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SENSOR CALIBRATION DESIGN BASED ON D-OPTIMALITY CRITERION

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Abstract

In this study, a procedure for optimal selection of measurement points using the D-optimality criterion to find the best calibration curves of measurement sensors is proposed. The coefficients of calibration curve are evaluated by applying the classical *Least Squares Method* (LSM). As an example, the problem of optimal selection for standard pressure setters when calibrating a differential pressure sensor is solved. The values obtained from the D-optimum measurement points for calibration of the differential pressure sensor are compared with those from actual experiments. Comparison of the calibration errors corresponding to the D-optimal, A-optimal and Equidistant calibration curves is done.

Keywords: sensor, calibration curve, calibration design, D-optimality criterion, least squares method.

1. Introduction

Accurate measurement is the basis of almost all engineering applications, since uncertainty inherently exists in the nature of each measuring sensor. The cost of a measuring sensor, on the other hand, increases with its accuracy. Therefore, constructing the low-cost accurate measurement sensors is one of the main goals for engineers. One way of reducing the cost of accuracy is the calibration process. Generally, in order to achieve the best possible accuracy, a sensor should be calibrated. Therefore this paper deals with the calibration of a sensor with reference standards.

The calibration method can be explained as follows. The reference standards, whose characteristics are a priori known, are applied to a low-cost sensor and outputs of the sensor are recorded. This experiment is repeated for a variety of reference standards and the results are tabulated [1, 2]. The calibration curves can be evaluated from this table. Interpolation techniques are used when this table does not contain the required data. From the practical point of view, these curves should be in a polynomial form. The accuracy of this polynomial depends on the noise-free data which were used to obtain the curves [3]. To reduce the effect of noise, excessive numbers of data are used. However, this requires more experiments that will increase the cost even more. Thus, the main question becomes the evaluation of accurate calibration curves with a limited number of experimental data. In the existing works [4–7] for calibration of measurement sensors the equidistant measurement points are used. On the other hand, though it is paradoxical, the application of equidistant measurement points to obtain the best calibration curve is erroneous. Therefore, it is required to find the optimal measurement points to determine the best calibration curves (the experiment planning problem).

An approach to design sensor calibration with the aim of reducing the calibration curve uncertainty is proposed in [8, 9]. This uncertainty reduction is achieved by minimizing the standard deviations of coefficients of either the regression curve or the estimated calibration curve. In particular, criteria for the choice of the number of calibration points and their optimum

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locations are theoretically identified when the response curve is a polynomial and the uncertainties of sensor outputs can be neglected. The proposed criterion is not directly applicable to non-linear and complex sensors. The tables and figures given in [9] are an aid in choosing an experimental plan in terms of the number of calibration points, number of repetitions for each calibration point and calibration point location.

The optimum calibration plan for measurement chain is presented in [10], by suitably elaborating the error propagation law suggested by the ISO Guide [11]. This approach has led to a different way of designing the calibration with noticeable advantages regarding a traditional equally-spaced methodology. The main advantages of the proposed approach include:

- calibration plans with a reduced number of calibration points;
- the calibration curve applicable to the whole operating range;
- a linear rather than complex regression technique always usable.

A genetic algorithm can be used to perform optimizations for both the computation of the optimal input to the sensor and the optimal constant feedback gain [12]. A calibration method, which uses a neural network and genetic algorithms together, is presented in [13]. The methods based on artificial neural networks and genetic algorithms do not have physical foundations. For different data sets, which correspond to the same event, the model gives different solutions. Therefore, the model should be continuously trained by using new data.

Correction of the nonlinearity error of its static characteristic is performed in [14], analysing the error sensitivity of the measuring system's models. The errors of the system are determined using the introduced model of real system and the model of ideal measuring system. The corrective function is determined as a relation between the input variable of the tested system and its chosen parameter. The correction method was presented on an example of a phase angle modulator.

