau_{1i} &= 0 \\ n_k \sigma_{ki} \tau_{2i} &= 0 \end{aligned} \, . \qquad (21)$$

The Γ surface is represented as two-dimensional two-parameter variety in the three-dimensional space, assigned by the vector equation [20]:

$$\vec{r} = \vec{r}(t,x,y) = (x, y, h(t,x,y)) \, . \qquad (22)$$

In case of holding the variable y fixed, we shall receive the first coordinate line on Γ, in case of holding x fixed we will obtain the second coordinate line on Γ. The vectors:

$$\vec{E}_x = \frac{\partial \vec{r}}{\partial x} = \left(1, 0, \frac{\partial h}{\partial x}\right), \quad \vec{E}_y = \frac{\partial \vec{r}}{\partial y} = \left(0, 1, \frac{\partial h}{\partial y}\right) \qquad (23)$$

are the basis on Γ and they are tangentials to coordinate lines on it. The condition of their normalising gives the expressions for unit tangents to Γ vectors:

$$\vec{\tau}_1 = \frac{\vec{E}_x}{|\vec{E}_x|}, \quad \vec{\tau}_2 = \frac{\vec{E}_y}{|\vec{E}_y|}. \tag{24}$$

Vector $\vec{N}$ normal to Γ equals to:

$$\vec{N} = \vec{E}_x \times \vec{E}_y = \left(-\frac{\partial h}{\partial x}, -\frac{\partial h}{\partial y}, 1\right), \tag{25}$$

and correspondingly a unit normal equals to:

$$\vec{n} = \frac{\vec{N}}{|\vec{N}|} = \left(-\frac{\partial h}{\partial x}, -\frac{\partial h}{\partial y}, 1\right)\frac{1}{|\vec{N}|}, \quad |\vec{N}| = \sqrt{1 + \left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2}. \tag{26}$$

From this point on we shall deal with the motion of the FM thin layer across smooth solid surfaces. In this case $\nabla_{xy} h = (\partial h/\partial x, \partial h/\partial y)$ is a vector the modulus of which is less than one and therefore the second-order terms are much less than the first-order terms $|\nabla h|^2 = o(|\nabla h|)$. From this point on we ignore the stated terms.

Thereafter the dynamic conditions (21) after apparent simplications can be written as following:

$$-2\left(\frac{\partial}{\partial x}h\right)\sigma_{xz} - 2\left(\frac{\partial}{\partial y}h\right)\sigma_{yz} + \sigma_{zz} + P_0 = 0, \tag{27}$$

$$-\left(\frac{\partial}{\partial x}h\right)\sigma_{xx} - \left(\frac{\partial}{\partial y}h\right)\sigma_{xy} + \sigma_{xz} + \sigma_{zz}\frac{\partial}{\partial x}h = 0, \tag{28}$$

$$-\left(\frac{\partial}{\partial x}h\right)\sigma_{xy} - \left(\frac{\partial}{\partial y}h\right)\sigma_{yy} + \left(\frac{\partial}{\partial y}h\right)\sigma_{zz} + \sigma_{yz} = 0, \tag{29}$$

$$(z = h(t, x, y)).$$

The pan bottom Σ_0:

The external normal:

$$\vec{n} = (0, 0, -1).$$

Tangent vectors:

$$\vec{\tau}_1 = (1, 0, 0), \quad \vec{\tau}_2 = (0, 1, 0).$$

Stress tangents:

$$\tau_x = \vec{n} \cdot \hat{\sigma} \cdot \vec{\tau}_1 = -\sigma_{xz},$$
$$\tau_y = \vec{n} \cdot \hat{\sigma} \cdot \vec{\tau}_2 = -\sigma_{yz}.$$

Therefore the conditions (13) for Σ_0 acquire the form:

$$\sigma_{xz} = C_s u, \tag{30}$$

$$\sigma_{yz} = C_s \mathrm{v}. \tag{31}$$

The right panel Σ_1:

$$\vec{n} = (0, -1, 0), \quad \vec{\tau}_1 = (1, 0, 0), \quad \vec{\tau}_2 = (0, 0, 1)$$
$$\tau_x = -\sigma_{xy},$$
$$\tau_z = -\sigma_{yz}.$$

