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Nonlinear damage accumulation of concrete subjected to variable amplitude fatigue loading

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Abstract. To account for the load sequence effect, damage fatigue models with nonlinearity in propagation and accumulation have been developed. This paper reviews five classical nonlinear fatigue models used to predict the life times of concrete under variable amplitude loadings. Experimental results from literature are used to validate the five classical prediction models. It can be found that Hilsdorf and Kesler model yields unsafe or conservative predictions, and the other four models are more suitable for predicting life times of concrete. In this paper, the author used a new nonlinear damage model based on the nonlinear continuum damage mechanics to predict fatigue life of concrete. The model considers fatigue limit, loading parameters, the unseparable characteristics for the damage parameter and the load sequence effect. The validity of the nonlinear fatigue damage model is checked against tests from literature.

Key words: nonlinear damage, variable amplitude loading, prediction model, concrete.

1. Introduction

Much research was carried out to obtain the tensile properties of concrete by monotonic tensile loading [1, 2]. However, concrete structures are frequently subjected to cyclic loading in their service life [3, 4]. The accumulation damage on the concrete structure reduces its service life as the repetitions of the applied loading increase. It is essential to predict fatigue life of concrete in design and calculation of concrete structure [5, 6]. The investigations on fatigue performance of concrete under cyclic loading can date back to the beginning of 20th century. By applying probabilistic procedure, a relationship between probability of failure and number of cycles to failure at a specific stress level was obtained by cyclic compression test on concrete [7]. As specimen under uniaxial tension has simple stress distribution, many researchers [8–11] have studied the fatigue behavior of plain concrete subjected to cyclic loading in uniaxial tensile loading. The effect of stress amplitude during the bending fatigue test was first reported by Ople and Hulsbos [12]. They found that as the stress amplitude increased, the number of cycles to failure decreased. Tepfers [13] concluded that the mechanism of fatigue is the same in both compression and tension by carrying out fatigue test on concrete specimen in both splitting and compression cyclic loading.

In practice, concrete structure is vulnerable to variable amplitude loadings. However, little is known of the fatigue behavior of concrete, as it is difficult to obtain the accumulation damage under variable amplitude loading [14]. Until the end of the last century, some researchers began to carry out relevant studies on fatigue behavior of concrete. Hilsdorf and Kesler [15] studied the fatigue strength of concrete subjected to repeated flexural stresses under various loading histories, showing that the fatigue behavior can be interpreted by variations of strength during a fatigue test due to the relief of shrinkage stresses and cumulative microcracking. Based on the continuum damage mechanism, Oh [16], Grzybowski and Meyer [17], Vega et al. [18] and Hamdy [19] proposed their own prediction models for fatigue life of concrete under various loading regimes.

Damage accumulation theory has been used in existing models, which can be mainly classified into two categories: linear accumulative damage and nonlinear accumulative damage. The Palmgren-Miner's rule as a classical linear model is commonly used in the fatigue analysis. A linear damage evolution is assumed and given by $D_i = {}^{N_i}\!/_{N_{\mathrm{F}i}}$. The linear damage evolution means that the relationship between damage D versus N/N_{f} is the same for all stress level. This does not reflect the reality. Consequently, a nonlinear accumulative damage theory was suggested. A damage variable representing the evolution of damage under the imposed condition must be selected. Some experimental research [10, 16, 18, 19] indicates that the maximum strain follows the same trend as evolution of damage in concrete. Strains were recorded for different stress levels and plotted versus normalized number of cycles. Three-stage damage propagation was observed from the above experimental results. There is a fast increase in maximum strain at stage 1, which is followed by the constant increase at stage 2. Stage 3 showed continuous increase in maximum strain till failure. Grzybowski and Meyer [17] defined the damage index as the ratio of dissipated energy to total dissipated energy till failure for a specific stress or strain level. Residual strength was

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also used as the damage parameter by some researchers [19, 20]. Additionally, the number of cycles to failure for different stress levels is essential to predict the fatigue life of concrete under variable amplitude loadings. The relationships between number of cycles to failure and stress level have been obtained by regression analysis based on test data [9, 17]. Modified prediction models have been proposed based on dissipated energy [21, 22]. The average dissipated energy was found to be a good estimator of fatigue life for fiber reinforcement concrete. Equivalent cumulative damage theory was commonly used to predict the remaining life at a specific stress with previous loading histories by many researchers [16, 17, 19].