The sensor calibration design problem can be solved using the A-optimality criterion, *i.e.*, by minimizing the sum of variances of the calibration coefficients' estimation errors (trace of the dispersion matrix of the estimation errors) [15]. That is, the A-optimality criterion attempts to minimize the standard deviations, or – in other words – to increase the accuracy of the estimates. However, the A – optimality criterion is strongly dependent on the units and is not scale-invariant [16]. This aspect needs to be carefully considered when designing the A-optimal calibration points.

The D-optimal experimental designs are constructed in [17] to propose a practical 12-position calibration procedure of *Dynamically Tuned Gyroscope* (DTG). It is said that the calibration accuracy of the deterministic error of DTG strongly depends on the multi-position calibration procedure design. The experiment results show that the compensation accuracy of the deterministic error model given by the D-Optimal 12-position calibration procedure is better than that given by the traditional 24-position calibration procedure taking half of the experiment time.

The paper [18] presents the usage of D-optimal design to improve the accuracy and robustness of solutions of the magnetometer calibration problem. It is shown that the accuracy of solutions for this problem strongly depends on the measurements' distribution. The authors propose to select the optimal set of measurement points using the D-optimal design method. The solution of the D-optimal design problem in the paper is obtained by using a standard *Particle Swarm Optimization* (PSO) based algorithm.

The optimal designs and adaptive sequential analysis are applied in [19] to solve the item calibration problem. The results indicated that the proposed optimal designs are cost-effective and time-efficient. In this study, the design points for various optimal designs (A-optimality, D-optimality, E-optimality and random design) are estimated and the accuracy and efficiency of item calibration in fully sequential analysis are discussed. Because the same stopping criterion was used for these four methods, the authors stated that no significant difference in the

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estimating parameters existed. However, the sample size used in the optimal designs was smaller than that used in the random design.

Different measures, all based on the dispersion matrix of the calibration coefficients' estimation errors, can be chosen as the criteria to design the optimal measurement points. In this study the D-optimality criterion is chosen as the optimality criterion to determine the optimal measurement points for calibration of measuring sensors. This criterion is the overall measure in terms of the volume of the estimation error ellipsoid, which is proportional to the determinant of dispersion matrix of estimation errors. Therefore, the measurement points determined using the D-optimality criterion minimize the determinant of dispersion matrix. In this case the optimum is invariant to scaling of the system states. Determination of the optimal measurement points for calibration of a differential pressure sensor for different numbers of calibration points is presented below.

2. Statement of sensor calibration design problem

From practical considerations the calibration curve is given in a polynomial form as follows:

$$y_i = a_0 + a_1 p_i + a_2 p_i^2 + ... + a_m p_i^m, \tag{1}$$

where: y_i is the output of low-cost transducer and p_i are the values of the reference standard; $a_0, a_1, ..., a_m$ are the calibration curve coefficients. The measurement contains random noise in Gaussian form:

$$z_i = y_i + \delta_i = a_0 + a_1 p_i + a_2 p_i^2 + ... + a_m p_i^m + \delta_i, \tag{2}$$

here: z_i is the measurement result; δ_i is the measurement error with zero mean and σ^2 variance. Let the calibration curve coefficients be denoted as $\tilde{\theta} = [a_0, a_1, ..., a_m]^T$. The coefficients in these polynomials were evaluated in [20] by the least squares method. The expressions used to make the evaluation had the form:

$$\tilde{\theta} = (\tilde{X}^T \tilde{X})^{-1} (\tilde{X}^T \tilde{z}), \tag{3a}$$

$$\tilde{D}(\hat{\theta}) = (\tilde{X}^T \tilde{X})^{-1} \sigma^2, \tag{3b}$$

where $\tilde{z}^T = [z_1, z_2, ..., z_n]$ is the vector of measurements:

$$\tilde{X} = \begin{bmatrix} 1 & p_1 & p_1^2 & . & . & . & p_1^m \\ 1 & p_2 & p_2^2 & . & . & . & p_2^m \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & p_n & p_n^2 & . & . & . & p_n^m \end{bmatrix} \tag{4}$$

is the matrix of known coordinates (here, $p_1, p_2, ..., p_n$ are the values that are obtained from the reference standard instrument), $\tilde{D}(\hat{\theta})$ is the dispersion matrix of the calibration coefficients' estimation errors.