Conditions on Σ_1

$$\mathrm{v} = 0, \tag{32}$$

$$\sigma_{xy} - C_s u = 0, \tag{33}$$

$$\sigma_{xz} - C_s w = 0. \tag{34}$$

The left panel Σ_2:

$$\vec{n} = (0, 1, 0), \quad \vec{\tau}_1 = (1, 0, 0), \quad \vec{\tau}_2 = (0, 0, 1).$$
$$\tau_x = \sigma_{xy},$$
$$\tau_z = \sigma_{yz}.$$

Conditions Σ_2:

$$\mathrm{v} = 0, \tag{35}$$

$$\sigma_{xy} + C_s u = 0,$$
$$\sigma_{yz} + C_s w = 0. \tag{36}$$

On the inlet section Σ_3 let us set the distribution of velocities, density and layer depth:

$$u(t, 0, y, z) = U^0(t, y, z), \quad \mathrm{v}(t, 0, y, z) = V^0(t, y, z),$$
$$w(t, 0, y, z) = W^0(t, y, z), \quad \rho(t, 0, y, z) = N^0(t, y, z),$$
$$h(t, 0, y) = H^0(t, y). \tag{37}$$

Volume flow rate of FM through the inlet section equals to:

$$Q = Q(t) = \int_0^{h(t,0,y)} \int_{-l_1/2}^{l_1/2} \mathrm{U}^0(\mathrm{t}, y, z)\,dydz. \tag{38}$$

This ratio allows to reconcile the characteristics of U^0 with the flow Q.

<u>Initial conditions</u>. For differential equations possessing the first-order term time derivatives the number of initial conditions equals to the number of unknown quantities and defined by the value of these unknown quantities at the start time ($t = 0$). In this case we have:

$$\rho(0, x, y, z) = \rho^0(x, y, z),$$
$$u(0, x, y, z) = u^0(x, y, z),$$
$$\mathrm{v}(0, \mathrm{x}, \mathrm{y}, \mathrm{z}) = \mathrm{v}^0(x, y, z),$$
$$w(0, x, y, z) = w^0(x, y, z). \tag{39}$$

The problems in which flow velocity field is defined to an accuracy of only average velocities on the verticals are called planned problems of flow motion. The basic premise of planned motion equations is the condition of small size of flow depth in comparison with its width. The theory describing the planned motion is called the shallow water theory.

The basic principles of the shallow water theory:

1. The layer depth is small in comparison with linear dimensions in the flow plane. The free surface is smooth.

2. The normal component of the medium velocity relative to the pan bottom is small $(w = 0)$. The liquid particles acceleration in the same direction is small $|dw/dt| = 0$.

3. The change of velocity components u, v and density in the direction of the axis Oz is small $(\partial u/\partial z = 0, \partial \mathrm{v}/\partial z = 0, \partial \rho/\partial z = 0)$.

4. The pressure distribution p along Oz is linear and corresponds to the hydrostatic one:

$$p(t, x, y, z) = p_0(t, x, y)(1 - z/h(t, x, y)) + $$
$$+ p_1(t, x, y)z/h(t, x, y), \tag{40}$$

where: p_0 – pressure at the layer bottom; p_1 – pressure at the corresponding point of the free surface.

5. Normal stresses on the areas with a normal which is directed along the axis Oz exceed its tangent stress $|\sigma_{zz}| > |\sigma_{xz}|, |\sigma_{yz}|$.

Under the made assumptions the dynamic equations are simplified, the equation (6) becomes a relation:

$$\frac{\partial}{\partial z} p + \rho g \cos(\theta) = 0 . \tag{41}$$

This equation can be integrated:

$$p(t,x,y,z) = p_0(t,x,y) - \rho(t,x,y) g \cos(\theta) z =$$
$$= p_1(t,x,y) + \rho(t,x,y) g \cos(\theta)\left(h(t,x,y) - z\right) . \tag{42}$$

The boundary conditions (21) on the free surface express:

$$p\left[t,x,y,h(t,x,y)\right] = P_0 , \tag{43}$$

$$\sigma_{xz} = (\sigma_{xx} - \sigma_{zz})\left(\frac{\partial}{\partial x} h\right) + \left(\frac{\partial}{\partial y} h\right)\sigma_{xy} ,$$

$$\sigma_{yz} = (\sigma_{yy} - \sigma_{zz})\left(\frac{\partial}{\partial y} h\right) + \left(\frac{\partial}{\partial x} h\right)\sigma_{xy} . \tag{44}$$