This paper mainly reviews five existing nonlinear models which are most often used to predict the fatigue life of concrete under variable amplitude loadings. These models are evaluated by comparison of predicted results with the test data. Finally, a nonlinear damage cumulative model which takes both nonlinear damage evolution and load sequence into account is proposed. It has been validated by comparing the predicted results to test data.

2. Existing prediction models

For convenience, fatigue damage is expressed with a suitable parameter such as D. The damage parameter must satisfy two boundary conditions: $D = 0$ with an initial non-damage state, and $D = 1$ represents the specimen fails completely under cyclic loading. In order to study the fatigue behavior of concrete under variable amplitude loading, the damage evolution must be obtained firstly under constant amplitude loading. At the same time, the relationship between fatigue life N_f and a single stress level S was necessary to predict the fatigue life of concrete.

Now some models proposed in previous literature are reviewed and evaluated by comparing the prediction of fatigue life to the experimental data.

2.1. Oh model [16]. Total strain has been selected to describe the evolution of damage. A three-stage damage evolution phenomenon was observed and described using a cubic equation expressed as:

$$D = a_1 x^3 + a_2 x^2 + a_3 x, \tag{1}$$

in which D represents the damage, $x = n/N_F$ represents the cycle ratio at a given stress level, and a_1, a_2 and a_3 can be determined by boundary conditions.

It is assumed that damage produced by n_i cycles of operation at any stress level S_i is exactly equivalent to n_1 cycles of stress level S_1, one may write:

$$n_{ie} = n_i \left(\frac{S_i}{S_1}\right)^p \tag{2}$$

in which n_{ie} is the number of operation at reference stress level S_1 to produce damage equivalent to n_i actual cycles at stress level S_i.

The damage ratio is then defined as:

$$D_i = \frac{n_{ie}}{N_1}. \tag{3}$$

The fatigue failure occurs when the summation of these damage ratios equals unity:

$$\sum D_i = D_1 + D_2 + ... + D_i = 1. \tag{4}$$

The remaining life n_{ir} at the stress level S_i may be determined as:

$$n_{ir} = (N_1 - n_1)\left(\frac{S_1}{S_i}\right)^p - .... - n_{i-1}\left(\frac{S_{i-1}}{S_i}\right)^p \tag{5}$$

where, the index p was determined by the experimental data in the study, and was equal to 18.21.

2.2. Grzybowski and Meyer model [17]. The damage evolution of concrete under constant stress level can be formulated by the following expressions:

$$\begin{aligned} D &= (1.7 - S)\bar{n} \quad \text{for} \quad \bar{n} \le 0.6 \\ D &= \bar{n}^{1.6S} \qquad\quad \text{for} \quad \bar{n} > 0.6 \end{aligned} \tag{6}$$

in which, $\bar{n} = {}^{n}/_{N_F}$ is the normalized number of cycles, n is the applied number of cycles. Eq. (6) allows for predicting damage accumulation for variable amplitude loadings with known or assumed amplitude spectrum. This can be achieved by converting the number of cycles n_i with a stress level S_i to an equivalent number of cycles n^{i^*} with another stress level S_j, by equating the respective accrued damage, i.e.

$$D_i = \left(\frac{n_i}{N_{fi}}\right)^{1.6S_i} = \left(\frac{n_j}{N_{fj}}\right)^{1.6S_j} = D_i^* \tag{7}$$

where, D_i is the damage due to n_i cycles with constant stress level S_i, D_i^* is the damage due to n_i^* cycles at a constant stress level S_j.