The problem can be stated as follows: Find such values of $p_1, p_2, ..., p_n$ that the calibration curve coefficients' values $a_0, a_1, ..., a_m$ are optimum with respect to the D-optimality criterion.

Let us take a look at the physical significance of D-optimality criterion. If the columns of dispersion matrix are independent, the determinant of the matrix will have a maximum value subject to the given constraints in terms of the postulated model. On the other hand, if parameters are correlated, the determinant will be smaller; in the worst case, the linear dependency of two parameters results in the zero determinant, indicating that the two parameters cannot be estimated independently. Thus, the D-optimality minimizes the redundancy and leads to better identifiable parameters [16]. Furthermore, differently from the A-optimality criterion, the D-optimality criterion is invariant to scaling of the system states. For these reasons the D-optimality criterion is used in this study as the optimality one, *i.e.*:

$$\min_{p_i}\left[\ \det(\tilde{X}^T\tilde{X})^{-1}\sigma^2\ \right] \tag{5}$$

is sought. The values of $p_1, p_2, ..., p_n$ found by solving the above equations should be in the range of $0 - p_{\max}$ ($p_{\max}$ is the maximum value that is obtained from the reference standard instrument). Otherwise the solution is invalid.

3. D-optimal sensor calibration design

Let the objective function written as in [21] be:

$$f(p_1, p_2, ..., p_n) = \det\left\{\tilde{D}(p_1, p_2, ..., p_n)\right\}. \tag{6}$$

As explained above the problem is a constrained optimization problem. The objective function is a multivariable, nonlinear, continuous and has a derivative in the considered interval.

Assume that the minimum of $f(p_1, p_2, ..., p_n)$ exists for the following values of $p_1, p_2, ..., p_n$: $p^* = \left[p_1^*, p_2^*, ..., p_n^*\right]^T$.

In order that p^* is the minimum of (6), the following conditions should be met [22]:

$$\nabla f(p^*) = 0, \tag{7}$$

$$\nabla^2 f(p^*) \text{ is semi-positive}, \tag{8}$$

where: ∇ denotes the gradient.

The extremum condition (7) can be written in the following form:

$$\frac{\partial\left[\det\left\{\tilde{D}(p_1, p_2, ..., p_n)\right\}\right]}{\partial p_i} = 0, \quad i = \overline{1, n}. \tag{9}$$

From (9) n algebraic equations in n unknowns there is obtained:

$$Q_i(p_1, p_2, ..., p_n) = 0, \quad i = \overline{1, n}, \tag{10}$$

where n denotes the number of calibration points.

For derivation of (10) in the case of $m = 2$ (the degree of calibration polynomial) the dispersion matrix of estimations errors (3b) is expanded. After multiplication and finding the inverse of the matrix, we obtain:

$$\tilde{D} = (\sigma^2/\det)\begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}, \tag{11}$$

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where $a_{11}, a_{12}, ..., a_{33}$ are the algebraic minors of $\widetilde{X}^T\widetilde{X}$ matrix:

$$\det = n\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} p_i^2 p_j^2 (p_i - p_j)^2 - \sum_{i=1}^{n} p_i \sum_{\substack{i,j=1 \\ i\neq j}}^{n} p_i^3 p_j (p_i - p_j) + \\ + \sum_{i=1}^{n} p_i^2 \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} p_i p_j (p_i - p_j)^2 \tag{12}$$

is the determinant of $\widetilde{X}^T\widetilde{X}$ matrix.

After appropriate mathematical transformations, the determinant of dispersion matrix $\widetilde{D}$ can be found. Having taken the corresponding derivatives one has a system of n algebraic equations in n variables. To find all the possible solutions of the system one can apply numerical methods for searching. It can be simply accomplished by a computer.

The sign of $\nabla^2 f(p^*)$ is determined in order to determine whether the found values correspond to a local minimum or a local maximum. Furthermore, those values which make $\det\left\{\tilde{D}(p_1, p_2, ..., p_n)\right\}$ minimal, should be in the range of $0 - p_{\max}$. The set of solutions which satisfy the above conditions can be used to calculate the calibration curve coefficients given in (1). This polynomial approximates the calibration curves between the low-cost sensor and reference standards in the best way (with respect to the selection criterion).