The equalities (44) provide support for the assumption 5) correctness, and the expression (43) allows to define the single-valued pressure:

$$p(t,x,y,z) = P_0 + \rho g \cos(\theta)\left[h(t,x,y) - z\right] . \tag{45}$$

To obtain the planned flow equations let us integrate the equations (3-5) along the variable z within $[0, h(t,x,y)]$. Let us use the known in mathematical analysis expression for differentiation of an integral having a parameter with a respect to a parameter [21]:

$$\frac{d}{d\xi} \int_{a(\xi)}^{b(\xi)} F(\xi, z) dz =$$

$$= \int_{a(\xi)}^{b(\xi)} \frac{\partial}{\partial \xi} F(\xi, z) dz + F[\xi, b(\xi)] \frac{db(\xi)}{d\xi} - F[\xi, a(\xi)] \frac{da(\xi)}{d\xi}$$

In our case the formula is slightly simplified:

$$\frac{\partial}{\partial x} \int_0^{h(t,x,y)} F(t,x,y,z) dz =$$

$$= \int_0^{h(t,x,y)} \frac{\partial}{\partial x} F(t,x,y,z) dz + F[t,x,y,h(t,x,y)] \frac{\partial h(t,x,y)}{\partial x}$$

$$\frac{\partial}{\partial y} \int_0^{h(t,x,y)} F(t,x,y,z) dz =$$

$$= \int_0^{h(t,x,y)} \frac{\partial}{\partial y} F(t,x,y,z) dz + F[t,x,y,h(t,x,y)] \frac{\partial h(t,x,y)}{\partial y} . \tag{46}$$

Let us assume:

$$\gamma = \gamma(t,x,y) = \int_0^{h(t,x,y)} \rho(t,x,y,z) dz ,$$

$$\prod = \prod(t,x,y) = \int_0^{h(t,x,y)} p(t,x,y,z) dz = h\left(P_0 + \frac{1}{2} \gamma cos\theta\right) , \tag{47}$$

$$T_{xx} = \int_0^h (\sigma_{xx} - p) dz = 2\mu h \frac{\partial u}{\partial x} ,$$

$$T_{xy} = \int_0^h \sigma_{xy} dz = \mu h \left(\frac{\partial u}{\partial y} + \frac{\partial \text{v}}{\partial x}\right) , \tag{48}$$

$$T_{yy} = \int_0^h (\sigma_{yy} - p) dz \; 2\mu \frac{\partial \text{v}}{\partial y} .$$

Let us integrate the equation (3) over z. We shall obtain:

$$\frac{\partial}{\partial t}\gamma + u \frac{\partial}{\partial x}\gamma + \text{v} \frac{\partial}{\partial y}\gamma + \gamma \frac{\partial}{\partial x} u + \gamma \frac{\partial}{\partial y} \text{v} -$$

$$- \rho \left(\frac{\partial}{\partial t} h + u \frac{\partial}{\partial x} h + \text{v} \frac{\partial}{\partial y} h - w\right)\Bigg|_{z=h} = 0 .$$

On account of the boundary condition (18) the latter additive component goes to zero. We come to the first dynamic equation of the thin layer:

$$\frac{\partial}{\partial t}\gamma + u \frac{\partial}{\partial x}\gamma + \text{v} \frac{\partial}{\partial y}\gamma + \gamma \frac{\partial}{\partial x} u + \gamma \frac{\partial}{\partial y} \text{v} = 0 . \tag{49}$$

Integrating the equation (4) and taking into consideration the condition (30), we shall obtain the relation:

$$\gamma\left(\frac{\partial}{\partial t} u + u \frac{\partial}{\partial x} u + \text{v} \frac{\partial}{\partial y} u\right) + \frac{\partial}{\partial x} \prod - \frac{\partial}{\partial x} T_{xx} - \frac{\partial}{\partial y} T_{xy} -$$

$$- \gamma g \sin\theta - P_0 \frac{\partial}{\partial x} h + C_s u = 0 .$$

Or in expanded form taking into account (47, 48) we come to the second dynamic equation:

$$\frac{\partial}{\partial t} u + u \frac{\partial}{\partial x} u + \text{v} \frac{\partial}{\partial y} u + \frac{g\cos\theta}{2} \frac{\partial}{\partial x} h + \frac{hg\cos\theta}{2\gamma} \frac{\partial}{\partial x} \gamma -$$