Equation (7) can be solved for n_i^*, the equivalent number of cycles at stress level S_j, producing the same damage as n_i cycles at stress level S_i.

$$n_i^* = \left(\frac{n_i}{N_{fi}}\right)^{\frac{S_i}{S_j}} N_{fj} \quad \text{for} \quad \bar{n} > 0.6. \tag{8}$$

2.3. Vega model [18]. Vega [18] defined a damage parameter $D_i = (\varepsilon_i - \varepsilon_0)/(\varepsilon_{ult} - \varepsilon_0)$ based on the continuum damage mechanics. The damage can be expressed as a function of the maximum and minimum stresses as the following general formula:

$$D = \alpha_0 {\sigma_{max}}^2 + \alpha_1 \sigma_{max} \sigma_{min} + \alpha_2 (\sigma_{max} - \sigma_{min}), \tag{9}$$

where D is damage at applied number of cycles ratio n; σ_{rmax} is the maximum relative stress equal to ${}^{\sigma_{max}}/_{f_t} > 0$; $\sigma_{r\min}$ is the minimum relative stress equal to ${}^{\sigma_{min}}/_{f_t} \ge 0$; σ_{max} is the applied

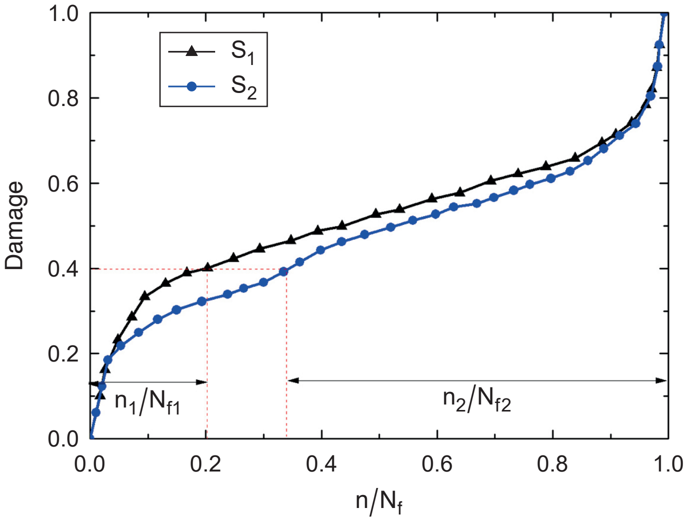

Fig. 1. Calculated damage evolution functions of concrete under variable amplitude loading

maximum stress; σ_{min} is the applied minimum stress; f_t is the tensile strength of concrete. The coefficients α_0, α_1, and α_2 are evaluated by fitting the experimental damage curves as shown in literature [18].

To calculate the accumulated damage under variable amplitude loading, the damage evolution curve under constant stress must be given previously. As shown in Fig. 1, the remaining life at stress level S_2 can be expressed as:

$$n_r = \frac{n_2}{N_{f2}} \times N_{f2}, \tag{10}$$

where, n_r is the remaining life of concrete at stress level S_2, ${}^{n_2}/_{N_{f2}}$ is the remaining cycle ratio for stress level S_2.

2.4. Hamdy model [19]. As in Vega model, the relationship between total strain variation and the cycle ratio is selected to represent the damage evolution of concrete under repeated loading. The damage evolution can be expressed as follows:

$$D = f(D, \sigma_{max}, \sigma_{min}, N) \tag{11}$$

where, D is the damage variable; σ_{max} is the maximum stress; σ_{min} is the minimum stress; N is the number of cycles at applied stress level.

The formulation for the $S - N_f$ curve is obtained by fitting test data that related fatigue life to stress level, and it is expressed as follows:

$$S = \mathrm{a} + \mathrm{b}\log(N_f) \tag{12}$$

where, N_f is number of cycles to failure at a given stress level S; a and b are empirical coefficients.

The damage evolution curves and $S - N_f$ curve equation were stored in the computer program for further use in the life prediction process. Fatigue life under variable amplitude loading can be expressed as follows:

$$N = \left(\frac{N}{N_f}\right)_1 \times N_{f1} + \left(1 - \left(\frac{N}{N_f}\right)_2\right) \times N_{f2}, \text{ 2-levels} \tag{13}$$

$$N = \left(\frac{N}{N_f}\right)_1 \times N_{f1} + \left(\frac{N}{N_f}\right)_2 \times N_{f2} + \\ + \left(1 - \left(\frac{N}{N_f}\right)_3\right) \times N_{f3}, \text{ 3-levels} \tag{14}$$

where, ${}^{N}/_{N_f}$ is partial life, and N_{f1}, N_{f2} and N_{f3} are the fatigue life at stress levels of S_1, S_2 and S_3.