4. Computational results of D-optimal design

The computational results of the D-optimal sensor calibration design for the cases of $m = 2$ and $n = 3$, $n = 4$, $n = 5$ are given in Tables 1, 2 and 3, respectively. In the calculations, the following data and initial conditions are used:

- Calculation of the optimum measurement points is performed for a differential Sapphir-22DD pressure sensor (‘‘Teplokontrol”, Kazan, Russia). The range of this sensor is $0 \le p_i \le 1600$ [bar]. The output sensor signal is an electrical signal in the unit of [mV]. The differential pressure sensor uncertainty is subjected to the normal distribution with zero mean and the standard uncertainty $\sigma = 2,6$ [bar] [20].
- The calibration curve of the examined sensor is described by the 2nd order polynomial as follows:

$$y_i = a_0 + a_1 p_i + a_2 p_i^2 . \tag{13}$$

The equation for measurements is written in the form:

$$z_i = a_0 + a_1 p_i + a_2 p_i^2 + \delta_i , \quad i = \overline{1,n} . \tag{14}$$

The optimum measurement points for calibration of the sensor are evaluated using (10). The method is used to obtain the optimum polynomial coefficients by employing the D-optimality criteria. The calculation is performed for the cases: $n = 3$, $n = 4$ and $n = 5$. The closed-form algebraic equations (10) are calculated to solve the equation given in (5). The software program MATHEMATICA is used to find the optimum values of $p_i^* \left(i \neq 1, i \neq n\right)$. The optimum calibration points for the cases of $n = 3$, $n = 4$ and $n = 5$are tabulated in Table 1, Table 2 and Table 3, respectively.

Table 1. The D-optimum and equidistant calibration points for $n = 3$.

Calibration points	Pressure values, [bar]		
	p_1	p_2	p_3
D-optimum points	0	800	1600
Equidistant points	0	800	1600

Table 2. The D-optimum and equidistant calibration points for $n = 4$.

Calibration points	Pressure values, [bar]			
	p_1	p_2	p_3	p_4
D-optimum points	0	228,5	848	1600
Equidistant points	0	533	1066	1600

Table 3. The D-optimum and equidistant calibration points for $n = 5$.

Calibration points	Pressure values, [bar]				
	p_1	p_2	p_3	p_4	p_5
D-optimum points	0	256	800	1344	1600
Equidistant points	0	400	800	1200	1600

5. Experimental verification of obtained results

The obtained values of D-optimum measurement points for calibration of the differential pressure sensor for the case of $n = 4$ are verified by actual experiments. In the experiments the measurements are taken with the differential Sapphir-22DD pressure sensor. The calibration of the differential pressure sensor is made with the help of the pressure standard (the piston gage set) [23]. The piston gage set reproduces the pressure signals at the corresponding optimum and equidistant calibration points. The calibration experiment results are presented in Table 4.

Table 4. The calibration experiment results corresponding to the D-optimum and equidistant calibration points.

Calibration points	D-optimum points, $p_i, [bar]$	$z_i, [mV]$	Equid. points, $p_i, [bar]$	$z_i, [mV]$
1	0	0.0051	0	0.0051
2	228,5	1395,3912	533	4097,8896
3	848	7908,3186	1066	11149,5862
4	1600	21172.8851	1600	21172.8851

The estimates of coefficients $\hat{a}_0, \hat{a}_1$ and $\hat{a}_2$ found by the estimation algorithm (3a) and their errors' variances determined by the (3b) are presented in Table 5.

Table 5. The calibration coefficients' estimates and variances of the estimation errors.