$$- \frac{2\mu h}{\gamma} \frac{\partial^2}{\partial x^2} u - \frac{\mu h}{\gamma} \frac{\partial^2}{\partial y^2} u - \frac{2\mu}{\gamma} \frac{\partial}{\partial x} h \frac{\partial}{\partial x} u - \tag{50}$$

$$- \frac{\mu}{\gamma} \frac{\partial}{\partial y} h \frac{\partial}{\partial y} u - \frac{\mu}{y} \frac{\partial}{\partial y}\left(h \frac{\partial}{\partial x} \text{v}\right) + \frac{C_s}{\gamma} u - g \sin\theta = 0.$$

Analogous actions with the relation (5) lead to the third equation:

$$\frac{\partial}{\partial t} \text{v} + u \frac{\partial}{\partial x} \text{v} + \text{v} \frac{\partial}{\partial y} \text{v} + \frac{\text{g}\cos\theta}{2} \frac{\partial}{\partial y} h + \frac{hg\cos\theta}{2\gamma} \frac{\partial}{\partial y} \gamma -$$

$$- \frac{\mu h}{\gamma} \frac{\partial^2}{\partial x^2} \text{v} - \frac{2\mu h}{\gamma} \frac{\partial^2}{\partial y^2} \text{v} - \frac{\mu}{\gamma} \frac{\partial}{\partial x} h \frac{\partial}{\partial x} \text{v} - \tag{51}$$

$$- \frac{2\mu}{\gamma} \frac{\partial}{\partial y} h \frac{\partial}{\partial y} \text{v} - \frac{\mu}{\gamma} \frac{\partial}{\partial x}\left(h \frac{\partial}{\partial y} u\right) + \frac{C_s}{\gamma} \text{v} = 0.$$

Three equations (49-51) possess four unknown functions h, γ, u, v. To close the given equation system let us invoke the kinematic boundary condition (18) (here $w = 0$):

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+\mathrm{v}\frac{\partial h}{\partial y}=0. \quad (52)$$

Boundary conditions

As it is clear from formulated thin layer dynamic equations, the field of definition of the unknown functions is $\Sigma_0=\{0<x<l,-l_1/2<y<l_1/2\}$. The boundary of the stated field consists of lines $L_1=\{0<x<l,y=-l_1/2\}$,

$L_2=\{0<x<l,y=l_1/2\}$,

$L_3=\{x=0,-1_1/2<y<l_1/2\}$,

$L_4=\{x=l,-1_1/2<y<l_1/2\}$.

On the boundary L_3 the distributions are predetermined (37):

$$h(t,0,y)=H^0(t,y),\ \gamma(t,0,y)=G^0(t,y),$$
$$u(t,0,y)=U^0(t,y),\ \mathrm{v}(t,0,y)=V^0(t,y). \quad (53)$$

On the lines L_1, L_2 the conditions are met:

$$\mathrm{v}(t,x,-l_1/2)=0,\quad \left.\frac{\partial u}{\partial y}\right|_{y=-l_1/2}-\left.\frac{C_s}{\mu}u\right|_{y=-l_1/2}=0, \quad (54)$$

$$\mathrm{v}(t,x,l_1/2)=0,\quad \left.\frac{\partial u}{\partial y}\right|_{y=l_1/2}+\left.\frac{C_s}{\mu}u\right|_{y=l_1/2}=0. \quad (55)$$

If we temporarily consider that functions γ,u,v are set then the equation (52) becomes the first-order equation in partial derivatives relative to the function *h*. For that kind of equations there has been developed a mathematical model [17]. For them we have the notion "characteristic" - a line in four-dimensional space of variables t,x,y,h which is defined by the system of ordinary differential equations:

$$\frac{dt}{ds}=1,\quad \frac{dx}{ds}=u(t,x,y),\quad \frac{dy}{ds}=\mathrm{v}(t,x,y),\quad \frac{dh}{ds}=0. \quad (56)$$

where: s - parameter.