2.5. Hilsdorf and Kesler model [15]. According to the Miner rule, if concrete is subjected to multi-amplitude loading, it fails when $M = \sum_{i=1}^{n} {}^{N_i}/_{N_{Fi}} = 1$. At first, the evolution of damage should be predicted accurately. Hilsdorf and Kesler [15] assumed that the damage after n_1 cycles at a stress level S_1 can be expressed as

$$d_1 = \left(\frac{N_1}{N_{f1}}\right)^{a_1} \tag{15}$$

where both fatigue life N_{f1} and a_1 are functions of stress level and failure occurs if $d_1 = 1$. It can be shown that with these assumptions, the failure criterion for specimens subjected to two-amplitude loading is:

$$N_2 = N_{f2}\left[1 - \left(\frac{N_1}{N_{f1}}\right)^A\right] \tag{16}$$

where, $A = {}^{a_1}/_{a_2}$, a_1, a_2 are coefficients of damage function for stress level S_1 and S_2.

$$A = \frac{a_1}{a_2} \tag{17}$$

When $A = 1$, (16) is identical to the Miner rule. If a proper value of exponent A is chosen, this model will probably predict the remaining fatigue life of concrete under stress level S_2.

2.6. Evaluation of existing models. To evaluate these five models in predicting fatigue life of concrete under variable amplitude loading, comparison of predicted results with the test data is presented in this section. Test data come from other literatures [16, 19, 23]. The details of tests are shown in Table 1. Two-stage and three-stage increasing and decreasing load were performed by Oh [16]. Hamdy [19] carried out bending fatigue test on concrete with two-stage increasing and decreasing loading with different pre-assigned value of damage (50% and

Table 1
Details of test from previous researchers

Researcher	Concrete type	Specimen size	Loading type	f (Hz)	Specimens
Jau (1986)	High Strength	$\phi 4\times 8$ in	Compression	3	161
Oh (1991)	Normal	$100\times 100\times 500$ mm	Four-point flexural	4.17	80
Hamdy (1997)	Normal	$4\times 4\times 48$ in	Bending	5	36
Hisdorf (1966)	Normal	$6\times 6\times 60$ in	Bending	7.5	185
Grzybowski and Meyer (1993)	Concrete with and without fiber	$\phi 152\times 304$ mm	Compression	1	144

75% of total damage). Data [23] tested on high strength concrete under cyclic uniaxial compression load with different frequencis and stress ratios (ratio of minimum stress to maximum stress). All test results represent nonlinear behavior of concrete regardless of the loading regime.

Hilsdorf and Kesler model has the simplest form and is convenient for predicting fatigue life of concrete. However, the damage variable has not been defined in this model and the damage evolution has not been given. It can be seen from eq. (16), the remaining cycle ratio N_2/N_{f2} is always less than unity when a two-stage loading test is performed on specimen. However, for some test results the value of N_2/N_{f2} is larger than unity have been observed. A similar inadequacy occurs when the model is applied to the specimen subjected to three- or multi- stage loading.

Oh model is developed based on a lot of test data. Cubic functions are developed to represent the evolution of damage in concrete for different stress levels firstly. A mean value for the total fatigue life of concrete under repeated constant stress level is selected as the fatigue life. Finally, a mathematical model is developed based on the equivalent cumulative damage theory, in which an index p is introduced. The representative value of index p is obtained by regression analysis for test data by Oh and needs further validation. Besides, the authors consider that the statistical scatter should be analyzed since the test data show larger discreteness.

Hamdy model used splines described the evolution of damage under any constant amplitude loading. Similar to Oh model, damage evolution with cycle ratio and relation between fatigue life versus stress level $S \sim N_F$ are necessary to develop the fatigue life model of concrete under various amplitude loading. To obtained reliable test result $S \sim N_F$, a group of tests at each stress level have been carried out on concrete. In general, a mean value of the groups is used in linear regression analysis. Weibull analysis has been performed on the repeated tests at each stress level to get the mean value of each group.

Grzybowski & Meyer model defined damage index as the ratio between the dissipated energy so far to the total dissipated energy till failure. The damage evolution curve was described by a piecewise function. A logarithmic expression was developed to predict the fatigue life of concrete. This model introduced an equivalent number of cycles n_i^* at any stress level S_j, which produce the same damage as n_i cycles at the reference stress level S_i. It is not convenient for calculating as the damage evolution function is piecewise. This model has never been validated by test data, it should be further examined.