Using sensor calibr. design method	$\hat{a}_0$ $[mV]$	$\hat{a}_1$ $[mV/bar]$	$\hat{a}_2$ $[mV/bar^2]$	$D_{\hat{a}_0}$ $[(mV)^2]$	$D_{\hat{a}_1}$ $[(mV/bar)^2]$	$D_{\hat{a}_2}$ $[(mV/bar^2)^2]$
D-opt.	–0,0251	4.9198	0.0052	2,3568	0.0085	$5{,}1176\times10^{-6}$
Equid.	–0.1826	4.9206	0.0052	2,6315	0.0079	$4{,}7453\times10^{-6}$

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After determining the calibration coefficients $\hat{a}_0, \hat{a}_1$ and $\hat{a}_2$, the polynomial:

$$y_i = \hat{a}_0 + \hat{a}_1 p_i + \hat{a}_2 p_i^2 \tag{15}$$

can be used as the calibration curve of differential pressure sensor. The calibration curve obtained in this way and corresponding to the D-optimum calibration points is shown in Fig. 1.

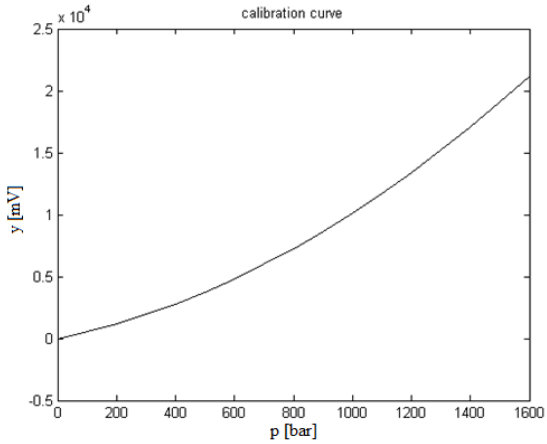

Fig. 1. The calibration curve (D-optimum calibration points are used).

In real conditions, after obtaining each measurement of z_i, in the microprocessor of differential pressure sensor the inverse problem is solved, *i.e.*, the roots of (15) are found:

$$\hat{p}_{(1,2)i} = \frac{-\hat{a}_1 \pm \sqrt{\hat{a}_1^2 - 4\hat{a}_2(\hat{a}_0 - z_i)}}{2\hat{a}_2} \tag{16}$$

and the root $\hat{p}_{1_i}$ is assumed to be the estimation of measured pressure. The second root $\hat{p}_{2_i}$ is negative or considerably different from the measurement z_i.

In the experiments the piston gages reproduce the pressure signals of $p_i (i = \overline{1,17})$ [bar] in the measurement interval of $0 \le p_i \le 1600$ [bar] with the step of 100 [bar] and the output signals for the differential pressure sensor z_i are registered. The experiment results are presented in [15].

5.1. Comparison of absolute and relative calibration errors

Using the calibration experiment results presented in [15], z_i, $i = \overline{1,17}$ and the equations of appropriate calibration curves, the values of $\hat{p}_i = \hat{p}_{1i}$, $i = \overline{1,17}$ are calculated using (16). Then the appropriate values of absolute Δ_{abs_i} and relative error δ_{rel_i} errors are determined by means of the known expressions:

$$\Delta_{abs_i} = \hat{p}_i - p_i \,;\; \delta_{rel_i} = \frac{\Delta_{abs_i}}{p_i} 100\% . \tag{17}$$

Since the true value cannot be determined, in the expressions (17) either a value obtained by the perfect measurement or a conventional true value may be used [11]. In this case, the pressure reproduced by the piston gage set is assumed to be the perfect measurement result. Therefore,

in (17) the values of the reference standard (the pressures reproduced by the piston gage set) p_i are assumed to be the true ones. The calibration values $\hat{p}_i$ obtained in this way and the values of absolute and relative calibration errors corresponding to the D-optimum and equidistant calibration intervals are presented in Table A1 in Appendix A, whereas the plots that show the relations $\Delta_{abs_i} = f(p_i)$ and $\delta_{rel_i} = f(p_i)$ are shown in Fig. 2 and Fig. 3, respectively.

As seen from the presented results, the absolute values of calibration errors corresponding to the D-optimum calibration points are considerably smaller than in the case of equidistant calibration points. The experiment results show that the theoretical results obtained here are correct.

Comparison of the calibration errors corresponding to the D-optimal (Table A1, Appendix A) and A-optimal [15] calibration curves, show that, at most of the experimental input points, the calibration curve obtained with the D-optimal design method gives the best results.