When establishing the initial condition:

$$t(s_0)=t^0,\quad x(s_0)=x^0,\quad y(s_0)=y^0,\quad h(s_0)=h^0, \quad (57)$$

and fulfilling the equations tractability conditions the latter ones lead to the single-valued solution – the characteristic. For the equation (52) we may set a Cauchy problem: to find the solution of the equation (52), which is geometrically represented as the surface in the space of variables t,x,y,h, passing through the curve $L_0=\{t^0(r),x^0(r),y^0(r),h^0(r)\}$. It is proved that if the tangent to a curve L_0 is not a tangent to any other characteristic of the equation (52), the Cauchy problem is to be solved. In these conditions the integral surface Σ_0 consists of the characteristics passing through the points of the curve L_0. The last equation of the system (56) shows that *h* is maintained along the characteristic. Excluding the parameter *s* and choosing *x* as an explanatory variable we can write the equation for the characteristic projection on the plane *x0y*:

$$\frac{dy}{dx}=\frac{\mathrm{v}}{u}. \quad (58)$$

If the character of the flow is such that $u>0$, and $\mathrm{v}=0$ where $y=\pm l_1/2$, then the possible character of projection characteristics on *x0y* is represented on fig.3.

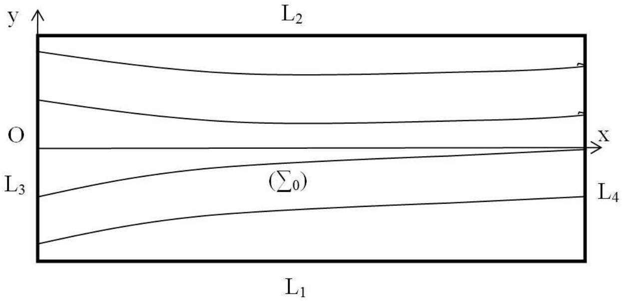

Fig. 3 The character of the projections of characteristics on x0y

The unique characteristic corresponds to each point of line L_3. Moreover, the lines L_1, L_2, corresponding to the side panels of the pan segment, where v=0 are the characteristics.

Similar reasoning can be given for the equation (49). From this it follows that the conditions for the functions *h*, *γ* are to be set only on the line L_3.

CONCLUSION

The system of planned motion equations of free-running grain mixture flow on the vibrating sieve has been achieved.

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Аннотация. В статье составлены уравнения, позволяющие в рамках теории мелкой воды моделировать движение потока сыпучей зерновой смеси на плоском наклонном виброрешете.

Сыпучая зерновая смесь рассматривается как гетерогенная система состоящая из двух фаз, одна из которых представляет твердые частицы, а другая газ. В состояние псевдоожижения смесь приводится посредством наложения высокочастотных вибраций. Коэффициенты внутреннего и внешнего трения, динамической вязкости уменьшаются по экспоненциальному закону при увеличении интенсивности колебаний.

При рассмотрении динамики сыпучей смеси (СС) выдвинуты следующие предположения: влиянием воздуха на динамику смеси пренебрегают, плотность частиц считается постоянной, сыпучая смесь подобна ньютоновской жидкости.

Основная система уравнений динамики СС является следствием законов механики сплошных сред. Уравнение неразрывности вытекает из закона сохранения массы, а уравнения динамики следует из закона изменения количества движения. Тензор напряжений равен сумме равновесного и диссипативного тензоров. Равновесная часть тензора напряжений представляется шаровым тензором, что соответствует выполнению закона Паскаля для жидкостей, а диссипативная часть, отвечающая за действие вязких сил, определяется законом Навье-Стокса.

Рассмотрены граничные условия на поверхностях, ограничивающих объем сыпучей зерновой смеси. На входном и выходном сечениях потока смеси задаются распределения объемной плотности и поля скоростей. На боковых стенках и решете нормальная составляющая скорости СС равняется нулю, что соответствует условию непротекания среды через стенку. Помимо этого здесь выполняется динамическое условие, характеризующее проскальзывание смеси по твердой стенке, а сила сопротивления движению потока представлена в виде линейной зависимости от средней скорости. На свободной поверхности слоя устанавливается кинематическое условие и два динамических. Одно из них выражает непрерывность потока массы через свободную поверхность, другое выражает собой непрерывность напряжений при переходе через свободную поверхность.

Главной предпосылкой уравнений планового движения является условие незначительных размеров глубины потока по сравнению с его шириной. При использовании основных положений теории мелкой воды уравнения динамики потока упрощаются, и для их решения может быть поставлена задача Коши.

Ключевые слова: теория мелкой воды, плановое движение потока, сыпучая смесь, тензор напряжений, скорость деформаций, вязкое трение.