Three-stage damage evolution was summarized by Vega. The damage index is expressed as a function of the maximum and minimum stresses. A relationship between total numbers of cycle and applied stress level is developed based on test data. The damage evolution curves for different stress levels should be plotted in the same diagram. Equivalent damage accumulative theory was used in this model, which means that when the applied stress converts from a stress level to the other stress level the damage remains constant at the shift point.

In summary, the existing models gave a damage evolution function and the relationship between total number of cycle to failure and applied constant stress level at first. And then predicted the remaining fatigue life of partially damaged concrete at a given stress level based on the equivalent damage accumulative theory. The major differences between those models are the damage evolution function with cycle ratio and the $S \sim N_F$. The remaining number of cycle can be expressed by a general expression as follow:

$$N_{re} = kN_{Fn} \tag{18}$$

in which, N_{re}, N_{Fn} represent the remaining life and the total number of cycles to failure at the last block applied stress level, respectively. k is a simplified coefficient, which is a function of load history and depends on the previous applied load stress level, the existing partial damage and some other parameters. k is directly related to the evolution of damage and total cycles to failure for each given stress level. The evolution of damage and $S \sim N_F$ are both semi-empirical, as well as the parameter k.

Comparison of predicted results with test data has been made and shown in Table 2. To directly evaluate the prediction models, both correlation coefficients and standard deviations are listed in Table 2. Hisdolf and Kesler model is a simple exponent function. The correlation coefficient between predicted results and test data is 0.72, which demonstrates the model has bad prediction efficiency. The standard deviation of Hilsdorf model is much larger than the other four models, which further demonstrates the prediction results of this model deviate far the test data. The correlation coefficients of the other four models are close to each other, while the standard deviations

Table 2
Comparison of the predicted fatigue life from these five traditional models to experimental data tested in 1986–1997

Tested by	Load stage					Test data N_f	Predicted N_f				
	S_1	S_2	S_3	$\frac{n_1}{N_{f1}}$	$\frac{n_2}{N_{f2}}$		Oh	Hamdy	Grzybowski and Meyer	Hilsdorf and Kesler	Vega
Oh (1991)	0.75	0.85		0.20		6670	6695	6262	6069	6302	6313
	0.85	0.75		0.20		13641	13616	16791	19842	15854	16016
	0.65	0.75	0.85	0.10	0.10	38654	37594	37176	36878	37118	37176
	0.85	0.75	0.65	0.10	0.10	254895	172509	175205	277669	22280	215050
Hamdy (1997)	0.8	0.85		0.04		1802	1790	2402	2112	2123	2120
	0.85	0.8		0.11		39002	38511	34244	34003	30620	30202
	0.8	0.85		0.34		12195	12191	12270	12430	12475	12462
	0.85	0.8		0.56		23464	23107	24016	17013	14706	15103
Jau (1986)	0.7	0.9		0.27		97020	97019	95852	95827	95859	96852
	0.7	0.9		0.69		253000	253005	250044	250025	250073	250045
	0.8	0.9		0.15		880	880	540	533	547	542
	0.9	0.7		0.50		149900	149749	142406	291874	74365	139834
	0.9	0.8		0.42		1820	1763	3143	6683	1877	3133
	0.8	0.9		0.09		3454	4060	2104	2089	2054	2204
	0.8	0.9		0.31		6420	7994	6544	6491	6454	6510

are different. The standard deviation of Vega model is least among the five models, and the correlation coefficient is the largest, which both demonstrate the Vega model has the finest goodness of fit. Except Hilsdorf and Kesler model, the other four models gave their own damage variables and the evolution of damage with cycle ratio was expressed by formulations or curves. Nonlinear in propagation and accumulation were considered in the models.