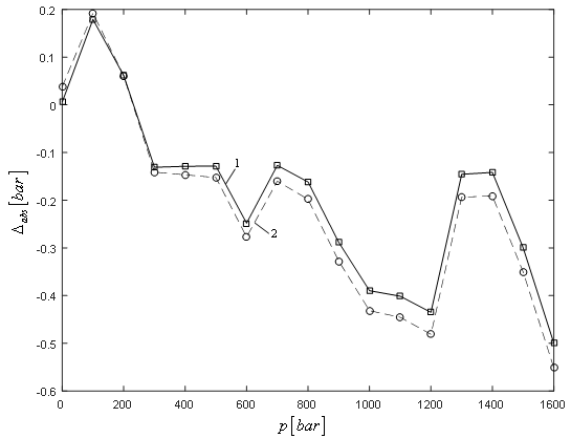

Fig. 2. The absolute calibration errors: 1 – calibration is performed by using the optimum calibration points; 2 – calibration is performed by using the equidistant calibration points.

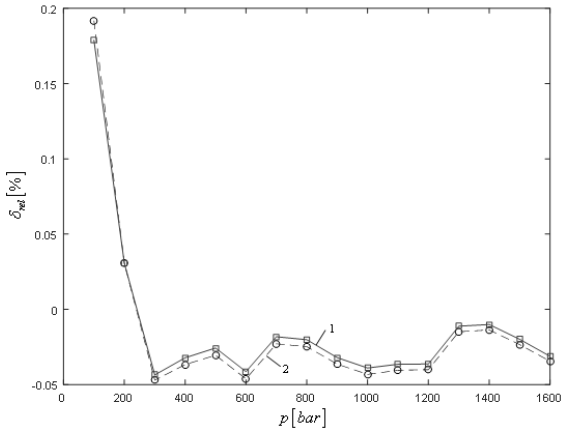

Fig. 3. The relative calibration errors: 1 – calibration is performed by using the optimum calibration points; 2 – calibration is performed by using the equidistant calibration points.

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5.2. Comparison of root-mean-square errors

The *Root-Mean-Square* (RMS) errors of calibration curves can be determined using the expression:

$$RMS = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{p}_i - p_i\right)^2}, \tag{18}$$

where N is the number of experiments.

The RMS errors of calibration curves obtained by the D-optimal, A-optimal and Equidistant design methods are presented in Table 6.

Table 6. The RMS errors of calibration curves obtained by the D-optimal, A-optimal and Equidistant design methods.

Using sensor calibration design method	RMS, [bar]
D-optimal	0.2608
A-optimal	0.2780
Equidistant	0.2944

As seen from Table 6, the D-optimal and A-optimal design methods give better results than the Equidistant design method. The RMS error of calibration curve obtained by the D-optimal design method is smaller than the errors obtained by the A-optimal and Equidistant design methods.

5.3. Uncertainty of calibration using reference standards

When a set of several repeated readings (statistics) has been taken (for the Type A estimate of uncertainty), the mean $\overline{\Delta}_{abs_i}$ and the estimated standard deviation S can be calculated for the set. From those, the estimated standard uncertainty u of the mean is calculated using [24, 25]:

$$u = \frac{S}{\sqrt{n}}, \tag{19}$$

where n is the number of measurements in the set.

The method for calculating uncertainties of calibrated values from a calibration curve (15) requires periodic measurements on the reference standards. In the experiments the piston gage set reproduces the pressure standard signals at the corresponding equidistant calibration points and the output signals of differential pressure sensor z_i are registered [15].

In real conditions, each measurement z_i from the differential pressure sensor is corrected by the calibration curve (15). The standard deviation of these values should estimate the uncertainty associated with the calibrated values. The standard deviation of calibrated values is calculated with the formula:

$$S = \sqrt{\frac{\sum_{i=1}^{n}\left(\Delta_{abs_i} - \overline{\Delta}_{abs_i}\right)^2}{n-1}}, \tag{20}$$

where $n = 17$ is the number of measurements, $\overline{\Delta}_{abs_i} = \frac{1}{n}\sum_{i=1}^{n}\Delta_{abs_i}$ is the sample mean of absolute calibration errors.