3. Nonlinear damage model

For the uniaxial problems, the fatigue damage is defined as originally proposed by Chaboche [24], by the following differential equation:

$$\delta D = \left[1-(1-D)^{\beta+1}\right]^{\alpha(\sigma_{\max},\sigma_{\text{med}})} \left[\frac{\sigma_a}{M_0(1-b\sigma_{\text{med}})(1-D)}\right]^{\beta}\delta n \tag{19}$$

where β, M_0 and b depend on material, $\sigma_{\max}$ and σ_{med} are respectively the maximum and the mean stress of cycle and $\sigma_a = \sigma_{\max} - \sigma_{\text{med}}$; the exponent α depends on the loading $(\sigma_{\max}, \sigma_{\text{med}})$ which results in non-separability between damage and loading. In this study, only oscillating stress has been considered, so the Eq. (19) becomes:

$$\delta D = \left[1-(1-D)^{\beta+1}\right]^{\alpha}\left[\frac{\sigma_a}{M_0(1-D)}\right]^{\beta}\delta n \tag{20}$$

The expression of α proposed by Chaboche [24] is:

$$\alpha = 1 - \frac{1}{H}\left\langle\frac{\sigma_a-\sigma_f}{\sigma_u-\sigma_a}\right\rangle \tag{21}$$

where symbol $\langle\,\rangle$ is defined as $\langle x\rangle = 0$ if $x < 0$ and $\langle x\rangle = x$ if $x > 0$. σ_f is the fatigue limit, σ_u is the ultimate static strength of material, and H is parameter to be experimentally determined. In quasi-brittle materials like concrete there is not the fatigue limit, the expression can be modified as:

$$\alpha = 1 - \frac{1}{H}\left(\frac{\sigma_a}{\sigma_u-\sigma_a}\right) \tag{22}$$

The exponent α is dependent on damage evolution of concrete. Damage is hard to be measures but it can be linked to some variable representing the fatigue degradation phenomenon. In this study, the exponent α can be determined by the experimental data from previous investigations. The main objective of this section is to build a nonlinear damage model to predict fatigue life of concrete.

Integrating the Eq. (20) in a generic instant before failure ($D < 1$, and $n < N_f$) the damage D can be expressed as a function of n/N_F:

$$n = \frac{1}{1-\alpha}\,\frac{1}{1+\beta}\left[\frac{\sigma_a}{M_0}\right]\left[1-(1-D)^{1+\beta}\right]^{1-\alpha} \tag{23}$$

$$\Rightarrow D = 1 - \left[1-\left(\frac{n}{N_f}\right)^{1/(1-\alpha)}\right]^{1/(1+\beta)} \tag{24}$$

For two-stage loading, the specimen is firstly loaded at a stress amplitude of σ_{a1} for n_1 cycles and then at a stress amplitude of σ_{a2} for n_2 cycles up to failure. Use the continuum damage mechanism, the partial damage caused by n_1 cycles at σ_{a1} should be the damage caused by N_2 cycles at σ_{a2}. The equivalence damage can be expressed as:

$$1-\left[1-\left(\frac{n_1}{N_{f1}}\right)^{\frac{1}{1-\alpha_1}}\right]^{\frac{1}{1+\beta}}=1-\left[1-\left(\frac{N_2}{N_{f2}}\right)^{\frac{1}{1-\alpha_1}}\right]^{\frac{1}{1+\beta}} \qquad (25)$$

$$\Rightarrow \frac{N_2}{N_{f2}}=\left(\frac{n_1}{N_{f1}}\right)^{(1-\alpha_2)/(1-\alpha_1)} \qquad (26)$$

The remaining life n_2 at stress σ_{a2} can be expressed as:

$$\frac{n_2}{N_{f2}}=\left[1-\left(\frac{n_1}{N_{f1}}\right)^{(1-\alpha_2)/(1-\alpha_1)}\right]. \qquad (27)$$

In the above damage evolution model, the expression for α is a monotonically decreasing function of stress, and take the load sequence into account. In fact, it obviously can be seen that in two-stage decreasing load, i.e. $\sigma_{a1}>\sigma_{a2}$, from Eq. (22), it can be obtained that:

$$\frac{1-\alpha_2}{1-\alpha_1}<1 \qquad (28)$$

$$\frac{n_2}{N_{f2}}=\left[1-\left(\frac{n_1}{N_{f1}}\right)^{(1-\alpha_2)/(1-\alpha_1)}\right]<1-\frac{n_1}{N_{f1}}. \qquad (29)$$

Therefore, $\frac{n_1}{N_{f1}}+\frac{n_2}{N_{f2}}<1$.

This model considered the load interaction effect. It is illustrated that in the case of decreasing loading the accumulated damage is less than unity while in the case of increasing loading the damage accumulation value is more than unity when it failures. This is correspondent with experimental results from Oh [16] and Vega [18].

In the case of multilevel loading, it is easy to get the continuum damage function through sequential accumulation calculation by introduce an auxiliary variable V:

$$D_i=1-\left[1-\left(\frac{N_i+n_i}{N_{fi}}\right)^{\frac{1}{1-\alpha_i}}\right]^{\frac{1}{1+\beta}}=1-\left[1-V_i\right]^{\frac{1}{1+\beta}} \qquad (30)$$

in which

$$V_i=\left(\frac{N_i+n_i}{N_{fi}}\right)^{1/(1-\alpha_i)}= \\ =\left\{\left[1-\left(1-D_{i-1}\right)^{1+\beta}\right]^{1-\alpha_i}+\frac{n_i}{N_{fi}}\right\}^{1/(1-\alpha_i)}. \qquad (31)$$

In the case of multilevel condition the exponent α is equal to 1 and Eq.(22) becomes:

$$\delta D=\left[1-(1-D)^{\beta+1}\right]\left[\frac{\sigma_a}{M_0(1-D)}\right]^{\beta}\delta n \qquad (32)$$

Integrating it and bringing in an auxiliary variable W, the damage expression becomes:

$$D_i=1-\left[1-W_i\right]^{1/(1+\beta)} \qquad (33)$$

where

$$W_i=\left[1-\left(1-D_{i-1}\right)^{1+\beta}\right]e^{(n_i/N_i^{'})(1+\beta)} \qquad (34)$$

and

$$N_i^{'}=\left(M_0/\sigma_{ai}\right)^{\beta} \qquad (35)$$

4. Verification of the nonlinear model

The nonlinear damage model considers the mean stress, the unseparable characteristic for the damage, and the effect of load sequence in concrete subjected to variable amplitude loading and some significant loading parameters. Table 3 gives the comparison of predicted fatigue life by the nonlinear damage model with test data. It can be seen that the prediction is safe and more conservative than the experimental results. The correlation coefficient is 0.94 and the standard deviation is 29749, and it can be found that the goodness of fit is a little worse than the above four models while the fitting degree is sufficient and is better than Hilsdorf model. In the case of three-stage loading condition, the predicted results have more error than that in two-stage loading, which indicates that the load interaction effect is not evaluated exactly and needs further study. The experi-

Table 3
Comparison of results predicted by the nonlinear damage model to experimental data

Tested by	Test data N_f	Predicted results N_f
Oh (1991)	6670	6290
	13641	16134
	38654	35658
	254895	183507
Hamdy (1997)	1802	2120
	39002	31054
	12195	12465
	23464	15100
Jau (1986)	97020	95859
	253000	250075
	880	623
	149900	60370
	1820	3368
	3454	2104
	6420	6527

mental data tested on high strength concrete by Jau [23] took the loading frequency into consideration, but it is neglected in this nonlinear damage model. Compared to the other empirical damage models, this model introduced a parameter α which results in unseparable between damage and loading history. The parameter α is a function of static strength, the maximum and the mean stress. In some extent, the nonlinear damage model shows a good agreement with the experimental results.

5. Conclusions

This paper reviewed five existing methods to predict the fatigue life of concrete under variable amplitude loading and then gave a new nonlinear damage model based on the continuum damage mechanism. The main conclusions can be drawn as follows.

The nonlinearity both in propagation and accumulation of damage in concrete are taken into consideration in the classical prediction models. The evolution of damage under constant stress level and relation between fatigue life and the applied stress level are necessary to predict the fatigue life of concrete under variable amplitude loading. The equivalent damage accumulative theory is generally used in those models. Except for the Hisdorf model, the predicted results from other four models show good agreement with test data.

A new nonlinear damage model is used to predict the fatigue life of concrete under sequential loading. In the nonlinear model, parameter α is introduced, which is a function of ultimate strength, maximum and mean stress. It considers the load sequence effect and nonlinear damage mechanism in concrete subjected to variable amplitude loading. The predicted results show good agreement with test data.

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