To determine the random uncertainty of the set, the standard deviation of the mean has to be evaluated using (19). The results for different calibration characteristics (D-opt, A-opt and Equidistance) are presented in Table 7.

Table 7. The uncertainty results of calibration obtained by the D-optimal, A-optimal and Equidistant design methods.

Using sensor calibration design method	Mean, [bar]	Standard Deviation, [bar]	Standard Deviation of the Mean, [bar]
D-optimal	–0.1929	0.1809	0.043876
A-optimal	–0.2159	0.1807	0.043827
Equidistant	–0.2210	0.2004	0.048605

As seen from Table 7, the equidistant calibration curve has a poorer uncertainty characteristic than the D-optimal and A-optimal calibration curves. The uncertainties of D-optimal and A-optimal calibrations are similar.

Appendix A

Table A1. Absolute and relative calibration errors corresponding to the D-optimum and equidistant calibration points.

Input Pressure, [bar]	$\hat{p}_i, [bar]$ D-opt. points	$\hat{p}_i, [bar]$ Equid. points	$\Delta_{abs}, [bar]$ D-opt. points	$\Delta_{abs}, [bar]$ Equid. points	$\delta_{rel}, [\%]$ D-opt. points	$\delta_{rel}, [\%]$ Equid. points
0	0.0061	0.0381	0.0061	0.0381	∞	∞
100	100.1792	100.1922	0.1792	0.1922	0.1792	0.1922
200	200.0615	200.0612	0.0614	0.0610	0.0307	0.0305
300	299.8691	299.8588	–0.1309	–0.1411	–0.0436	–0.0470
400	399.8712	399.8533	–0.1288	–0.1467	–0.0322	–0.0367
500	499.8715	499.8476	–0.1285	–0.1524	–0.0257	–0.0305
600	599.7518	599.7229	–0.2482	–0.2771	–0.0414	–0.0462
700	699.8729	699.8400	–0.1270	–0.1600	–0.0181	–0.0229
800	799.8382	799.8019	–0.1617	–0.1981	–0.0202	–0.0248
900	899.7115	899.6721	–0.2885	–0.3279	–0.0321	–0.0364
1000	999.6103	999.5683	–0.3897	–0.4316	–0.0390	–0.0432
1100	1099.5986	1099.5545	–0.4013	–0.4455	–0.0365	–0.0405
1200	1199.5649	1199.5188	–0.4351	–0.4812	–0.0363	–0.0401
1300	1299.8544	1299.8065	–0.1456	–0.1935	–0.0112	–0.0149
1400	1399.8583	1399.8089	–0.1417	–0.1911	–0.0101	–0.0137
1500	1499.7003	1499.6496	–0.2996	–0.3504	–0.0200	–0.0234
1600	1599.5004	1599.4484	–0.4996	–0.5516	–0.0312	–0.0345

6. Conclusion

The paper shows that the accuracy of calibration curves of measuring sensors substantially depends on the selection of calibration points. A procedure for the optimal selection of sample measurement points to find the best calibration curves (with respect to the selection optimality criterion) for a measuring sensor is proposed. As an optimality criterion in this study the D-optimality criterion is used. This criterion is the overall measure in terms of the volume of the estimation error ellipsoid, which is proportional to the determinant of dispersion matrix of estimation errors. In this case the optimum is invariant to scaling of the system states.

As an example, the problem of optimal selection of the standard pressure setters (the piston gages) during calibration of a differential pressure sensor is solved. Comparison of the absolute and relative calibration errors and RMS errors corresponding to the D-optimal, A-optimal

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and Equidistant calibration curves, shows that the calibration curve obtained with the D-optimal design method gives the best results. Moreover, the D-optimal and A-optimal design methods give better results than the Equidistant design method.

The uncertainties of D-optimal and A-optimal calibrations are similar. The equidistant calibration curve has a poorer uncertainty characteristic than the D-optimal and A-optimal calibration curves. The experiment results correspond to those obtained theoretically, proving their correctness.

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